# 2018 National Astronomy Olympiad (NAO)

## Instructions (Please Read Carefully)

(Start the exam after reading the instructions carefully and affirming that the contestant understood the instructions given.)

The top 5 eligible scorers on the NAO will be invited to represent USA at the next IOAA. In order to qualify for the national team, you must be a high school student with US citizenship or permanent residency.

This exam consists of 2 parts: Short Questions (5 points each) and Long Questions (15 points each). The maximum score is 100 points.

The test must be completed within 2.5 hours (150 minutes). The proctor should mark the start and end time of the exam on the front page.

Please solve each problem on a blank piece of paper and mark the number of the problem at the top of the page. The contestants full name in capital letters should appear at the top of each solution page. If the contestant uses scratch papers, those should be labeled with the contestants name as well and marked as ”scratch paper” at the top of the page. Scratch paper will not be graded. Partial credit will be available given that correct and legible work was displayed in the solution. This exam document, solution, and all the scratch paper used should be turned in to the proctor at the end of the exam.

This is a written exam. Contestants can only use a scientific calculator for this exam. A table of physical constants will be provided. Discussing the problems with other people is strictly prohibited in any way until the end of the examination period on April 6th. Receiving any external help during the exam is strictly prohibited. This means that the only allowed items are: a scientific calculator, the provided table of constants, a pencil (or pen), an eraser, blank sheets of papers, and the exam. No books, notes, laptops, mobile phones, or any other devices are allowed while taking the exam.

After reading the instructions, please make sure to sign at the bottom of this page, affirming that:

1. All work on this exam is mine.
2. I took this exam under a proctors supervision.
3. I did not receive any external aids besides the materials provided.
4. I am not allowed to discuss the test with others throughout the period of this examination.
5. Failure to follow these rules will lead to disqualification from the exam.

To be completed by the proctor:

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Full name (First, Last): ______________________________________________________________
Position (e.g. Physics Teacher at High School x): ___________________________________
Email address: _______________________________________________________________________
Student began exam at (hh:mm):         _________________
Student submitted the exam at (hh:mm): _________________
Signature:___________________________________________________ Date (mm/dd/yy):________
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To be completed by the student:

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Last name:  _________________________________
First name: _________________________________  Middle name: _________________________
Date of birth (mm/dd/yy): ___________________  Email address: _________________________
Signature:___________________________________________________ Date (mm/dd/yy):________
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# I: Short Questions

1. [5 pt] In order to detect an Earth-twin, we need significant advances in the precision of spectrographs to detect the periodic Doppler shift of nearby stars. Estimate the radial velocity semi-amplitude, in m/s, that a planet with the mass, radius, and semi-major axis of Earth would cause in the motion of a star with the mass of the Sun. Assume that the Earth-twin has zero eccentricity. Note that the mass of Earth is $5.97 \times 10^{24}$ kg, and the distance from the Earth to the Sun is $1.5 \times 10^{8}$ m.

2. [5 pt] Planet Nine is a hypothesized planet in the outer Solar System that may explain the clustering of orbital elements of distant trans-Neptunian objects. The hypothesized periapse of Planet Nine is 200 AU, and the apoapse is expected to be at approximately 1200 AU. What would the eccentricity of Planet Nine be? How does this eccentricity compare to that of the 8 major planets in the Solar System?

3. [5 pt] What is the main-sequence lifetime of a star with a mass of 0.1 solar masses, and a star with a mass of 10 Solar masses? Assume that stellar luminosity, $L \propto M^{3.5}$, where $M$ is stellar mass, and that the main-sequence lifetime of the Sun is 10 billion years.

4. [5 pt] The Very Large Array radio interferometer ($\lambda$=1 m) has maximum baseline of D = 36.4 km. How large will an optical telescope have to be to achieve a similar angular resolution in visible light ($\lambda$=5500 A°)?

5. [5 pt] An amateur astronomer observes the Moon with 20-cm telescope, and accomplishes 160x magnification with an eyepiece with focal length 10 mm. What is the f-number of the telescope?

6. [5 pt] The average person has 1.4 m$^2$ of skin. What is the energy per second radiated by the average person in the form of blackbody radiation? What is the peak wavelength of emitted radiation? Why cant we see it with our eyes?

