

LIGO (5 points).

The first detection of gravitational waves GW150414 was announced in 2016 by the collaboration LIGO (Laser Interferometer Gravitational-Wave Observatory). The detected signal corresponds to the merger of two black holes with masses of $35M_{\odot}$ and $30M_{\odot}$, which when joined formed a black hole of $62M_{\odot}$. Ignoring the rotational energies of the black holes, you may assume that the energy released by this process (E_{GW}) is emitted solely in the form of gravitational waves, that were observed by the interferometer in 2015. You are given that the explosion of a supernova (SN) releases $E_{SN} = 2 \times 10^{44} \text{ J}$.

- | | | |
|-----|--|-------|
| 1.1 | To find out which of these two events (SN, GW) releases more energy, estimate the energy ratio $\frac{E_{SN}}{E_{GW}}$. | 5.0pt |
|-----|--|-------|

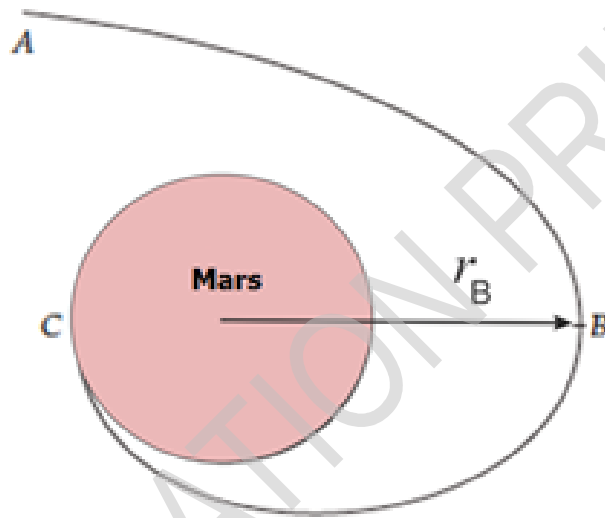
Temperature of the Earth (10 points).

For at least the last few million years, the Earth has been in roughly thermal equilibrium with the radiation from the Sun at the Earth's orbital distance.

- | | | |
|-----|--|-------|
| 2.1 | Assuming our planet to be an ideal black body, calculate what the Earth's equilibrium temperature (in Celsius) would be. | 4.0pt |
| 2.2 | The Earth's albedo is approximately 30%. Calculate the Earth's surface temperature (in Celsius) considering its albedo. | 2.0pt |
| 2.3 | The Earth's absorbed radiation is reemitted as black body radiation from its surface, but its atmosphere re-absorbs 58% of that energy, causing a greenhouse effect. Considering this effect, calculate the Earth's surface temperature (which will be the same as the temperature of the lower atmosphere). Give your answer in Celsius.
For simplicity, consider the reabsorption effect as happening only once, and do not consider the atmosphere as a separate black body. | 4.0pt |

Mars (10 points).

A spacecraft of mass $m = 5.0 \times 10^4 \text{ kg}$ approaches in a parabolic orbit AB , with respect to Mars. When the spacecraft reaches point B of least distance to the center of Mars, $r_B = 6.8 \times 10^6 \text{ m}$, it undergoes an instantaneous deceleration using its rockets and goes into a perfectly calculated orbit so that it will touch the Martian surface exactly at point C , diametrically opposite B , as shown in the figure.



- | | | |
|-----|---|-------|
| 3.1 | Determine the speed (km s^{-1}) of the spacecraft at point B just before the deceleration. | 3.0pt |
| 3.2 | Calculate the total energy (J) of the spacecraft as it is moving between points B and C . | 4.0pt |
| 3.3 | Calculate the speed (km s^{-1}) of the spacecraft at point C . | 3.0pt |

ALMA - Calculating photons (10 points).

ALMA is a radio observatory with a revolutionary design. It consists of 66 high-precision antennas, operating in the wavelength range from 0.32 mm to 8.60 mm . The principal array has fifty antennas of 12 m diameter each that can work together as a single telescope in the so-called interferometric mode. There is also another array of four 12 m antennas, and twelve smaller antennas of 7 m diameter each.

