Student's name

Problem 1. Which of the following relates the intrinsic luminosity of a spiral galaxy with its asymptotic rotation velocity?

Solution: **B: The Tully-Fisher Relation**

Fundamental Plane: Relates radius, surface brightness, and velocity dispersion of elliptical galaxies
Tully-Fisher: Relates intrinsic luminosity/mass with rotation velocity
Press-Schechter: Predicts number of objects of a certain mass within a given volume of the Universe
Faber-Jackson: Relates mass and luminosity of galaxies

□

Problem 2. Which of the following correctly gives the location of Population I vs. Population II stars in the Milky Way?

Solution: **A: Population I - Thin Disk, Spiral Arms; Population II - Halo, Bulge**

Older stars (Population II) are generally found in the Bulge and Halo, younger stars (Population I) are typically found in the thin disk and spiral arms

□

Problem 3. A quasar with a bolometric flux of approximately 10^{-12} erg s^{-1} cm^{-2} is observed at a redshift of 1.5, i.e. its comoving radial distance is about 4.4 Gpc. What is the bolometric luminosity of the quasar?

Solution: **B**: $\boldsymbol{6.0 \times 10^{11} L_\odot}$

Luminosity distance: $d_L = d_C(1+z)$, Flux $= 4\pi L(d_L)^2$ □

Problem 4. Now, lets assume that the quasar in the previous question is observed to have a companion galaxy which is 5 arcseconds apart. What is the projected linear separation of the companion galaxy from the quasar?

Solution: **D: 43 kpc**

$$\text{Linear separation} = d_A d\theta = d_C \frac{d\theta}{(1+z)}$$

where Angular diameter distance is denoted as d$_A$

□

Problem 5. An observer is standing atop the Burj Khalifa, the tallest building on earth (height = 830m, latitude = 25.2N, longitude = 55.3E). Which of the following options is the closest to the shortest and longest shadow on the ground at the local noon time due to the building in a given year?

Solution: **B: 25, 950m**

The maximum and minimum declination of the sun in a given year is 23.5 to -23.5 deg. At noon, the zenith angle of the sun at the given place will range from (25.2,-23.5) to (25.2,+23.5). The shadow length is given as $h\tan(z)$ where z is the zenith angle and h is the height of the tower.

The maximum approximates to 940 not 950. Though it depends what effects get included. With no atmospheric refraction and just a pure sun declination + latitude analysis, it should be 940.

□

Problem 6. Which of the following is closest to the ratio of the farthest distance to the horizon that can be seen by an observer standing top of the Mount Everest on Earth (height = 8.8 km) and Olympus Mons on Mars (height = 25 km)?

Solution: **B: 1**

$D = \dfrac{R}{R+h} \times \sqrt{2hR+h^2}$, and since $h \ll R$, we can approximate D such that $D \propto \sqrt{hR}$ (and this proportionality relationship is all that matters)

□

Problem 7. An observer measures the black-body spectrum for a variety of bodies as a function of temperature and wavelength in the long wavelength limit ($\frac{hc}{\lambda} \ll k_B T$) and finds that his data approximately fits the relationship $\log(I) = a + b\log(T) + c\log(\lambda))$. Here, I is the spectral intensity in terms of wavelength, T is the temperature of the body and λ is the wavelength. Which of the following are the expected values of b and c?

Solution: **A: 1,-4** □

Problem 8. Suppose a spacecraft were orbiting in a low Earth orbit at an altitude of 400 km. The spacecraft makes a single orbital maneuver to place it into a Mars transfer orbit. Delta-v (Δv) refers to the change in velocity during an orbital maneuver. What is the Δv required for this trans-Mars injection? The semimajor axes of the orbits of Earth and Mars are 1.496×10^8 km and 2.279×10^8 km, respectively.

