

GeCAA – Theory

Final Question Sheet

(Total 150 points)

1. Astrophotography

An astrophotographer, based on the Equator, uses a good digital camera on a tripod, but with no tracking. The camera has a telescopic lens with focal length of 150 mm and aperture (objective diameter) of 84 mm . The sensor has an effective light collecting diameter of 22.5 mm . The photographic target is a star field at the observer's Zenith.

- a) **(2 points)** Calculate the field of view (the angular width of the image) which can be captured on the sensor using this equipment.
- b) **(5 points)** The pixels in the camera's sensor are separated by a distance of $3.23\ \mu\text{m}$. What is the maximum possible exposure time for a single frame, so that no star trails appear on the exposed image?
- c) **(3 points)** For a better-quality image of the star field, the photographer decides to take multiple shots at the exposure time calculated in b) and then to stack them together. The total time for all these shots is 30 seconds (ignore any time taken to write data to the memory card) What proportion of the total field of view is possible at the higher signal to noise ratio?

2. Flat Earth

(10 points) A new model of the world is gaining in popularity among some people. These people believe in the “Flat Earth” view of the world, where the Earth is not a spheroid, but rather a circle with radius R_{\oplus} . The central axis of the Earth (normal to the circle passing through its centre C) passes through the observer's zenith.

This model must at least remain consistent with the observed phenomena, as listed below:

- The value of the solar constant is $S_{\odot} = 1366 \text{ W/m}^2$.
- The Earth's central axis precesses in a circle with a period 25800 years.
- The radius of the precession circle is 23.5° .

Assume that (1) the Earth is a perfect blackbody radiator and the Sun is sufficiently far away such that all solar rays are parallel and (2) the Sun's current (initial) location is at the zenith,

Determine how many years it will take for the Earth's equilibrium temperature to decrease by 1°C .

3. Mirror

A bored cosmologist comes up with a thought experiment to determine the Hubble constant (H_0) for his model of a Steady-State-Universe. In this experiment, a large, fully reflecting flat mirror - carrying several gyroscopes that would maintain its spatial orientation in the same plane - would be placed at a distance D from the Solar System in a region without gravitational influences. From the Earth, a laser beam would be directed towards that region for a long period of time. After a time T , the radiation would return and be detected, allowing the determination of the fixed constant H_0 .

- a) **(7 points)** Find an expression for H_0 as a function of D , c (speed of light) and T .

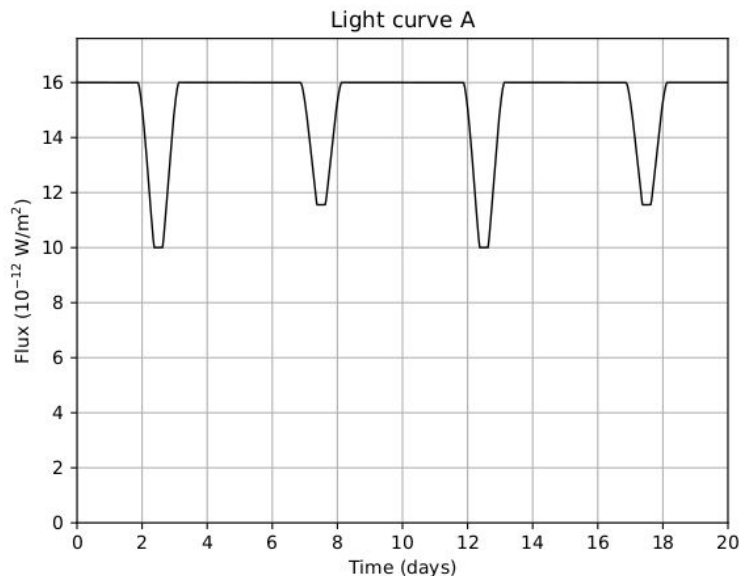
Consider that the separation S between the Solar System and the mirror increases only due to the expansion of the universe according to the law $S = s \cdot e^{H_0 t}$, where s is the initial separation. You may use $e^x \approx 1 + x$ for $x \ll 1$, if necessary.