Imagine that a single 12 m antenna is being calibrated, pointing to a source with a known incident flux of $1 \times 10^{-20}\text{ W/m}^2$

- | | | |
|-----|--|-------|
| 4.1 | Assuming that all the flux arrives at the shortest wavelength of ALMA sensitivity, determine the average number of photons that would reach the detector every second. | 2.0pt |
| 4.2 | Compare it to the average number of photons that would have reached the detector, if all the flux arrived at the longest wavelength of operation. | 2.0pt |
| 4.3 | What is the angular resolution (in arcsec) of a single 12 m antenna, operating at 74.9 GHz ? | 2.0pt |
| 4.4 | Imagine the principal array operating at 74.9 GHz in the interferometric mode. Assuming for simplicity that the spatial resolution is solely given by the longest baseline (largest distance between any pair of antennas), which turns out to be $D_{\text{max}} = 16\text{ km}$, what would be the angular resolution (in arcsec) in this case? Treat this case as a single slit aperture instead of a circular one. | 2.0pt |
| 4.5 | For a radio antenna, the term SEFD refers to 'System Equivalent Flux Density', which is a characteristic energy flux density of the antenna, depending on its temperature and size. We also note that for energy estimation of radio photons, Rayleigh-Jeans approximation is valid. Assuming a system temperature of 691 K , what would be the SEFD of the full ALMA observatory in Jansky if all the 66 antennas could work together? | 2.0pt |

Under pressure (10 points).

Magnetic fields in the Sun are constantly shaping the structure of various different features in the Solar atmosphere. Inside any feature, the magnetic field (B) adds to the total pressure exerted by the gas. This so-called magnetic pressure is a function of the height z and can be expressed as:

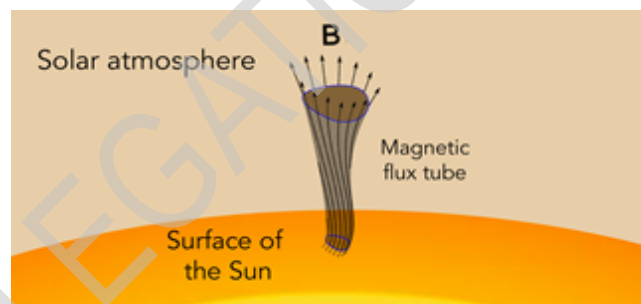
$$P_{mag}(z) = \frac{B^2(z)}{2\mu_0}$$

On the other hand, the gas can be considered to be in hydrostatic equilibrium and hence the gas pressure decays exponentially from an initial pressure value P_0 with increasing z . It can be expressed as,

$$P_{gas}(z) = P_0 e^{-z/H}$$

where H is the scale height, i.e. the height at which the pressure falls to $\frac{P_0}{e}$.

Consider one type of feature, a magnetic flux tube rising from the Solar surface up into an unmagnetized environment (see Figure below). Assuming that the total pressure of the material inside the tube and of the material outside it is in equilibrium,