Solution: **B: 3.57 km/s**

A Hohmann transfer orbit from Earth to Mars would have a semimajor axis of

$$a = \frac{1}{2}\left(1.496 \times 10^8 + 2.279 \times 10^8\right) = 1.888 \times 10^8 \text{ km}$$

On the Earth side of the transfer orbit, the spacecraft would need a velocity of

$$v_1 = \sqrt{GM_\odot\left(\frac{2}{1.496 \times 10^{11}} - \frac{1}{a}\right)} = 32.722 \text{ km/s}$$

The orbital velocity of the earth around the sun is

$$v_\oplus = \sqrt{\frac{GM_{odot}}{1.496 \times 10^{11}}} = 29.779 \text{ km/s}$$

The hyperbolic excess velocity is therefore

$$v_\infty = v_1 - v_\oplus = 2.943 \text{ km/s}$$

However, the spacecraft must first escape the gravitational influence of the earth. Its current orbital velocity is

$$v_{orb} = \sqrt{\frac{GM_\oplus}{1000 \times (6371 + 400)}} = 7.670 \text{ km/s}$$

The escape velocity is

$$v_{\text{esc}} = \sqrt{2} v_{orb} = 10.847 \text{ km/s}$$

Hyperbolic excess velocity is related to escape and perigee velocities by

$$v_p^2 = v_{esc}^2 + v_\infty^2$$

Therefore,

$$v_p = \sqrt{v_{esc}^2 + v_\infty^2} = 11.239 \text{ km/s}$$

Finally, $\Delta v = v_p - v_{orb} = 3.569$ km/s.

□

Problem 9. After entering Mars orbit, the spacecraft finds that over the course of the martian year, the position of Star A varies by 613.7 milliarcseconds (mas) due to the movement of the spacecraft around the sun. Determine the distance to Star A.

Solution: **D: 4.965 pc**

A parsec is defined as the distance at which an object produces a parallax of 1 arcsecond as seen from Earth. The parallax angle is half of the angular variation in position over one year, so

$$p = 0.6137/2 = 0.30685$$

This translates to a distance of

$$1/0.30685 = 3.259 \text{ pc}$$

(assuming the spacecraft were orbiting the earth). However, because the baseline used is

$$\frac{2.279 \times 10^8}{1.496 \times 10^8} = 1.523 \text{ AU}$$

while the parsec is defined relative to a baseline of 1 AU, the distance must be scaled appropriately. Therefore, the actual distance to the star is

$$1.523 \times 3.259 = 4.965 \text{ pc}$$

$\square$

Problem 10. Star A, of mass $3.5M_\odot$, shows radial velocity variations 24.2 m/s in amplitude and 23.22 years in period, suggesting the presence of an orbiting exoplanet. Which of the following is closest to the mass of the exoplanet? Assume the exoplanet's orbit is circular and has inclination 90°. The mass of Jupiter is 1.898×10^{27} kg.

Solution: **C: 5.6 M$_J$**

By Kepler's third law, the semimajor axis of the exoplanet's orbit is

$$a = \left(3.5 \times 23.22^2\right)^{1/3} = 12.358 \text{ AU}$$

Its orbital velocity is

$$v_{\text{orb}} = \sqrt{\frac{G \times 3.5M_\odot}{a}} = 15.853 \text{ km/s}$$

Finally, by conservation of momentum,

$$3.5M_\odot \times 24.2 = m_p \times v_{\text{orb}}$$

so

$$m_p = \frac{3.5M_\odot \times 24.2}{v_{orb}} = 1.063 \times 10^{28} \text{ kg} = 5.6M_J$$

$\square$

Problem 11. Whether or not a diffraction-limited optical system is able to resolve two points as distinct can be determined by the Rayleigh criterion. β Pictoris b is one of the first exoplanets discovered using direct imaging. The star system is located 19.44 pc away, and β Pictoris b is located 9.2 AU from the host star. When viewing in infrared ($\lambda = 1650$ nm), what is the minimum telescope diameter that is able to resolve β Pictoris and its exoplanet under the Rayleigh criterion?

