2026 National Astronomy Competition

1 Instructions (Please Read Carefully)

The top 15 eligible scorers on the NAC will be invited to the training camp. A third round will be conducted during the camp to select the national team for the IOAA 2026. In order to qualify for camp, you must be a high school student with US citizenship or permanent residency.

The test must be completed within 2.5 hours (150 minutes) and the maximum number of points is **300 points**. The point breakdown per problem is as follows.

Problem	P1	P2	P3	P4	P5	P6	P7	P8	P9	**Total**
Points	10	15	15	20	20	40	40	55	85	**300**

Please solve each problem on a blank piece of paper and mark the number of the problem at the top of the page. Some problems may require a designated answer sheet; these will be provided. The contestant's full name in capital letters should appear at the top of each solution page. If the contestant uses scratch papers, those should be labeled with the contestant's name as well and marked as "scratch paper" at the top of the page. Scratch paper will not be graded. Partial credit will be available given that correct and legible work was displayed in the solution.

This is a written exam. Contestants can only use a scientific calculator (non-programmable and non-graphing) for this exam. A table of physical constants will be provided. **Discussing the problems with other people is strictly prohibited in any way until the end of the examination period on April 20, 2026.** Receiving any external help during the exam is strictly prohibited. This means that the only allowed items are: a calculator, the provided table of constants, a pencil (or pen), an eraser, blank sheets of papers, and the exam. No books or notes are allowed during the exam. Students must be proctored during the entire duration of the exam.

This exam sheet and your answer sheets (including scratch papers) should be returned to your proctors once the exam ended. Your proctors should upload your answer sheets to the Google Form provided to them.

We acknowledge the following people for their contributions to this year's exam:

Srihari (Hari) Balaji, Feodor Yevtushenko, Ferdinand, Hagan Hensley, Joe McCarty, Lucas Pinheiro, Vincent Bian, Elizabeth Lee

After reading the instructions, please make sure to sign, affirming that:

1. All work on this exam has been completed by me.
2. I took this exam under the supervision of a proctor.
3. I did not receive any external help beyond the materials provided.
4. I will not discuss the contents of this exam with anyone until April 20, 2026.
5. Failure to follow these rules will result in disqualification from the exam.

Student Signature: ______________________ Date: ______________________

1. **(10 points)** *A "gravitational atom"*

   Bohr's semiclassical model of the hydrogen atom assumed that the ground state angular momentum of the electron is $\hbar = h/(2\pi)$. While this model is incorrect, the Bohr radius does give the correct lengthscale of the hydrogen atom. By analogy to the Bohr model, consider a bound system consisting of a neutron and an electron interacting purely gravitationally. If the orbital angular momentum is $\hbar$, what would be the radius of this "atom"?

2. **(15 points)** *Celestial Sphere: Night at the Observatory*

   An observer is located at latitude $\phi = +42.0^\circ$ (Northern Hemisphere). Assume the Earth is a perfect sphere and ignore atmospheric refraction.

   Star A has equatorial coordinates

   $$\alpha = 6^{\mathrm{h}}40^{\mathrm{m}}, \qquad \delta = +62.0^\circ.$$

   On a particular night, the local sidereal time (LST) is

   $$\mathrm{LST} = 5^{\mathrm{h}}10^{\mathrm{m}}$$

   at 10:00 PM local clock time.

   You may assume that one sidereal day equals $23^{\mathrm{h}}56^{\mathrm{m}}$ of clock time.

   (a) **(2 points)** Compute the hour angle H of star A at 10:00 PM. State whether the star is east or west of the meridian at that moment.

   (b) **(5 points)** Compute the altitude h of star A at 10:00 PM. Give your answer in degrees.

   (c) **(3 points)** How much clock time will pass after 10:00 PM until star A reaches upper culmination? Express your answer in hours, minutes, and seconds.