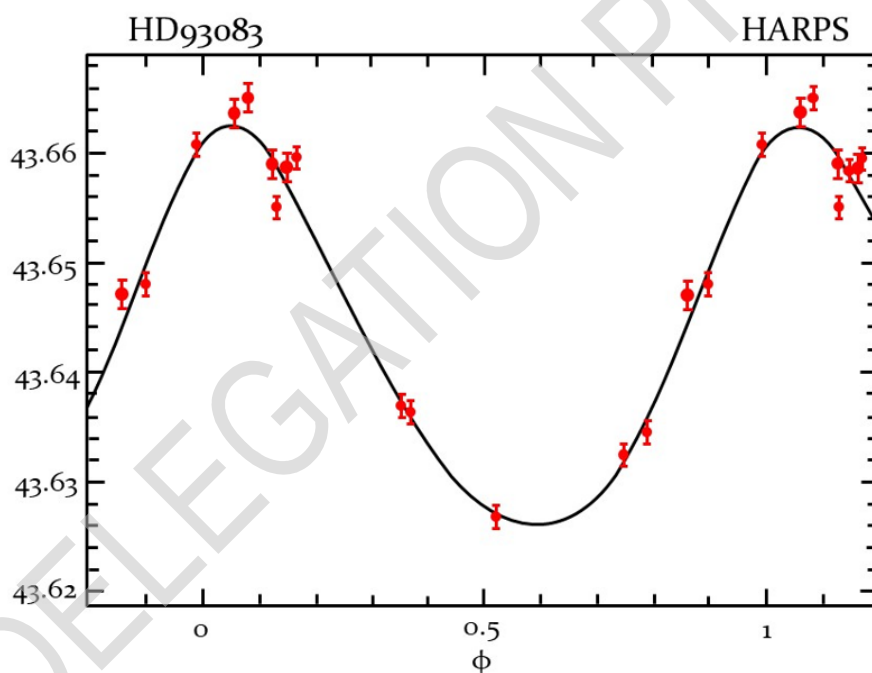


- | | | |
|-----|---|-------|
| 5.1 | Find an expression for the magnetic field strength as a function of height z . | 7.0pt |
| 5.2 | If the magnetic field at the base of a flux tube is $0.3T$, and scale height H in a given solar model is 150 km , at what height will the magnetic field be reduced to $0.03T$? | 3.0pt |

Macondo and Melquiades (12 points).

In 2019, as a part of the NameExoWorlds campaign of the International Astronomical Union, Colombia was granted an opportunity to select a name for the star HD 93083 and its planetary system. HD 93083 is a K – type dwarf star and has one extrasolar planet orbiting it. Today they are officially known as Macondo (star) and Melquiades (planet), from the literary ideas of the Colombian writer Gabriel García Márquez.

This star has an effective temperature of 4995 K and an apparent visual magnitude of 8.3. As per GAIA DR2, the parallax for Macondo is 35.03 milliarcseconds. You may assume the orbit of Melquiades is perfectly circular. In the figure you can see the plot of radial velocity of Macondo with respect to the phase.



Radial velocity of Macondo (Y-axis in km s^{-1}) as a function of the phase (X-axis).

- | | | |
|-----|---|-------|
| 6.1 | Find the wavelength (in nm) of peak emission for Macondo in its rest frame (i.e., ignoring Doppler shifts). | 2.0pt |
| 6.2 | Find the distance of this system from the Earth (in parsecs) and the absolute visual magnitude (M_V) of the star. | 2.0pt |
| 6.3 | Calculate the mean radial velocity of Macondo (in km s^{-1}). | 2.0pt |

Theory



Q6-2

English (Official)

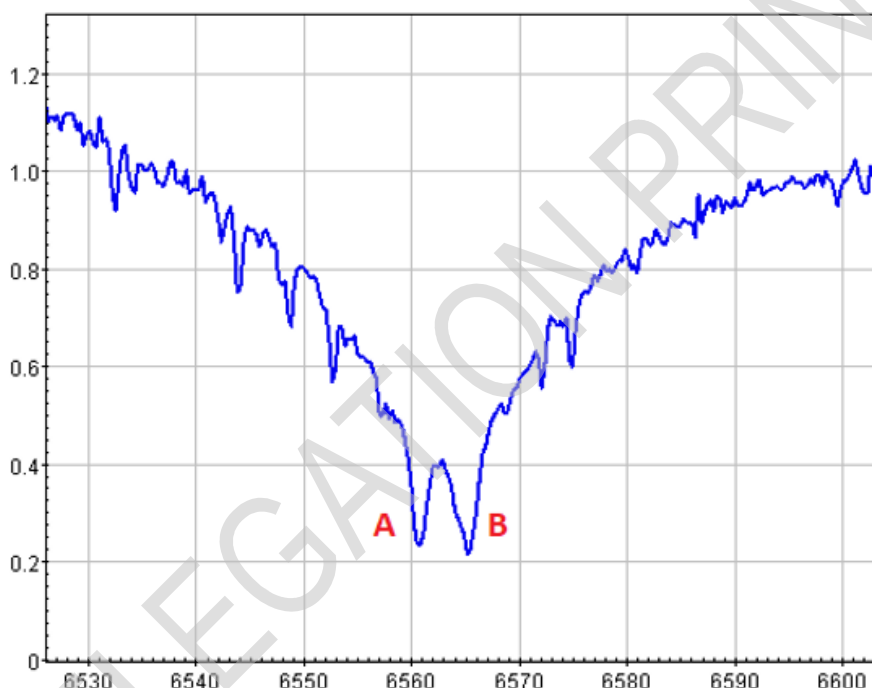
6.4	Calculate the orbital velocity (in km/s) of Melquiades (v_p), if mass of the star (m_s) is $0.7 M_\odot$ and the mass of exoplanet (m_p) is $7 \times 10^{26} \text{ kg}$. Assume that the orbital plane of the system is edge-on with respect to our line-of-sight.	2.0pt
------------	---	-------

6.5	Find the orbital radius of Melquiades (in au) and its orbital period (in days).	4.0pt
------------	--	-------

DELEGATION PRINT

Menkalinan (β Aurigae) (13 points).