Solution: **B: 0.877 m**

The Rayleigh criterion states that two objects can be resolved if their angular separation θ exhibits the relation

$$\theta > \frac{1.22\lambda}{D}$$

where D is the telescope diameter, so

$$D > \frac{1.22\lambda}{\theta}$$

Under the small angle approximation, the angular separation between β Pictoris and its exoplanet is

$$\theta \approx \frac{a}{d} = \frac{9.2 \text{ AU}}{19.44 \text{ pc}} = 2.294 \times 10^{-6} \text{ rad}$$

where d is the distance from Earth to the system. Therefore,

$$D > \frac{1.22 \times 1650 \times 10^{-9}}{2.294 \times 10^{-6}} = 0.877 \text{ m}$$

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□

Problem 12. The celestial coordinates of the Orion Nebula are RA $05^{\mathrm{h}}35^{\mathrm{m}}$, dec $-05°23'$. Which of the following is closest to the time (local solar time) when the Orion Nebula would cross the meridian on the night of February 1st 2019? The date of the vernal equinox of 2019 is March 20th.

Solution: **A; 08:40 PM**

February 1st is 47 days before the vernal equinox, so the RA of the sun is

$$-\frac{47}{365} \times 24 = -3^{\mathrm{h}}05^{\mathrm{m}}$$

The Orion Nebula therefore crosses the meridian

$$5^{\mathrm{h}}35^{\mathrm{m}} + 3^{\mathrm{h}}05^{\mathrm{m}} = 8^{\mathrm{h}}45^{\mathrm{m}}$$

past noon, or 08:40 PM.

□

Problem 13. A yellow hypergiant located 1.04 kpc away has an apparent visual magnitude of 1.49 and a $B-V$ color excess of 0.29. Assuming R_V, the ratio of V-band extinction to $B-V$ color excess, is 3.1, determine the absolute visual magnitude of the star.

Solution: **A: -9.5**

$R_V = \frac{A_V}{E_{B-V)}}$, so

$$A_V = 3.1 \times 0.29 = 0.90$$

Using the distance modulus,

$$m_V - M_V = 5 \log d - 5 + A_V$$

so

$$M_V = 1.49 - 5 \log 1040 + 5 - 0.90 = -9.5$$

□

Problem 14. The pp chain is a primary energy generation mechanism in the Sun. Each run of the process $2H + e \to D + \nu$ releases 26.73 MeV of energy. Calculate the neutrino flux on the surface of Mars (in neutrinos per m^2), assuming that the pp chain is responsible for 100% of the Suns energy generation. (Mars is at a distance of 1.52 AU)

Solution: **C: $\mathbf{1.37 \times 10^{14}}$**

$$\text{Neutrino flux} = \frac{3.828 * 10^{26}}{(26.73 * 10^{6} * 1.602 * 10^{-19})} * \frac{1}{(4 * \pi * (1.52 * 1.498 * 10^{11})^2)}$$

$$= 1.37 \times 10^{14} \text{ neutrinos/m}^2$$

□

Problem 15. A relation between which of the following pairs of properties of Cepheids variables makes Cepheids variables, specifically, useful objects for determining stellar distances?

Solution: **B: Period and Luminosity**

Cepheid variables have a period-luminosity relation

□

Problem 16. Assuming that the Chandrasekhar Limit is 1.4 Solar masses, estimate the maximum average density (in kg/m^3) of a Chandrashekhar mass black hole.

Solution: **D: $\mathbf{9.4 \times 10^{18}}$**

$$\text{Density} = \frac{M}{(4/3 * \pi * (2GM/c^2)^3)} = \frac{3 * (3 * 10^8)^6}{(32 * \pi * (6.67 \times 10^{-11})^3 * (1.4 * 1.99 * 10^{30})^2)}$$

$$= 9.4 \times 10^{18} \text{ kg/m}^3$$

□

Problem 17. The Suns differential rotation can be estimated with the equation $\omega = X + Y sin^2(\phi) + Z sin^4(\phi)$, where ω is the angular velocity in degrees per day, ϕ is solar latitude, and X, Y, and Z are constants (equal to 15, -2.5, and -2 degrees per day respectively). Two sunspots are spotted along the same solar meridian, one at 0° and the other at 40°. Assuming that the sunspots do not disappear or change latitude and move with the same velocity as the surface of the sun, after how many days will the sunspots be aligned once again? Round your answer to the nearest day.