   Suppose star B has declination

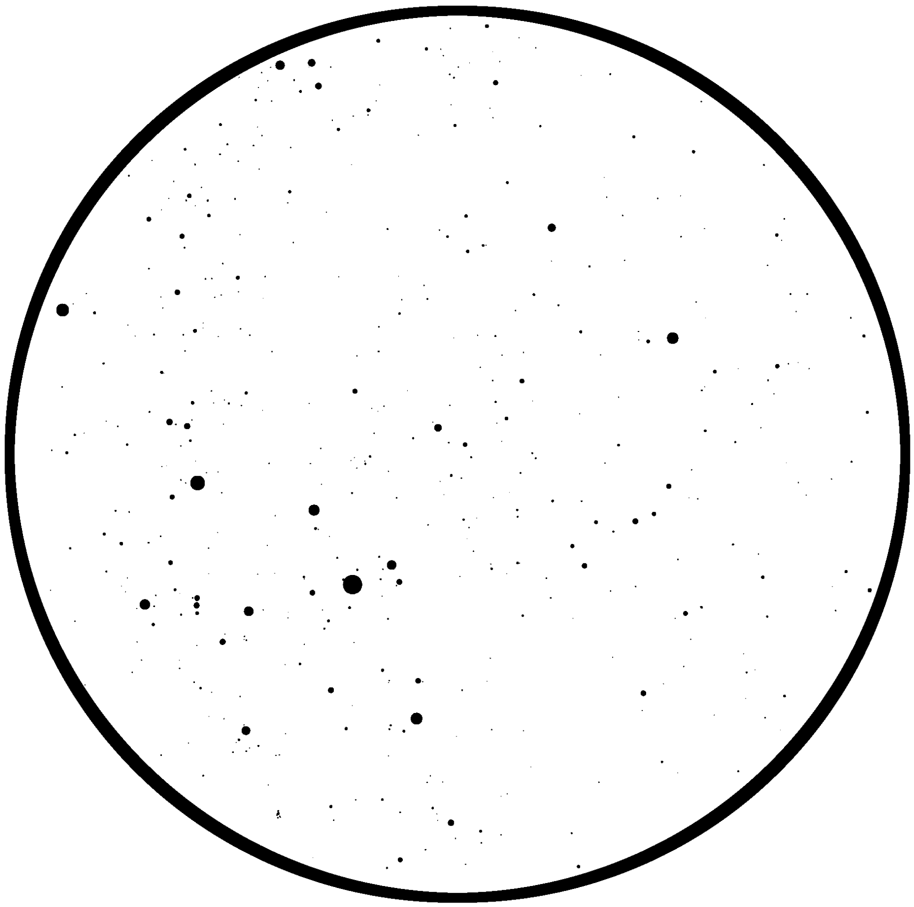
   $$\delta = +30.0^\circ.$$

   (d) **(5 points)** For star B, compute the total clock-time duration during which the star is above the horizon during one sidereal day. Express your final answer in hours and minutes of clock time.

3. **(15 points)** *Sky Map Attack!*

Please answer this question in the provided Question 3 Answer Sheet

Before IOAA 2026, you and your friends decided to go on vacation to a deserted island. One night, you looked up at the sky and it looked like the image below:



As the sky was bright, you and your friends gave each other quizzes about the sky.

(a) **(3 points)** What is the estimated latitude of the island?
Hint: The all sky map above is a stereographic projection, which means the zenith distance would follow: $z = 2\arctan(r/R)$*, where* r *is the distance of an object to the center of the map and* R *is the radius of the all sky map.*

(b) **(2 points)** Mark and label the cardinal directions: North, East, South, and West.

(c) **(2 points)** Draw and label the celestial equator.

(d) **(2 points)** Draw and label the ecliptic.

(e) **(3 points)** Mark and label (North/South) the available Celestial, Ecliptic, and Galactic Poles.

(f) **(1.5 points)** Are there any planets seen in the sky? If yes, mark and label the planets.

(g) **(1.5 points)** Circle and label the Pleiades (M45).

4. **(20 points)** *Solar Observations*

An astronomer who enjoys solar observations has the habit of recording the Sun's right ascension and declination every day. These coordinates are always recorded at the same civil time (the time on the clock). Between two consecutive observations, which happened when the Sun was in the Northern Celestial Hemisphere, the declination of the Sun increased by 0.3650°. Estimate the days in which these two observations occurred.

Neglect the eccentricity of Earth's orbit.

5. **(20 points)** *Enceladus geysers*

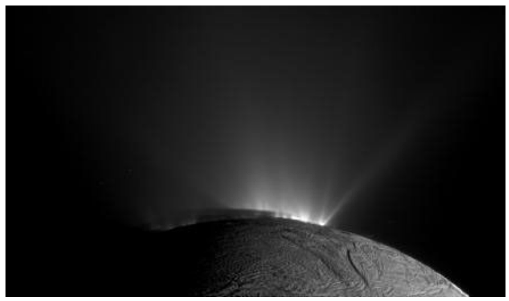


Figure 1: Cassini image of geysers from Enceladus. From NASA.