- b) **(3 points)** Imagine that such a mirror is located in the vicinity of the star Vega (which also features on the logo of the 1st GeCAA). Vega was the first star outside the Solar System to be photographed and one of the first stars whose parallax ($p = 0.125''$) was accurately measured in 1840 by G. W. von Struve.

Estimate the total duration of this H_0 measurement experiment.

4. Light Curves

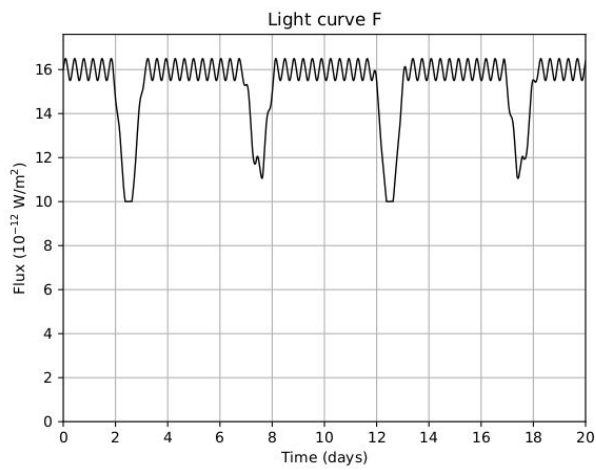
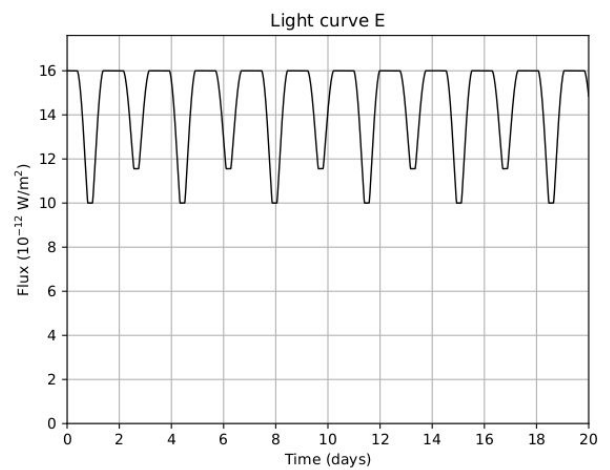
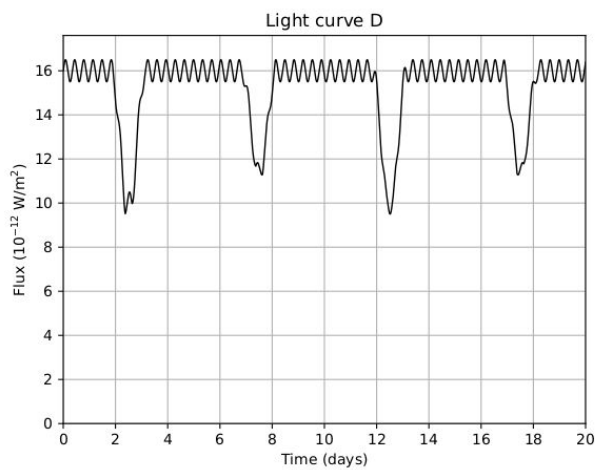
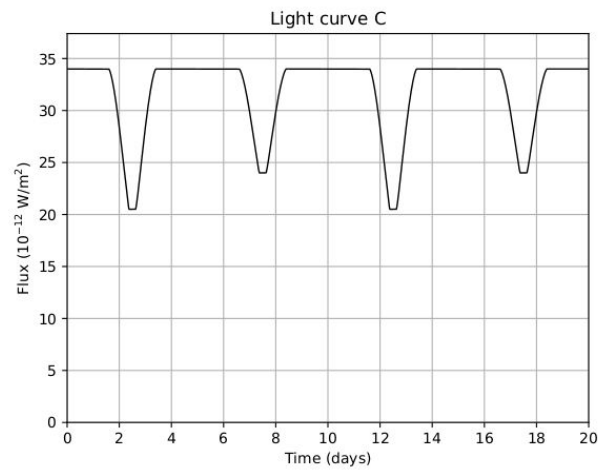
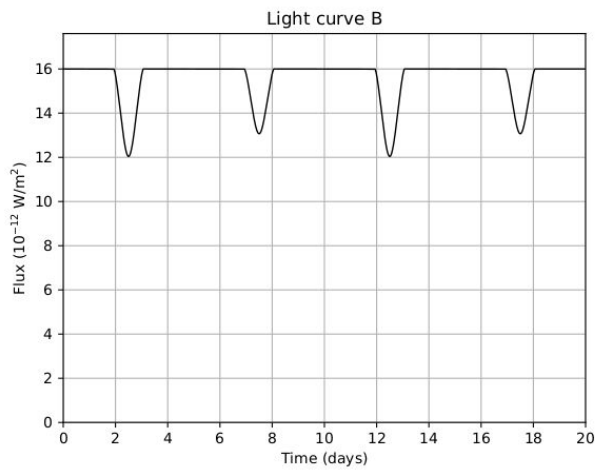
The light curve A shown below, shows a fictional edge-on eclipsing binary system containing stars X (radius r_X , luminosity L_X) and Y (radius r_Y , Luminosity L_Y). Assume that star X is