Solution: **C: 262**

ω for 0° is 15° per day and ω for 40° is 13.626° per day. The faster sunspot must travel exactly one rotation more than the slower sunspot for them to be aligned again, so

$$15t - 360 = 13.626t$$

Thus, we have

$$\boxed{t \approx 262 \text{ days}}$$

□

Problem 18. An observer generates a light curve of a binary system, and notices two different minima that repeat periodically (in an alternating fashion). The time between when the light curve reaches the first minima and the second minima is 285.7 days. In solar masses, estimate the total mass of the binary system if the two stellar bodies are separated by a mean distance of 4.1 AU.

Solution: **C: 28**

Period is $285.7 * 2/365$ years. Total mass in solar masses $= a^3/t^2 \approx 28$.

□

Problem 19. Eltanin, the brightest star in Draco, has the approximate coordinates RA: 17h 56m, Dec: $+51.5°$. Given that at the observers location, the latitude is $+50°$ and the local sidereal time is 14:00, how far above the horizon will Eltanin appear? Round your answer to the nearest degree.

Solution: **B: 54**

The LHA is the negative of the time until upper culmination, or $14 - 17.93 = -3.93$ hours, or -59°. Using spherical trig rules, we have

$$\sin(a) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(LHA) \approx 0.806$$

Taking inverse sine, we have approximately 54 degrees.

□

Problem 20. Stellar bodies located in the top left of a Hertzsprung-Russell diagram necessarily have which properties?

Solution: **B: Low absolute magnitude, High effective temperature**

More negative values of absolute magnitude and higher temperatures are characteristic of the top-left of the H-R diagram.

□

Problem 21. Which of the following correctly orders the following distance indicators from the smallest to largest scale?

Solution: **A: Stellar parallax, spectroscopic parallax, RR Lyrae variables, Hubble constant**

Stellar parallax (∼100 pc), spectroscopic parallax (∼10000 pc), RR Lyrae (∼1 Mpc), Hubble constant (∼13 billion ly)

□

Problem 22. As seen from Mars, what phase will Earth appear to be in at quadrature?

Solution: **C: Quarter**

The sun, Earth, and Mars form a right angle at quadrature, so half of Earth will appear to be lit (i.e. quarter).

□

Problem 23. Which of the following stars is almost always never visible to observers in the Northern hemisphere?

Solution: **D: Sigma Octantis**

For a star to almost always never be visible to observers in the Northern hemisphere, it would have to be close to the southern celestial pole (e.g. An observer at $< 5°$ North could see stars outside of the range 85-90° South). Sigma Octantis is widely accepted as the southern pole star.

□

Problem 24. Two amateur astronomers A and B living in Ecuador are standing on the Equator at the Galapagos Islands (Height 0m, Longitude 91° W) and Volcan Cayambe (Height 5790m, Longitude 78° W) respectively. What are the differences (in degrees) of the altitudes from the horizon and zenith distances of the Sun measured by these two astronomers on March 20, 2019 when it is local noon for observer B? Neglect refraction and give your answer to the nearest tenth of a degree.

Solution: **D: Difference in altitudes: 11.3, Difference in zenith distances: 13°**

20th March is the Spring Equinox and thus the Sun is on the celestial equator which passes through the zenith for both observers. When it is local noon for Observer B, the sun is at zenith. At the same time, the sun will appear (91-78)° or 13° away from zenith along the celestial equator towards West for Observer A. Thus, the difference in zenith distances is 13°. The altitude observed by Observer B must be corrected for height by adding the factor $\sqrt{\dfrac{2h}{R_E}}$ radians or 1.7° and thus the difference in altitudes must be (13-1.7)° or 11.3°.