Enceladus is a moon of Saturn which has an outer crust of ice and is believed to have a subsurface liquid water ocean. Here are some useful physical properties:

- Radius of Enceladus (including the ice shell) $R = 250\,\text{km}$
- Ice shell thickness $H = 25\,\text{km}$
- Density of ice $\rho_i = 920\,\text{kg/m}^3$
- Surface gravity $g = 0.11\,\text{m/s}^2$ (you can assume this is constant throughout the icy shell to the top of the subsurface ocean)
- Semi-major axis of Enceladus' orbit around Saturn $a = 2.4 \times 10^8\,\text{m}$
- Mass of Saturn $M = 5.7 \times 10^{26}\,\text{kg}$

(a) **(10 points)** The Cassini spacecraft observed that geysers of water erupt from the surface at a speed of $v = 400\,\text{m/s}$ relative to Enceladus. Calculate whether this water has sufficient speed to escape the gravitational field of Enceladus. If so, will it remain gravitationally bound to Saturn or escape into solar orbit?

(b) **(6 points)** Assuming that the water in the geysers starts from rest at the top of the subsurface ocean, calculate the pressure necessary to accelerate the water to $v = 400\,\text{m/s}$ at the surface of Enceladus, where there is no ambient pressure. Assume that viscous losses are negligible and the geyser material has a constant density of $1000\,\text{kg/m}^3$, ignoring any phase changes.

(c) **(4 points)** Estimate the hydrostatic pressure due to the weight of the icy shell at the top of the subsurface ocean. Is that pressure sufficient to explain the speed of the geysers?

6. **(40 points)** *Build your Own H-R Diagram*

The following table contains the apparent visual magnitudes m_v and color indices (B-V) for a sample of stars in a cluster.

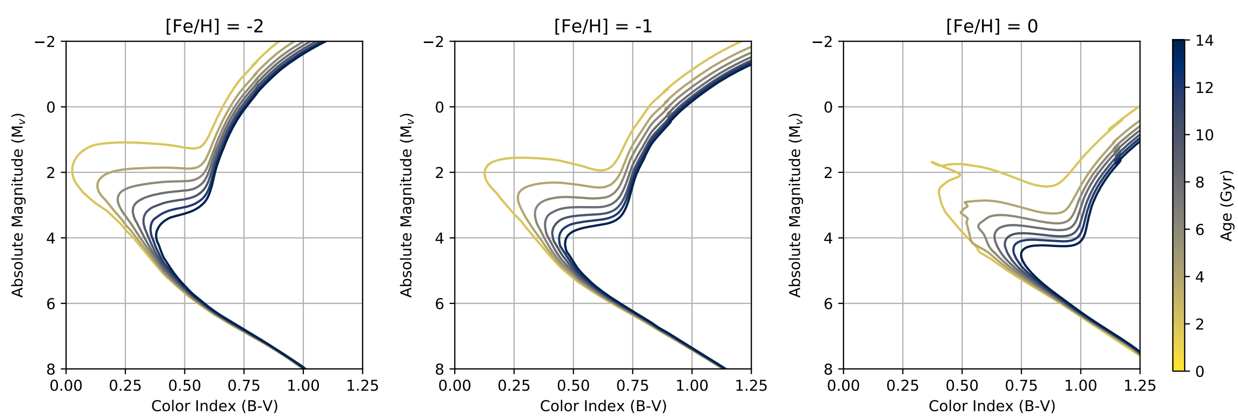
Star #	B-V	m_v	Star #	B-V	m_v	Star #	B-V	m_v	Star #	B-V	m_v
1	0.66	20.0	11	0.80	20.5	21	0.93	21.0	31	0.53	16.5
2	0.45	18.3	12	0.46	16.8	22	0.39	17.0	32	0.43	18.1
3	0.73	19.9	13	0.12	21.1	23	1.01	11.9	33	0.62	19.4
4	0.72	14.2	14	0.20	13.7	24	0.60	19.5	34	0.70	12.3
5	0.38	17.5	15	0.77	20.0	25	-0.03	14.3	35	0.85	13.0
6	0.03	13.9	16	0.51	19.3	26	0.65	15.5	36	0.06	20.9
7	0.55	18.9	17	0.74	13.9	27	0.49	18.9	37	0.61	16.2
8	0.70	14.3	18	0.99	12.0	28	0.61	13.2	38	0.72	20.0
9	0.57	16.3	19	0.53	13.1	29	0.62	15.0	39	0.68	19.7
10	0.91	12.3	20	0.50	16.4	30	0.53	19.0	40	0.11	13.9

For the purposes of this problem, the values have been corrected for extinction and reddening from the interstellar medium.