Almost half of the stars that we see are either binary or multiple star systems. A well-known example of this is Menkalinan (Beta Aurigae), which was initially thought to be a single star, but today recognised as a binary system comprising two stars that we will refer to as Menkalinan A and B. In the following figure, a spectrum of the system (obtained by the observatory of the Universidad de los Andes, in Bogotá) is shown:



Spectrum of Menkalinan binary system in the region of $H\alpha$. Y-axis is for the relative flux, and X-axis measures wavelengths. Menkalinan A is marked as A in the graph, and Menkalinan B is marked as B.

Answer the following questions using the plot and noting that the wavelength of $H\alpha$ line in the laboratory frame is 656.28 nm . Assume circular orbits, and assume that the binary system as a whole is at rest with respect to the observer.

7.1 In the spectrum, we can see the $H\alpha$ line for each star in the system. Calculate the line-of-sight velocity of each star (km/s) and determine, at the time of this observation, which of the two stars is moving towards us. 5.0pt

7.2 The binary system is located 81.1 light years from Earth and has an orbital period of 3.96 days. The semi-major axis for Menkalinan B (smaller star) was measured to be $3.35 \text{ milliarcseconds}$. If the mass ratio of the two components is 1.026, find the total mass of the system (in solar masses). 4.0pt

Theory



Q7-2

English (Official)

7.3	Calculate the individual masses of Menkalinan <i>A</i> and <i>B</i> in solar masses.	2.0pt
------------	--	-------

7.4	Since Menkalinan <i>A</i> and <i>B</i> are main sequence stars, use the relation:	2.0pt
------------	---	-------

$$\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}}\right)^{3.5}$$

to estimate the luminosity of each star (in solar luminosity).