7. [5 pt] On March 21st at true noon, length of the shadow of a vertical rod was equal to its height. On which geographic latitude did this happen?

8. [5 pt] In stars like the Sun, helium nuclei are formed by fusing hydrogen nuclei together in a process known as the proton-proton chain. One step of the proton-proton chain consists of a deuterium nucleus ($m_d$ = 2.01410 u) fusing together with a hydrogen nucleus ($m_H$ = 1.00783 u) to form a helium-3 nucleus ($m_{He}$ = 3.01603 u), where $u = 1.6605 \times 10^{-27}$ kg. How much energy is released during this fusion reaction?

9. [5 pt] Solar wind consists of protons that fly with the speed of 300 km/s and they fill the space of interplanetary matter around Earth with 10 particles/cm$^3$. With what force is this "wind" hitting the Moon? Recall that mass of a proton is $m_p = 1.610^{-24}$ g. Radius of the Moon is $R_m$=1737 km.

10. [5 pt] Mars orbits the Sun at an average distance of $2.28 \times 10^{11}$ m and has a radius of $3.39 \times 10^{6}$ m. The Sun has a luminosity of $3.828 \times 10^{26}$ W. How much solar energy falls on the surface of Mars each second? Ignore any effects of Mars' thin atmosphere.

11. [5 pt] When a gravitationally bound system (such as a galaxy) forms, it transitions from a just bound state ($E_{\rm kin} = |E_{\rm pot}|$) to a virialized state ($E_{\rm kin} = 0.5|E_{\rm pot}|$) and the excess binding energy has to be radiated away. Consider an idealized disk galaxy with an exactly flat rotation curve with a rotation speed of $v_{\rm circ} = 220$ km/s (you can neglect the kinetic energy in random motions). Its density profile cuts off abruptly at a radius of $R_{\rm max}$ = 50 kpc. Assume that it took 500 million years for this galaxy to collapse to its present state. What was its mean luminosity (in units of solar luminosity) due to the release of the binding energy during that period?