brighter, but star Y is hotter.

- a) **(1 point)** Which of the two stars is likely to be on the main sequence? (Write "X" or "Y")
- b) Based on light curve A, estimate:
 - (i) **(2 points)** $\frac{r_X}{r_Y}$, the ratio of the radii of the two stars.
 - (ii) **(2 points)** $\frac{L_X}{L_Y}$, the ratio of the Luminosity of the two stars.
- c) **(15 points)** For light curves B to F, in each case only one parameter of the binary system has been changed from the case in light curve A. For each case, choose the description from the following list that best corresponds to the change (Write the appropriate roman numeral in the answer sheet).
 - i) Star X increased in size.
 - ii) Star X increased in luminosity.
 - iii) Star X decreased in size.
 - iv) Star X decreased in luminosity.
 - v) Star Y increased in size.
 - vi) Star Y increased in luminosity.
 - vii) Star Y decreased in size.
 - viii) Star Y decreased in luminosity.
 - ix) Star X is a variable star.
 - x) Star Y is a variable star.
 - xi) The inclination of the system relative to the Earth has changed.

- xii) The distance of the system from the Earth has decreased.
- xiii) The distance of the system from the Earth has increased.
- xiv) The orbital period of the system increased.
- xv) The orbital period of the system decreased.



5. HII region

Luminous Blue Variables (LBV) are massive, unstable, supergiant stars that can undergo episodes of very strong mass loss, due to an instability in their atmospheres. After such an event, a dense nebula is formed around the star. LBV are also very hot stars and produce a large amount of high-energy photons that are able to ionise hydrogen atoms ($E_{ph} > h\nu_0 = 13.6 \text{ eV}$) creating a roughly spherical region of ionized hydrogen (HII region).

In this problem, we consider a static, homogeneous, pure hydrogen nebula with a concentration of $n_H = 10^8/m^3$ and temperature $T_{HII} = 10^4 K$, ionized by photons from a single LBV star with a stable rate of ionizing photons $Q = 10^{49} \text{ ph/s}$. Assume that each photon can ionise only one hydrogen atom. At a particular location within an HII region, the rate of photoionization is balanced by the rate of recombination per unit volume. This sets the radius of the fully ionized region and this region is called the Stromgren sphere with the radius R_S .