DELEGATION PRINT

Theory

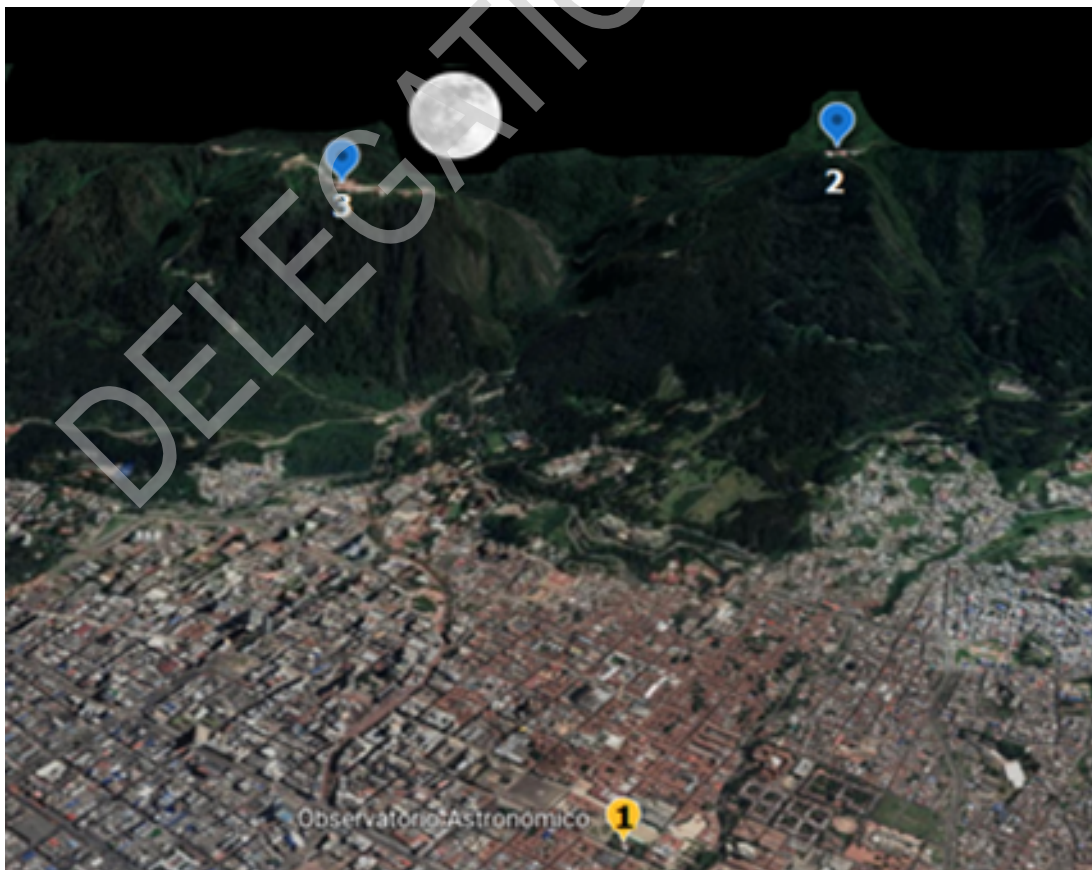


Q8-1

English (Official)

IOAA Logo (15 points).

The IOAA2021 logo is formed by the acronym IOAA, where the first letter is represented by the silhouette of the building of the National Astronomical Observatory (OAN) of Colombia, the oldest observatory in America. This observatory is located in Bogota, where it was founded in 1803. The capital city of Colombia is bordered by two famous hills, Monserrate and its neighbor Guadalupe, which are icons of Bogota's cityscape that decorate the logo's background.



Aerial view of Bogota City. Numbers show locations for the quoted places: 1 is for OAN; 2 is for Guadalupe; and 3 is for Monserrate.

Theory



Q8-2

English (Official)

Point	Latitude	Longitude	Elevation (m.a.s.l)
1	$4^{\circ} 35' 53'' N$	$74^{\circ} 04' 37'' W$	2607
2	$4^{\circ} 35' 30'' N$	$74^{\circ} 03' 15'' W$	3296
3	$4^{\circ} 36' 18'' N$	$74^{\circ} 03' 19'' W$	3100

8.1 Estimate the distance (in km), between points 2 (Guadalupe) and 3 (Montserrat). 3.0pt

8.2 Estimate the angular separation (in degrees) between Guadalupe (2) and Monserrate (3) as observed from the National Astronomical Observatory of Colombia (1). 6.0pt

8.3 From the OAN, on September 21 at 8:00 p.m. the Moon was observed towards the eastern hills (between Monserrate and Guadalupe). The measured ecliptic coordinates (longitude and latitude) of the Moon are shown in the table. Determine the equatorial coordinates of the Moon at the time of observation. 6.0pt



Theory



Q8-3

English (Official)

Local Time: 8:00 p.m.
$Az : +90^{\circ}42'59'' / Alt : +19^{\circ}01'42''$
$\lambda : +12^{\circ}20'16'' / \beta : -04^{\circ}24'14''$

Note: Azimuth measured from North to East.

DELEGATION PRINT

Pluto Satellites (15 points).

- 9.1** The mass of Charon, the biggest satellite of Pluto, is $1/8$ th the mass of Pluto. 15.0pt
Both bodies move in a circular orbit around a common center of mass. In addition, they both are tidally-locked.
The distance between the center of the planet and the center of the satellite is $R = 19\,640\text{ km}$ and radius of the satellite is $r = 593\text{ km}$.

Let g_0 be the gravitational acceleration on the surface of Charon due only to its mass. Let A be the point on Charon surface directly facing Pluto, and B the point diametrically opposite. Compute the percentage difference between gravitational acceleration at A and B respect to g_0 .

Terrestrial Transit (15 points).

Note: Assume perfect circular orbits in both questions below.

10.1	An alien astronomer from a distant planetary system is observing the Sun. Suddenly, the brightness of the Sun drops due to the transit of the Earth in front of it. What is the maximum duration that this transit may last (in hours)? Assume that the planet where the astronomer observes from, does not move relative to the Sun.	5.0pt
-------------	---	-------

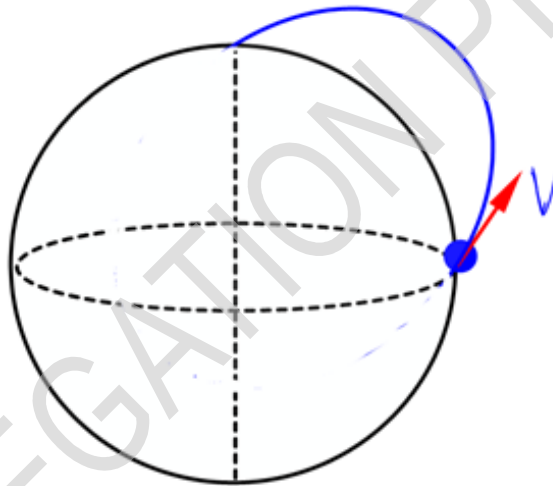
10.2	Imagine that the transit of a given exoplanet as seen from Earth lasts 31 minutes. The host star is a red dwarf, with mass and radius that are 10% of the mass and radius of the Sun. What is the minimum orbital period this exoplanet may have (in days)?	10.0pt
-------------	---	--------

Minimum velocity of a projectile (15 points).