(a) **(18 points)** Plot the data on an H-R diagram using the provided grid paper. Label at least three of the main features.

Any population of stars that formed together (such as the stars in a cluster) should all be of about the same age and metallicity and should lie along a given curve on the H-R diagram, because their position on the H-R diagram is only a function of their initial mass. These curves are known as *isochrones*.

The following figure shows a selection of modeled isochrones for different ages and metallicities [Fe/H]. The isochrones range in age from 2 Gyr (lightest) to 14 Gyr (darkest) in 2 Gyr steps.



Notes:

- The isochrones plotted here use absolute magnitude M_v on the y-axis, rather than apparent magnitude m_v.
- The isochrones do not include every phase of stellar evolution.
- The data you plotted in part (a) will not perfectly fall along a single curve due to measurement scatter.

(b) **(17 points)** By comparing your H-R diagram to the isochrones, estimate:

  (i) **(5 points)** the metallicity of the cluster (to the nearest integer).

  (ii) **(5 points)** the age of the cluster (to the nearest Gyr).

  (iii) **(7 points)** the distance to the cluster (to the nearest kpc).

(c) **(5 points)** Is this cluster more likely to be an open cluster or a globular cluster? Explain your reasoning.

7. **(40 points)** *Interstellar Flight*

Kirara is traveling to some stars in the constellation Crux, starting from Earth, and then visiting Gacrux (γ Crucis), Ginan (ε Crucis), Acrux (α Crucis), and Mimosa (β Crucis) in that order. To pass the time, she does some astronomy on each leg of the trip.

(a) **(5 points)** Suppose Kirara starts at rest in the reference frame of the barycenter of the solar system, and that Gacrux, in the same reference frame, has a radial velocity of 20.6 km/s away from Kirara. She has a magic warp drive that can teleport her rocket straight to Gacrux (and preserves her velocity).

However, if she directly teleports, she'll have a large velocity relative to Gacrux once she arrives! She plans to use a normal rocket to first accelerate to a point where she has no velocity relative to Gacrux. If her unfueled rocket weights 1.20×10^5 kg, how much fuel would she need? The exhaust velocity of her rocket is 10.0 km/s.

The Tsiolkovsky rocket equation says that $\Delta v = v_e \ln \frac{m_0}{m_f}$, where Δv is the change in velocity, v_e is the exhaust velocity, m_0 is the initial mass of the rocket (including fuel), and m_f is the final mass of the rocket.

Ignore the proper motion of Gacrux.

(b) **(10 points)** When Kirara gets to Ginan, she takes a break on a (fictional) exoplanet orbiting the star. Ginan has a mass of $1.5\,M_\odot$, and suppose the exoplanet has a mass of $2M_\oplus$, a radius of $1.2R_\oplus$, and a circular orbit with radius 10 AU. She rigs a catapult to give her rocket a one-time boost of Δv in any direction. What is the minimum Δv required for her to escape the combined gravitational field of both Ginan and the exoplanet?

(c) **(5 points)** Kirara arrives at Acrux, which is actually a system of 6 stars. She gets distracted taking measurements of them, and doesn't realize that she's drifting too close to one of the stars! Specifically, she is close to α Crucis Aa, which has a mass of 15.17 $M_\odot$.

She currently is at rest, at a distance R from α Crucis Aa. The free-fall time for an object at distance R to fall into a star with mass 15.17 $M_\odot$ is 24.0 hours, meaning that if Kirara doesn't act, her ship will be pulled into the star in 24 hours!

She repositions to a distance of $2R$ from α Crucis Aa, again with no velocity relative to the star. What is the free-fall time for an object at a distance $2R$ to fall into a star of mass 15.17 $M_\odot$?

Assume α Crucis Aa is at rest, and ignore the gravitational effects of the other stars in the system.