The total number of recombinations per volume is proportional to the concentration of protons n_p , the concentration of electrons n_e and the recombination coefficient for hydrogen $\alpha(T_{HII}) = 10^{-19} m^3/s$. For simplification, ignore the fact that the process of recombination can also release ionising photons.

- (5 points)** Derive an algebraic expression for the radius of the Stromgren sphere and calculate its value for the given parameters. Express your answer in units of parsecs (pc).
- (3 points)** The photoionization cross-section of H-atoms in the ground state encountering photons with frequency ν_0 is equal to

$$\sigma \approx 10^{-21} m^2$$

Calculate the mean-free path l_{ν_0} of an ionising photon. Compare l_{ν_0} to R_S to determine if this ionized nebulae is sharp-edged or not? (answer "YES" or "NO")

- (4 points)** On what timescale (in years) do you expect the Stromgren sphere to form?
- (4 points)** Radiation from an ionized hydrogen cloud (HII region) is often called free-free emission because it is produced by free electrons scattering off the ions without being captured: the electrons are free before the interaction and remain free afterwards. In this process, the electron retains most of its pre-scattering energy.
An electron, while passing by a much more massive singly ionized hydrogen atom, produces a radio photon of $\nu = 10 \text{ GHz}$. Calculate the mean electron thermal energy in the HII region, for the given temperature of the Stromgren sphere. Is this an example of free-free emission? (answer "YES" or "NO")
- (4 points)** Since the HII region is in local thermodynamic equilibrium, one can calculate the absorption coefficient that is proportional to the optical depth $\tau_\nu \propto \nu^{-2.1}$ and it turns out that at sufficiently high radio frequencies, the hot plasma is nearly transparent and hence $\tau_\nu \ll 1$.

The flux density of photons has power-law spectra of the form $S_\nu \propto \nu^\beta$.

Find β for the radio frequencies.

6. Occultation of an X-ray Source

Consider a satellite observing x-ray sources, while orbiting the Earth in the equatorial plane with orbital radius r , and orbital time period P . Assume that this satellite is pointed in one fixed direction in space for a given length of time. Take the radius of the Earth as R .

When the satellite moves 'behind' the Earth in its orbit, the x-ray source is 'occulted' and the measured x-ray flux from the source drops to zero. However, due to Earth's atmosphere, this drop is gradual. If the line of sight to the source passes through the atmosphere, the extent of the attenuation depends on the air-mass (i.e. length of air column) along the line of sight.

- a) **(1 point)** Let us assume that the satellite is pointing towards a fixed source at 0° declination. We consider that the source is occulted when 50% of the light coming from the source gets attenuated due to the atmosphere. Let us say that this happens when the minimum height of the line of sight from the surface of the Earth is h .

If θ_0 is the angle between the direction of the source and the direction of the Earth, as measured from the spacecraft, find an expression for θ_0 .

- b) **(4 points)** The time duration Δt between the source getting attenuated from 90% of the pre-occultation flux to 10% is defined as the 'occultation time' for the source. Assume the flux attenuates to 90% when the minimum height of the line of sight ($h + 0.5\Delta h$) and similarly the flux attenuates to 10% at ($h - 0.5\Delta h$), where $\Delta h \ll R$.

Find the expression for Δt in terms of r , P , Δh and θ_0 .

- c) **(15 points)** If the satellite was pointing towards a source at an arbitrary declination β instead (where β is not too large), what will be the expression for Δt ?

Note: If the satellite was not in the equatorial plane, then the problem could have been simply rephrased by assuming the satellite's orbital plane to be the equatorial plane. In that case, β becomes 'relative declination'.

7. Radiant of a Meteor Shower

A stargazer in Chiayi, Chinese Taipei (23.5°N , 120.4°E , GMT+8) saw two meteors streaking through the sky at 21:00 (Chinese Taipei time) on 25th September 2020. One of the meteors originated directly above the northern horizon at 15° altitude and streaked to meet the horizon exactly due east. The second meteor originated at an altitude of 23.5° and an azimuth of 210° and ended at an altitude of 75° and an azimuth of 255° .

