USAAAO Round 2 Qualification Exam 2016

Instructions (Read completely before you begin)

This is the second round qualification test for the USAAAO team. The test is short answer, consisting of 12 short questions and 2 longer, multipart questions. Once you begin, you will have two and a half hours (150 minutes) to complete the test. You may use the provided table of constants and a scientific calculator. No other materials are permitted.

Please show all your work. Answers without work will not receive credit. You will need to write your solutions on a separate sheet. Please begin a new page for each question, and number your pages. Please write as clearly as possible. Your test will be scanned for grading, so make sure that your writing is dark enough to be legible once scanned.

When you are done, please have your teacher scan your work and send it to usioaateam@gmail.com. We prefer to receive your work as a single pdf, named lastname_firstname.pdf.

Good luck.

Short Problems

Problem 1 [6]
A comet has an orbital period of 300 years. If at perihelion, the comet is 0.05 AU from the Sun, what is the aphelion distance? Determine the orbital velocity of the comet at its closest approach to the Sun.

Problem 2 [6]
What is the maximum distance at which the E-ELT (with a 39.3 m diameter primary mirror) could detect an Earth-like planet around a Sun-like star in the L-band (3.0 - 4.0 microns)?

Problem 3 [6]
A star displays a variation in radial velocity with a period of 297 days. The radial velocity curve has a maximum of 1289 m/s above baseline and a minimum of 373 m/s below baseline. If the variation is due to the presence of a planet, determine the eccentricity of the planet.

Problem 4 [6]
What physical factors cause some nebulae to display dark spectra with bright emission lines while others display bright spectra with dark absorption lines? Explain your reasoning with a diagram of each case for full credit.

Problem 5 [6]
Prove that a Lagrange point exists at the vertex of the equilateral triangle whose base is the line between the primary and secondary body.

Problem 6 [6]
The Sun has an effective temperature of 5778 K. If the spectral radiance is given by Planck's Law, $B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$, and the Sun's V-band magnitude is -26.7, determine the Sun's apparent magnitude in the L band (centered on 3.5 microns).

Problem 7 [6]
Alpha Centauri is a binary system located 4.37 ly from Earth with a total apparent magnitude of -0.27. If Alpha Centauri A and B have temperatures of 5790 and 5260 K, respectively, and radii of 1.227 and 0.865 solar radii, respectively, find the apparent magnitude of Alpha Centauri A and B individually.

Problem 8 [6]
Muons are subatomic particles similar to electrons with a mass of 105.7 MeV/c^2 and lifetime of 2.2 microseconds produced by cosmic ray collisions in the upper atmosphere (approximately 20 km above the Earths surface). If the muons are produced with a velocity of 0.998c, and 10,000 muons/m^2/minute are observed at sea level, what is the muon flux when the muons are created?

Problem 9 [6]

A star is observed with UBV magnitudes $m_u = 16.31$, $m_b = 14.52$, $m_v = 13.76$. Spectral analysis gives $M_bol = 7.31$, BC = -1.02, $(U - B)_0 = 1.222$. Determine the distance to the star.

Problem 10 [6]

A binary star system has a primary eclipse lasting 1 hour 19.2 minutes in total, with minimum brightness lasting 44.16 minutes. Determine the ratio of the radius of the larger star to the smaller star.

Problem 11 [6]

An eclipsing binary system has a magnitude $m_p = 14.2$ during the primary transit and $m_s = 13.7$ during the secondary transit. Find the normal (non-eclipsed) apparent magnitude of the system?

Problem 12 [6]

A star has a radius $R_* = 0.897 \pm 0.091 R_{sun}$. An orbiting exoplanet causes a 0.37 percent dip in brightness. Calculate the size of the planet in solar radii, including error bounds.

Long Problems

Long Problem 1 [30]

Use the given table to answer the following questions.

Star	Right Ascension	Declination
Aldebaran	$4^{\mathrm{h}}\,36^{\mathrm{m}}$	$16^\circ\,31'$
Miaplacidus	$9^{\mathrm{h}}\,13^{\mathrm{m}}$	$-69^\circ\,42'$
Deneb	$20^{\mathrm{h}}\,41^{\mathrm{m}}$	$45^\circ\,20'$
Regulus	$10^{\mathrm{h}}\,08^{\mathrm{m}}$	$12^\circ\,18'$
Hadar	$14^{\mathrm{h}}\,04^{\mathrm{m}}$	$-60^\circ\,24'$

a. If the star Deneb crosses your local meridian, how long must you wait until Miaplacidus crosses your local meridian? [3]

b. The coordinates of Jakarta, Indonesia is 6.17 degrees South and 106.82 degrees East . Which of these stars would never be visible to an observer in Jakarta? [4]

c. At the instant Miaplacidus crosses the local meridian of an observer in Jakarta, what is its altitude and azimuth? [3]

d. At the instant Miaplacidus crosses the local meridian of an observer in Jakarta, what is the latitude and longitude of an observer who will see Aldebaran at the zenith? [3]

e. For an observer at Jakarta, how long will Aldebaran be above the horizon, from sunset to sunrise? [6]

f. What is the angular separation between Regulus and Hadar? [6]

g. When Aldebaran is rising, what is the altitude of Miaplacidus as seen by an observer in Jakarta? [5]

Long Problem 2 [37]

The energy density of blackbody radiation is gien by $u = 4\sigma T^4/c$, where σ is the Stefan-Boltzmann constant. Assuming the temperature of the CMB today is $T_{CMB} = 2.73K$ and the critical density today is $\rho_{crit} = 0.921 * 10^{-29} h_{70}^2 gcm^{-3}$.

a. What is the energy density of the CMB? What is the corresponding Ω_{CMB}? [6]

b. What are the typical photon energies of the CMB, and the corresponding number density of photons per cm^3? [6]

c. Assuming $\Omega_{CMB} = 0.3$ what was the redshift z_{eq} of the transition from the radiation dominated to the matter dominated universe? [7]

d. Assuming the age of the universe at the epoch of decoupling to be 380,000 years, and knowing its redshift, estimate the ate at z_{eq}. You can assume the expansion laws appropriate for an $\Omega = 1$ universe. [8]

e. How old was the universe at the time when the temperature was comparable to the temperature in the Solar core, $T_c = 1.5 * 10^7 K$? [5]

f. What were typical photon energies then? [5]