(d) **(20 points)** When Kirara is halfway from Acrux to Mimosa (in linear Euclidean distance), she looks at Mimosa through a telescope. What is the apparent magnitude she observes for Mimosa? The following information will be helpful:

Designation	Name	RA	Dec	Apparent magnitude	Distance to Earth
α Crucis	Acrux	$12^h\ 26^m\ 36^s$	$-63^\circ\ 06'$	0.76	321 ly
β Crucis	Mimosa	$12^h\ 47^m\ 43^s$	$-59^\circ\ 41'$	1.25	352 ly

Assume that Acrux and Mimosa are stationary.

8. **(55 points)** *Watch Out!*

Starting at time $t = 0$, two spaceships A and B begin traveling apart from each other at a relative speed of $v = \beta c$. Let a given ship's reference frame be the frame in which it is constantly at position $x = 0$. For a given event, if it occurs at position x and time t in A's reference frame (call this frame S), let x' and t' be the position and time, respectively, of the event in B's reference frame (call this frame S'). Assume that the spaceships start at the same position, at which point their clocks are initially synchronized.

(a) **(5 points)** Using only Newtonian mechanics, and neglecting all relativistic corrections, write x' and t' in terms of x, t, and v.

(b) **(2 points)** Explain the primary physical contradiction that arises in the above system if β is significantly larger than 0.

Let us now account for relativistic effects. When switching between reference frames moving at a speed v relative to each other, the relativistically correct transformations are

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

Here, x, t, x', and t' are defined as before. You may also find the relativistic momentum ($\vec{p} = \gamma m\vec{v}$) and total energy ($E = \gamma mc^2$) formulas helpful.

The spaceships have previously agreed to communicate with each other through light signals. In particular, red light ($\lambda = 700$ nm) indicates that there is imminent danger ahead. Spaceship B, equipped with better radar system, detects a cluster of asteroids along the path that spaceship A is taking and sends a laser signal in red light with a wavelength of λ that lasts a time Δt_0. Both λ and Δt_0 are as measured in B's reference frame, and let the overall emitted power in B's reference frame be P. Use $\beta = 0.4$ for all numeric answers.

You may refer to the following table of color versus *approximate* wavelength for parts (c)(v) and (c)(vi):

λ (nm)	**Color**
$\leqslant 390$	ultraviolet
390 – 420	purple
420 – 480	blue
480 – 560	green
560 – 585	yellow
585 – 610	orange
610 – 720	red
$\geqslant 720$	infrared

(c) **(28 points)** By virtue of relativity, both the observed duration of the signal and the observed wavelength of the signal change. We will now examine this.

(i) **(5 points)** In terms of Δt_0 and $\beta = v/c$, how long does B spend emitting the signal, in A's reference frame? (Denote this as Δt_1.)

(ii) **(7 points)** Due to the finite travel time of light, and given that B is moving away from A, A observes a different length Δt_2 for the signal. *(This is analogous to the non-relativistic Doppler effect.)* Calculate Δt_2 in terms of Δt_0 and β.

(iii) **(3 points)** In terms of λ and β, what wavelength λ' does A observe for the signal?

(iv) **(8 points)** Using your answers to (ii) and (iii), calculate the power P' that A observes for the signal in terms of P and β. (Assume the entire beam emitted by B hits A.)

(v) **(2 points)** Evaluate λ' numerically. What part of the electromagnetic spectrum does this correspond to?

(vi) **(3 points)** In order for A to observe red light at a wavelength λ, numerically evaluate the wavelength λ'' that B would have to emit light at. What part of the electromagnetic spectrum does this correspond to?

An asteroid (of mass $5m$) is traveling directly toward spaceship A (of mass m) at a speed of $0.5c$ in spaceship A's reference frame. From spaceship A's perspective, the asteroid and spaceship B are diametrically opposite each other, and both are moving in the $+x$ direction. Let F_0 be the reference frame in which spaceship A is initially stationary and at the origin. *(Again, use $\beta = 0.4$ for all calculations, and from now on, express all answers in terms of m, c, and numeric prefactors. Onwards, p and E are not referring to four-vectors, rather, p refers to the momentum along the x-axis, E refers to the total energy (rest mass-energy + relativistic kinetic energy), and the "mass" refers to rest mass.)*

(d) **(5 points)** In F_0, what are the combined initial momentum p_i and initial total energy E_i of the asteroid and spaceship A?