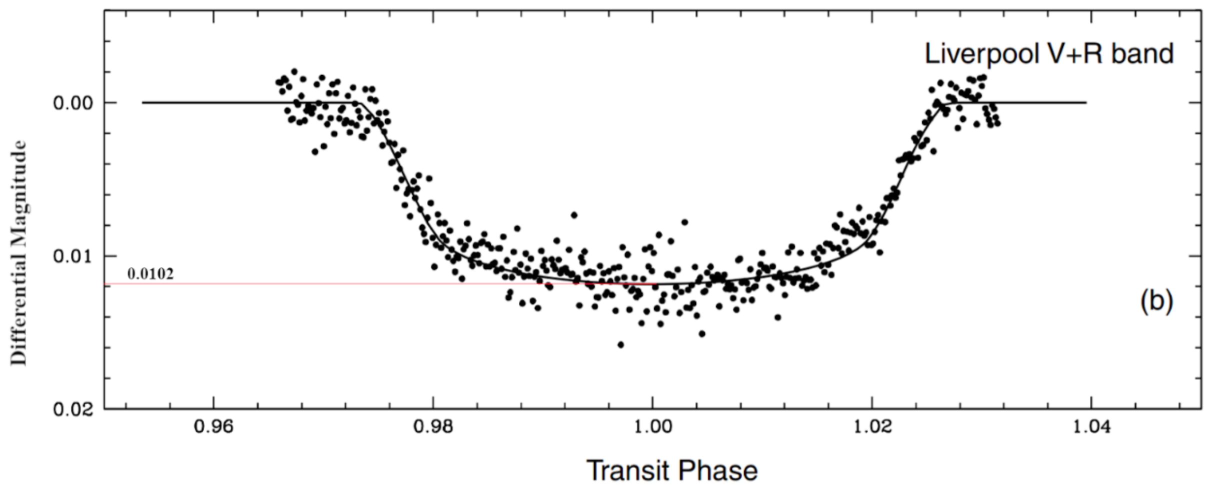
## II. Long Questions

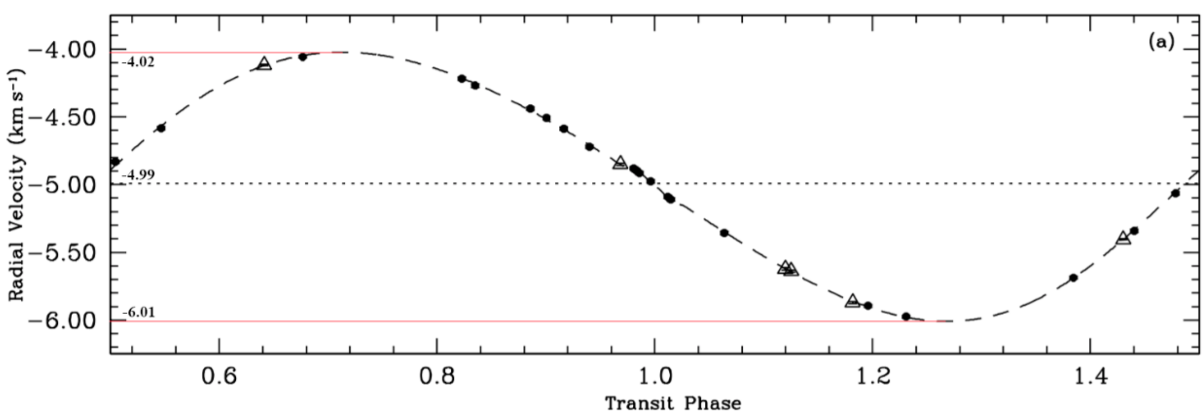
1. [15 pt] In 2008, while observing WASP-14, a main sequence star of mass 1.211 $M_{\odot}$ and radius 1.306 $R_{\odot}$, an exoplanet called WASP-14b was discovered via the transit method. Photometry as well as radial velocity data are shown in the figures. Transits occur once every 2.243753 days. The radial velocity of the center of mass of WASP-14 and its planet is -4.99 km/s. Fitting of the radial velocity curve indicates that the argument of periastron of the orbit of WASP-14b is 254.9°.

(a) [5 pt] Determine the length of the semi-major axis of the orbit of WASP-14b.

(b) [5 pt] Determine the density of WASP-14b.

(c) [5 pt] Determine the eccentricity of the orbit of WASP-14b.




2. [15 pt] The star Sualocin (RA: $20^{\mathrm{h}}\ 39.6^{\mathrm{m}}$, Dec: 15° 54.7', absolute magnitude: -0.4) is about 78 pc away from our solar system, and the star Rotanev (RA: $20^{\mathrm{h}}\ 37.5^{\mathrm{m}}$, Dec: 14° 35.7', absolute magnitude: 1.6) is about 31 pc away. An alien astronomer is on a planet with Earth's mass and radius orbiting Rotanev. The planet has a uniform albedo of 0.3.

(a) [4 pt] What is the angular distance between Sualocin and Rotanev?

(b) [3 pt] What is the distance between these stars in parsecs?

(c) [3 pt] On the alien's planet, what is the angular separation in the sky between Sualocin and our Sun?

(d) [5 pt] How much greater is the flux received by the planet from Sualocin than that received from our Sun?

3. [15 pt] Suppose that at some time in the very recent past all the hydrogen and helium (baryon density $\rho_b = 4.2 \times 10^{-31}$ g/cm$^3$, about 75% hydrogen (=1 baryon) by mass and 25% helium (=4 baryons) by mass) in the universe had been instantly fused into iron in stars, and the released energy thermalized into black body radiation. (Note that the binding energy per nucleon of $^{56}_{26}$Fe is 8.8 MeV and that of $^{4}_{2}$He is 7.1 MeV.)

(a) [5 pt] Calculate the current temperature of this black body radiation.

(b) [5 pt] At what wavelength would the black body spectrum peak, and what region of the electromagnetic spectrum would this be?

(c) [5 pt] The mean bolometric luminosity per unit volume emitted by stars in the universe today is about $3 \times 10^8 \ L_\odot/\mathrm{Mpc}^3$. How long would it takes stars at this rate to fuse all the hydrogen and helium in the universe? Compare to the present age of the universe.