11.1	What is the minimum speed with which a projectile must be launched from the Earth's surface at the equator such that the projectile reaches the north pole?	12.0pt
------	---	--------

11.2	Find the eccentricity of the trajectory described by the projectile	3.0pt
------	---	-------

You may ignore the rotation of the Earth. Also assume the earth surface is spherical.



Reference Chart

Hodograph (15 points).

In curvilinear motion of a planet around a star, the direction of the velocity vector changes continuously. This can be represented by a so-called "trajectory in velocity space" and is obtained as follows: for each point on the spatial trajectory, the corresponding velocity vector is drawn so that its starting point is at the origin of the velocity space, and its magnitude and direction is the same as the velocity vector at that point. The tip of this variable velocity vector generates a curve in velocity space. (The name 'hodograph' was given to this curve by Hamilton in 1846.)

As an example, see figures 1 and 2 below. For a circular orbit (Figure 1), the magnitude of the velocity is constant and therefore, the hodograph (Figure 2) of the velocity vector for Keplerian circular motion is also a circle, the center of which is located at the origin of the velocity space. The radius of this circle is equal to the constant magnitude of the circular velocity.

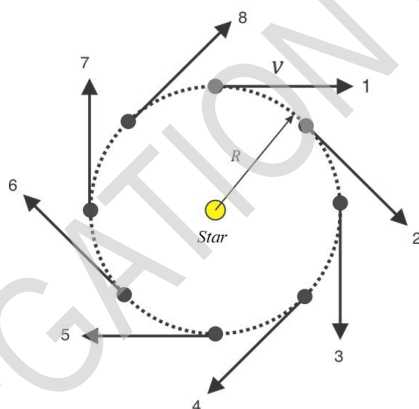


Fig. 1 Spatial trajectory of the Planet with Uniform Circular Motion around the star.

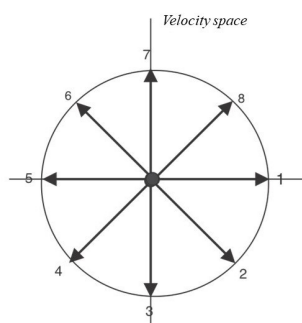


Fig. 2 Corresponding hodograph

- 12.1** Write an expression for the radius of the hodograph in Fig. 2, as a function of the mass M of the star, and the radius R of the circular orbit of the planet's motion. 1.0pt

- 12.2** For a planet in a Keplerian trajectory, write the expression for centripetal acceleration vector ($\vec{a}$) and the magnitude of angular momentum (L). For any Keplerian trajectory, it is true that 4.0pt

$$|\Delta v| = k\Delta\theta \quad (1)$$

Where k is a constant for each type of Keplerian trajectory. Find the expression for the constant k as a function of the masses M and m of the star and the planet, respectively, and the angular momentum, L .

(Eq.1) allows us to conclude that for any Keplerian trajectory, the hodograph (v as a function of θ) is a circle, but except for circular motion, the centre of the hodograph does not coincide with the star. It is not necessary to prove this result, you may simply accept it as a given. For the hodograph of uniform circular motion, the compliance with (eq.1) is completely obvious, as evidenced in Fig. 3

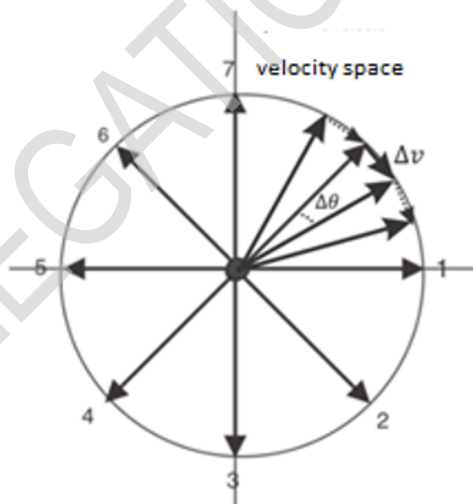


Fig. 3

- 12.3** Determine the expression of the constant k for the hodograph of circular planetary motion. 2.0pt