Spaceship A and the asteroid undergo a completely inelastic collision, with no mass or energy escaping the system. According to relativity, both p and E are conserved.

(e) **(11 points)** Following the collision, what is the mass M of the resulting object, and what is its velocity v_0 in frame F_0?

(f) **(5 points)** Switching to the reference frame of spaceship B, what are the mass M' and velocity v_0' of the resulting object? Is it moving toward or away from spaceship B?

The relativistic velocity addition formula is given as follows:

$$v' = \frac{v+u}{1+vu/c^2}$$

where

- v' = velocity of the object in the stationary frame,
- v = velocity of the object in the moving frame,
- u = velocity of the moving frame with respect to the stationary frame.

9. **(85 points)** *Neutron Star Dynamics!*
*Note that the provided **Answer Sheet** for this question is only for subtasks (c)(vi) and (d)(v). Do all other work on blank pages.*
In this question, we do a detailed dive into several elements of neutron star dynamics and energy losses. While understanding the internal nature and makeup of a neutron star requires quantum chromodynamics and general relativity, other properties, such as rotation rate, are far more elementary to model. Over time, a neutron star's rotation rate slowly decreases due to factors such as energy loss and mass accretion. By measuring changes in a neutron star's rotation rate, astronomers can make inferences about the physics surrounding a neutron star and its environment. Neglect more complex behavior (such as QED/QCD/GR corrections), instead assuming Newtonian mechanics holds unless otherwise stated.

Let $\Omega(t)$ be the star's angular velocity around its axis as a function of time. In several scenarios, we can observe a proportionality relation of the form $\dot{\Omega} \propto -\Omega^n$, where n is the neutron star's **braking index**. *(Throughout this problem, dots denote time derivatives, i.e. $df/dt = \dot{f}$ and $d^2f/dt^2 = \ddot{f}$.)*

(a) **(10 points)** We will begin by examining the basic behavior of the time evolution of Ω.

(i) **(5 points)** Express n in terms of Ω, $\dot{\Omega}$, and $\ddot{\Omega}$.

(ii) **(3 points)** If $n > 1$, then we can deduce that $\Omega(t) \propto t^{-\alpha}$ for some α. Compute α in terms of n.

(iii) **(2 points)** What type of curve does $\Omega(t)$ versus t trace if $n = 1$?

This relation allows astronomers to measure n for a given star. We will now examine several mechanisms for rotational slowdown and the subsequent values for n, treat each of these mechanisms as **independent** of each other unless otherwise stated. (In other words, do not combine mechanisms until part (d)(v).)

(b) **(8 points)** First, we will look at a primitive model for mass accretion. Let M be the neutron star's mass and R be its radius. Assume the rate of mass accretion $\dot{M}$ is constant and that the infalling matter has no initial angular momentum relative to the rotation axis. Additionally, assume (slightly unrealistically) that the neutron star is a uniformly dense sphere of matter.

(i) **(2 points)** Write down the expression for the angular momentum of the neutron star L in terms of M, R, and Ω, up to a numerical constant. Use the fact that L stays constant for problems (ii) and (iii) of this subpart.

(ii) **(3 points)** Now, assume that as the star accretes mass, the density ρ stays constant over time. Calculate the value of n.

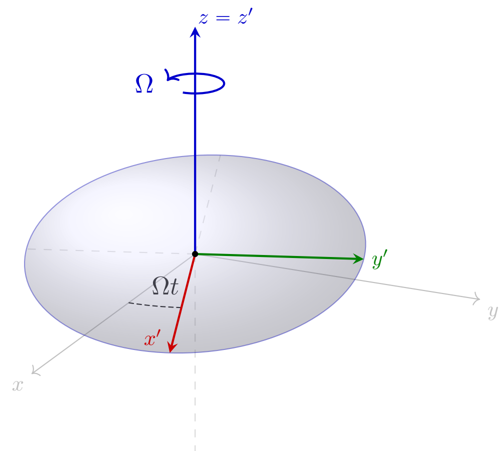
(iii) **(3 points)** Frontier QCD simulations currently appear to imply that, in contrast to the above, R stays relatively constant as M changes. Treating this new relationship as exact, calculate the new value of n.