- a) **(6 points)** What is the Local Sidereal Time (LST) at the time of observation?
- b) **(16 points)** Find the alt-az coordinates of the apparent radiant of the two meteors.
- c) **(6 points)** Find the equatorial coordinates of the apparent radiant.
- d) **(2 points)** Which of the following constellations is closest to the radiant?
Crux / Dorado / Pavo / Tucana / Triangulum Australes
(choose one and write the same name in the answer box)

Notes:

- Azimuths are measured from the North (0°) towards the East.
- The Greenwich Sidereal Time (GST) at 00:00 UT on 1st January 2020 is $6^{\text{h}} 40^{\text{m}} 30^{\text{s}}$.

8. Jupiter's Great Red Spot

In the following problem, the fluid mechanics of Jupiter's Great Red Spot (GRS) is studied based on the velocity field data. The diagram on the next page shows a map of relative velocity for GRS and the surrounding region. The arrows are oriented and scaled as per the directions and magnitudes of winds at different points.

Due to the combined effects of gravity and rotation, Jupiter is slightly flattened at its poles. The equation of a spheroid approximating for the shape of Jupiter can be stated as:

$$\frac{x^2+y^2}{R_e^2} + \frac{z^2}{R_p^2} = 1$$

where $R_e = 7.15 \times 10^7 m$ is the equatorial radius of Jupiter, and $R_p = 6.69 \times 10^7 m$ the polar radius. The radii of curvature of this spheroid in any direction can be calculated by the following equations ($\varepsilon = \frac{R_e}{R_p}$):

$$r(\phi) = R_e(1 + \varepsilon^{-2}\tan^2\phi)^{-1/2}$$

$$R(\phi) = R_e\varepsilon^{-2} \left(\frac{r(\phi)}{R_e\cos\phi} \right)^3$$

where $r(\phi)$ and $R(\phi)$ are the zonal (aka in the zone of a particular latitude) and meridional (aka longitudinal) radii of curvature, respectively, as a function of planetographic latitude ϕ . The sidereal rotation period of Jupiter is $P = 3.57 \times 10^4 s$.

- (4 points)** Calculate the zonal and meridional radii values (\bar{r} and \bar{R} respectively) at the location of the centre of the GRS.
- (5 points)** Estimate the eccentricity of the GRS.
- (6 points)** 'Vorticity' at any point is a measure of local spinning of the fluid as measured by an observer situated in the reference frame of the fluid. Mathematically, it is calculated as 'curl' (vector derivative product) of the velocity field. In this case, the average relative vorticity may be estimated by the equation:

$$\xi = \frac{V_w L_{GRS}}{A_{GRS}}$$

where V_w is the maximum value of winds as per the velocity field, L_{GRS} is the length of the circumference of the GRS and A_{GRS} is the area of the GRS.

Estimate average relative vorticity of the GRS.

Hint: The circumference of an ellipse is well approximated by an average of circumferences of the corresponding auxiliary and minor circles.

- (2 points)** Find the absolute vorticity $\xi_a = (\xi + f)$ by adding the Coriolis parameter

$$f = 2\Omega\sin\phi$$

where Ω is the angular velocity of the Jupiter (due to axial rotation) and ϕ is the appropriate latitude.

- (1 point)** If the absolute vorticity has the same sign as the latitude, we call the storm a 'cyclonic storm'. If they have opposite signs, the system is 'anticyclonic'. Is the GRS cyclonic or anticyclonic?
- (12 points)** Imagine that the GRS moves to another latitude ϕ_1 , where the absolute vorticity changes the sign (changes from anti-cyclonic to cyclonic or vice versa). Assuming minimum possible displacement of the GRS, at what value of ϕ_1 do we expect this change?

In your analysis, assume that the GRS at the new location would occupy the same angular span in latitude, as well as have the same wind velocities and eccentricity as the original.