- 12.4** Given that the hodograph of the Keplerian elliptical motion is a circle, determine the radius of this hodograph and the distance between the center of the hodograph and the position of the star, as a function of the velocities at periastron and apoastron. Draw a rough sketch of the hodograph in the answer sheet as per the schematic shown in Fig. 4. The black circle is the star. 4.0pt

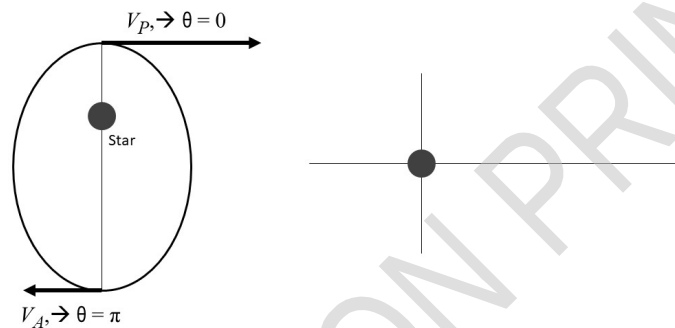


Fig. 4

- 12.5** Similarly, for the parabolic Keplerian trajectory, determine the radius of the corresponding hodograph and the distance from the center of that hodograph circle to the star. Express the radius as a function of the velocity at periastron. Draw a rough sketch of the hodograph circle in the answer sheet. 4.0pt

Lucy: The First Mission to the Trojan Asteroids (15 points).

CCD cameras on space probes are very sensitive and exposed to space weather conditions. Intense radiation passing through the CCD produces electron-hole pairs in the silicon of the CCD chip. The rate at which these pairs are produced is an important parameter when operating cameras on board spacecraft and can be calculated for radiation of any given energy.

A high energy particle or photon of radiation passing through the CCD will deposit some energy in the chip with each electron-hole pair it creates. The 'stopping power' of silicon for a given type of particle can be measured as the energy per areal density (*areal density = mass per unit area*) that the silicon 'takes away' from the travelling particle.

NASA's Lucy mission will be the first to study the Trojan asteroids and will revolutionize our understanding of the formation of the Solar System. One of the instruments on board is L'LORRI (Lucy LOng Range Reconnaissance Imager), which contains a sensitive CCD in order to produce detailed images of the Trojan asteroids. Unfortunately, the radiation around Jupiter is very intense and it can generate a lot of 'noise' in the pixels of the CCD.

Let us assume that an average charged particle trapped in Jupiter's magnetic field has an energy of 15 MeV and that the flux of such particles in this region is equivalent to about $600 \text{ electrons } s^{-1} \text{ cm}^{-2}$. Also assume that for each electron-hole pair which a particle passing through a pixel creates, it deposits exactly the excitation energy of the pair in that pixel. After the pixel crosses a threshold number of electron-hole pairs it is 'excited' and no more pairs can be produced in that pixel. Any remaining energy in the particle is passed to the next pixel (and so on).

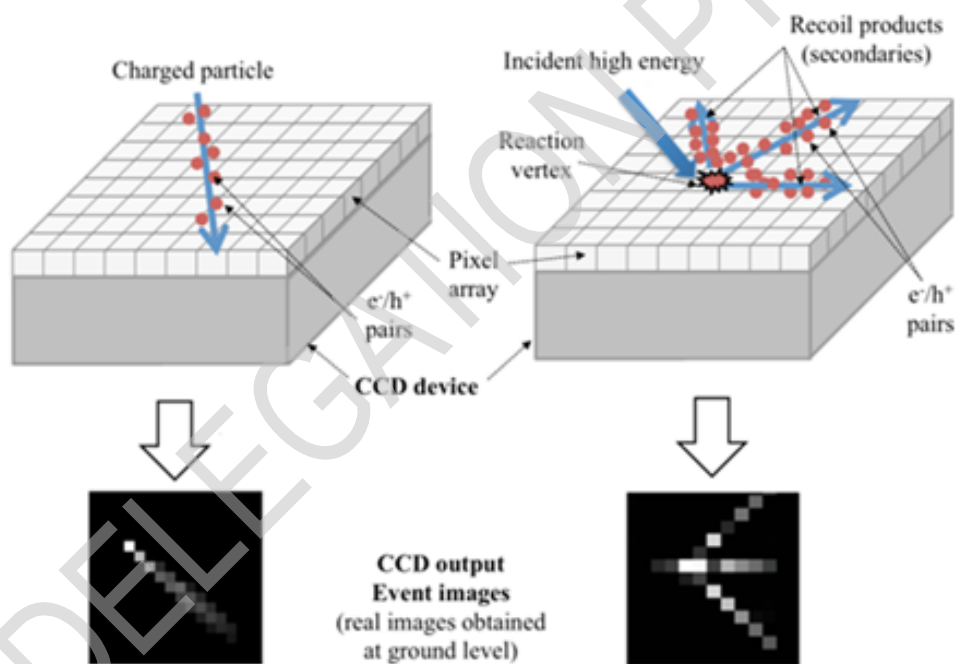
Using the data given below for the CCD chip in the L'LORRI camera, answer the following questions:

13.1	How many pixels will be excited by one such particle of radiation passing through the CCD when the spacecraft is near Jupiter's orbit?	10.0pt
-------------	--	--------

13.2	Given the radiation flux near Jupiter, what percentage of the total number of pixels in an image will be excited?	5.0pt
-------------	---	-------

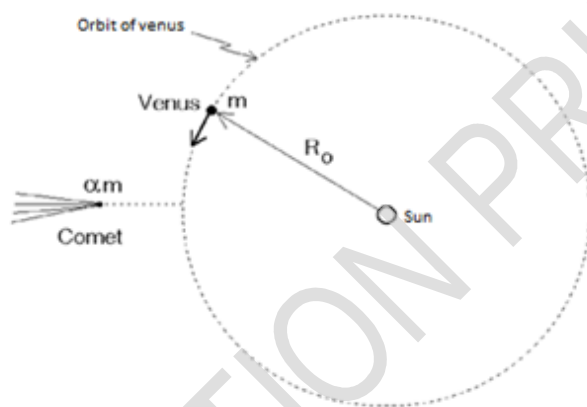
CCD Data:

- Exposure time of an image = 30 ms
- Pixels on the CCD = 1024 x 1024
- CCD Area = 13 mm x 13 mm
- CCD chip thickness = 0.06 cm
- Density of silicon, $\rho = 2.34 \text{ g cm}^{-3}$
- Excitation energy of single pair = 2.36 eV
- Excitation threshold of a single pixel = 250 pairs
- Stopping power' of silicon for a 15 MeV electron = $3.012 \text{ MeV g}^{-1} \text{ cm}^2$



Formation of the Venus-2 (35 points).

A comet of mass αm is heading ("falls") radially towards the Sun. It is known that the total mechanical energy of the comet is zero. The comet crashes into Venus, whose mass is m . We further assume that the orbit of Venus, before the collision, is circular with radius R_0 . After the crash, the comet and Venus form a single object, called "Venus-2".



- | | | |
|------|---|--------|
| 14.1 | Find the expression in terms of M_{sun} and R_0 for the orbital speed, v_0 , of Venus before the collision. | 1.0pt |
| 14.2 | Find an expression for the total mechanical energy of Venus in its orbit before colliding with the comet. | 1.0pt |
| 14.3 | Find an expression for the radial velocity, v_r , the angular momentum, L , of "Venus-2" immediately after the collision. | 10.0pt |
| 14.4 | Find an expression for the mechanical energy of the combined object "Venus-2" and express it in terms of energy before the collision, E_i , and α . | 5.0pt |
| 14.5 | Show that the post-collision orbit of "Venus-2" is elliptical and determine the semi-major axis of the orbit. | 5.0pt |
| 14.6 | Determine if the year for the inhabitants of "Venus-2" has been shortened or lengthened because of collision with the comet. Write the ratio between the period of Venus-2 and Venus. | 3.0pt |

Theory



Q14-2

English (Official)

14.7 What should be the value of α such that the post-collision orbit of Venus-2 would make it crash in the Sun? We will call this as α_c 5.0pt

14.8 A comet with $\alpha = \alpha_c$ collided with Venus. Calculate the percentage change in the magnitude of Venus' velocity (δv) and the change in the direction of the velocity vector ($\delta\theta$) immediately after the collision. 5.0pt

DELEGATION PRINT