(c) **(40 points)** Next, we will examine gravitational wave emission from the neutron star. To leading order, gravitational wave emission is caused by oscillations in the *mass quadrupole moment*, which is the deviation from spherical symmetry when projected onto an axis, of the neutron star. The general formula for the traceless quadrupole moment terms is

$$Q_{ij} = \int \rho(\mathbf{r})(r_i r_j - \frac{1}{3} r^2 \delta_{ij}) d^3\mathbf{r},$$

where $\mathbf{r}$ is the displacement from the center of the ellipsoid, $i, j \in \{x, y, z\}$ are coordinate axes, r_i are projections of $\mathbf{r}$ onto these axes (so $r_x = x$), and $\delta_{ij} = 1$ for $i = j$ with $\delta_{ij} = 0$ otherwise. **Such integrals are over all space** unless otherwise specified. *(Formally, the quadrupole moment is a 3 by 3 matrix Q with 9 terms Q_{ij}, but you will not need any knowledge of linear algebra for this problem.)*

We model the neutron star as a rotating ellipsoid (which deviates very slightly from a perfect sphere).



We establish a fixed xyz coordinate system for reference, which the neutron star rotates at angular speed Ω relative to. The neutron star has body symmetry axes x', y', and $z' = z$, as per the diagram. By the definition of body symmetry axes,

$$\int \rho(\mathbf{r}')(r'_i r'_j) d^3\mathbf{r}' = I_i \delta_{ij}.$$

In other words, the moments of inertia around the body axes x', y', and z' are I_x, I_y, and I_z, respectively, and all "cross" terms between different axes vanish. (Let $I_x > I_y$.) Some elementary trigonometry yields

$$\begin{cases} x = x'\cos(\Omega t) - y'\sin(\Omega t), \\ y = x'\sin(\Omega t) + y'\cos(\Omega t), \\ z = z'. \end{cases}$$

We will now compute the time evolution of Q_{ij}. Assume Ω is constant until part (v).

(i) **(12 points)** Using the above coordinate transformations and inertia integrals, calculate all terms Q_{ij}, expressing your answer in terms of the body axis moments of inertia and trigonometric functions of Ωt. For example,

$$Q_{xz} = \int \rho(\mathbf{r})(xz - \frac{1}{3}r^2\delta_{xz})d^3\mathbf{r} = \int \rho(\mathbf{r}')(xz)d^3\mathbf{r}' = \int \rho(\mathbf{r}')(x'\cos(\Omega t) - y'\sin(\Omega t))z' d^3\mathbf{r}',$$

$$= \cos(\Omega t)\int \rho(\mathbf{r}')x'z'd^3\mathbf{r}' - \sin(\Omega t)\int \rho(\mathbf{r}')y'z'd^3\mathbf{r}' = 0,$$

since both integrals vanish as stated.

(ii) **(7 points)** For the first step of our calculations, we will need the second time derivative $\frac{d^2Q_{ij}}{dt^2}$ of the quadrupole moment. Calculate these terms and indicate which ones are nonzero. *(You may find one or more of the identities* $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$, $\sin 2\alpha = 2\sin\alpha\cos\alpha$, *and* $\cos^2\alpha + \sin^2\alpha = 1$ *helpful.)*

Detecting gravitational waves involves measuring the *strain* h, or the fractional change in length $\Delta L/L$ of spacetime caused by the gravitational wave. For the purposes of this problem, we ignore nuances such as viewing angle and GW polarization regimes. As such, we approximate the behavior as

$$h_{ij}(t) = \frac{2G}{c^4 d}\frac{d^2Q_{ij}}{dt^2},$$

where d is the distance from the observer to the neutron star. *(Note that there is a time offset due to the nonzero time it takes gravitational waves to reach the observer, but we ignore this, as it is not highly relevant in understanding the qualitative behavior of the system.)*

(iii) **(3 points)** For convenience, we only look at one term of h_{ij} to capture the underlying behavior. Calculate the amplitude h_0 and angular frequency Ω_{GW} of the oscillations in h_{xx}. Express your answers in terms of I_z, $\epsilon = \frac{I_x - I_y}{I_z}$, Ω, and fundamental constants.