□

Problem 25. The spectra of two stars A and B peak at wavelengths 500 nm and 250 nm respectively. What is the ratio of their luminosities if they form black holes with Schwarzschild radii in the ratio 8:1? Assume that their densities were uniform and identical before they collapsed to form a black holes and that they did not lose any mass while forming the black holes.

Solution: **C: 1:4**

The Schwarzschild radius is directly proportional to the mass of the star that forms the black hole. Thus, the ratio of masses is 8:1. Thus, the ratio of their radii is 2:1. Their surface temperatures are in the inverse ratio of their peak wavelengths (Wien's Law). Thus, the ratio of their temperatures is 1:2. Using the Stefan-Boltzmann Law, the ratio of their luminosities is 1:4.

□

Problem 26. Two stationary observers at a distance 100 AU from the sun observe transits of Mercury across the diameter of the Sun's disk when Mercury is at perihelion and aphelion respectively. Which of the following is closest to the ratio of the aphelion transit time to the perihelion transit time? You are given that the semi-major axis and eccentricity of Mercury's orbit are 0.387 AU and 0.21 respectively.

Solution: **B: 1.5:1**

The perihelion and aphelion distances of Mercury are 0.3057 AU and 0.4683 AU respectively. Thus, the speeds of Mercury at these positions are 38.69 km/s and 59.26 km/s respectively and these will be directly proportional to the angular speeds of Mercury as seen by the observers. Thus, the ratio of the times of transit will be 59.26:38.69 or approximately 1.5:1.

□

Problem 27. Find the total sum of the binary system of the star Capella, if semi-major axis between them is 0.85 AU, and period of 0.285 years.

Solution: **C: 7.6 solar masses**

□

Problem 28. The New Horizons spacecraft completed a flyby of 2014 MU69 on New Years day of this year. 2014 MU69 is a Kuiper Belt Object with a semi-major axis of 44.58 AU. Estimate the maximum temperature at the surface of 2014 MU69, in Kelvin, assuming the object has zero albedo.

Solution: **B: 58.9 Kelvin**

$T_{\text{irr}} = T_{\text{star}} \times \sqrt{\frac{R_{\text{star}}}{2A}} * 4^{1/4}$, where T_{star} is the temperature of the Sun (5770 K), R is the radius of the sun (6.95×10^8 m), and A is the distance from the object and star (44.58 AU).

This is because the expression WITHOUT the factor of $4^{1/4}$ gives the equilibrium temperature assuming that the energy is fully distributed across the entire planet. However, since we want the maximum temperature here, we assume that only the daytime side re-emits, which means that the surface area in question doesn't have the 4 factor (in the $4\pi r^2$), and that therefore T_{irr} requires an extra $4^{1/4}$ factor (see the following Wikipedia page for the full explanation: https://en.wikipedia.org/wiki/Planetary_equilibrium_temperature)

□

Problem 29. HD 209458b is an extrasolar gas giant planet with a radius of 1.38 Jupiter radii and a mass of 0.69 Jupiter masses (1 Jupiter radius = 6.99×10^7 m, 1 Jupiter mass = 1.90×10^{27} kg). Which of the following is closest to the pressure at the very center of HD 209458b, in bars?

Solution: **B:** 10^6 **bars**

Answer: see http://burro.case.edu/Academics/Astr221/SolarSys/hydrostat.html.
Full estimation:

$$P \sim \rho * g * R = \frac{M}{\frac{4}{3}\pi R^3)} \frac{GM}{R^2} R = \frac{GM^2}{\frac{4}{3}\pi R^4} = 3.16 \times 10^{11} Pa \sim 10^6 \text{ bars}$$

□

Problem 30. Imagine that our Sun was suddenly replaced by an M-dwarf with a mass half that of the Sun. If our Earth kept the same semi-major axis during this change, what would Earths new orbital period be around the M-dwarf?

Solution: **C: 1.414 years**

Keplers third law:

$$M \propto T^{-2} \longrightarrow T \propto M^{-1/2} \rightarrow (1/2)^{-1/2} = 1.414$$

□