(iv) **(6 points)** According to general relativity, the radiated power is given by

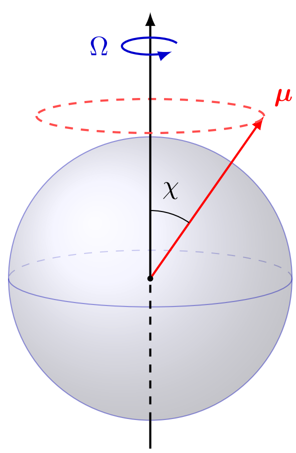
$$P = \frac{G}{5c^5} \sum_{ij} \left(\frac{d^3 Q_{ij}}{dt^3} \right)^2,$$

where the sum runs over all 9 pairs of indices $i, j \in \{x, y, z\}$. Calculate P in terms of I_z, ϵ, Ω, and fundamental constants.

(v) **(6 points)** The power for the gravitational waves comes purely from the rotational kinetic energy $E = \frac{1}{2} I_z \Omega^2$ of the neutron star. Assuming its mass, radius, and inertia moments stay constant, calculate the value of $\dot{\Omega}$ in terms of Ω, I_z, ϵ, and fundamental constants. From this, extract the value of n.

(vi) **(6 points)** As the neutron star slowly loses rotational kinetic energy while keeping its inertia moments constant, oscillations of the observed gravitational wave strain h change in both period and amplitude. On your **Answer Sheet**, **sketch** a graph for $h_{xx}(t)$ versus time t that reflects these changes. *(Exaggerate your graph so that significant visible changes in both period and amplitude occur over the course of several periods. Your graph need not be to scale.)*

(d) **(27 points)** Finally, we will consider magnetic dipole radiation, which is electromagnetic radiation induced by the rotation of the star and subsequent movement of its magnetic field. This is the main source of energy loss in pulsar systems. Let the magnetic dipole moment be μ with an inclination χ to the rotation axis (here, the z-axis). Take $\chi \in (0, \frac{\pi}{2})$.



Some simple geometry gives

$$\boldsymbol{\mu}(t) = \mu(\sin\chi\cos(\Omega t), \sin\chi\sin(\Omega t), \cos\chi)$$

in component form.

(i) **(4 points)** Calculate the second derivative $\ddot{\boldsymbol{\mu}}(t)$ in component form. Using this, evaluate its magnitude $|\ddot{\boldsymbol{\mu}}|$.

(ii) **(5 points)** According to the Larmor formula, the power radiated by the magnetic dipole is given by

$$P = \frac{\mu_0}{6\pi c^3} |\ddot{\boldsymbol{\mu}}|^2,$$

where μ_0 is the permeability of free space. As in the previous section, assume that this power comes from a change in the rotational kinetic energy $E = \frac{1}{2} I_z \Omega^2$, where we again take I_z as the (constant) moment of inertia around the rotation axis. Calculate $\dot{\Omega}$ as a function of Ω and any other relevant variables. Using this, calculate the value of n. (Refer to this value as n_0.)

(iii) **(9 points)** Nevertheless, neutron stars are sometimes observed to have n-values that significantly differ from n_0. We will now model this. Specifically, take your same $P = -\dot{E}$ expression as before, but now assume that μ and χ are no longer fixed, instead potentially having small nonzero rates of change. Using your result from part (a)(i), calculate a new value for n. Express your answer in terms of Ω, $\dot{\Omega}$, μ, $\dot{\mu}$, χ, $\dot{\chi}$, and fundamental constants. Again, assume that the mass, radius, and composition of the neutron star stay constant.

(iv) **(2 points)** Assume that the neutron star's magnetic moment decays exponentially according to

$$\mu(t) \propto e^{-t/\tau_B}.$$

Rewrite your expression for n from part (iii) in terms of Ω, $\dot{\Omega}$, τ_B, χ, $\dot{\chi}$, and fundamental constants.

(v) **(7 points)** The Crab Pulsar is observed to have a long-term average braking index of $n_{obs} = 2.51 \pm 0.01$. For each of the following, mark on your **Answer Sheet** whether it would cause n to deviate **toward** n_{obs}, **away from** n_{obs}, or have **no change**, assuming a default value of $n = n_0$ (remember $\dot{\Omega} < 0$ and $\chi \in (0, \frac{\pi}{2})$):

(A) A gradual increase in the inclination χ,

(B) A gradual decrease in the internal magnetic dipole moment μ,

(C) Gradual accretion of a nearby star (use your result from part (b)(iii) of the problem).

(D) Significant gravitational wave emission (use your result from part (c) of the problem).