Singapore Astronomy Olympiad 2023

Theory

Instructions

1. The theory portion of this Olympiad is worth a total of **82 marks**.
2. When asked to do so, check that you have **9** printed pages.
3. Write your **final answers** in the summary answer sheet, but write your workings clearly on the other answer sheets provided.
4. Submit both the summary answer sheet and all used answer sheets.
5. Fill in these details on each side of your answer sheet:
   - Year of competition
   - Your participant code
   - The page number – which should be continuous from 1 to N
   - The section of the paper, and the question number
6. Cross out all workings or answers you do not wish to be evaluated.
7. If you require assistance (e.g. to visit the restroom, enquire about an ambiguity or possible errata, etc.), please get the attention of the invigilators.

Competition Rules and Regulation

1. Only the use of scientific calculators is permitted. No graphing or programmable calculators are allowed.
2. Disruptive behaviour, cheating, collusion to cheat or any integrity-related offences are grounds for immediate disqualification.
3. You may opt to retain the question paper and constants sheet for personal use. Return all unused answer sheets to the Organising Team.

This page has been intentionally left blank

1 Multiple Choice Questions [20]

This section contains a total of **8** multiple choice questions. Write all responses in the summary answer sheet provided.

1.1 Hertzsprung-Russell [2]

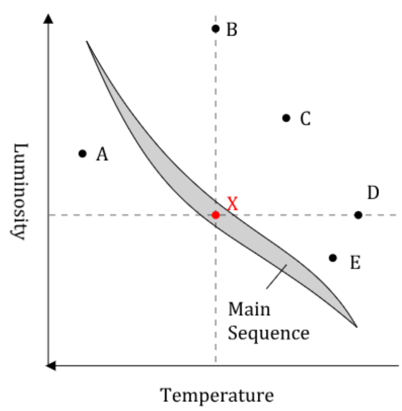


Figure 1: Hertzsprung-Russell Diagram *(Not drawn to scale)*

Stars X and Y started their evolution as main-sequence stars. Furthermore,

- Star Y started hydrogen fusion a substantial amount of time before Star X
- Star X and Star Y currently have similar radii

The current position of Star X are shown in the figure above. What is a possible position of Star Y?

1.2 Cosmological Conundrum [2]

An SAO 2023 participant calculates the distance d to a quasar with redshift $z = 4.72$ with the following equations:

$$v = cz \qquad v = H_0 d$$

Hence determining that this quasar is approximately 20.8 Gpc away. What is wrong with this calculation?

i) The quasar is traveling faster than the speed of light and does not physically exist

ii) The quasar's redshift is too high to use these equations

iii) Hubble's law only works for objects within the Hubble sphere

iv) The velocity term v refers to different velocities in each equation

A) (i) only

B) (ii) only

C) (ii) and (iii) only

D) (iii) and (iv) only

E) (ii), (iii) and (iv) only

1.3 Star Death [2]

Which of the following statements regarding the final phases of a star's evolution is **NOT** true?

A) A type Ia supernova typically occurs when a white dwarf accretes enough matter from a close companion.

B) Stars with a radiative core will most likely end their evolution as a white dwarf after ejecting their outer layers as a planetary nebula.

C) Massive stars ($< 10M_\odot$) may form a neutron star or black hole following a core-collapse supernova.

D) Neutron stars are supported against their own gravity by electron degeneracy pressure.

E) Low-mass stars ($0.3M_\odot$) eventually form white dwarfs after burning their entire hydrogen content to helium.

1.4 Rotation Curve [2]

An astronomer obtains an observed rotation curve for a galaxy through spectroscopic measurements of its velocity at various radii. He also predicted a theoretical rotation curve for the galaxy by using the light emitted to calculate the mass distribution with radius.

To his surprise, he found that the observed rotational velocity was consistently lower than the predicted velocity at all radii. What are some possible reasons for this discrepancy?

i) He was viewing the galaxy edge-on but did not correct for this when analyzing the spectroscopic data.

ii) The stars in the galaxy produced less light per unit mass than he had accounted for in his calculations for the theoretical model.

iii) The galaxy contains dark matter which is not detectable through this method.

A) (i) only

B) (iii) only

C) (ii) and (iii) only

D) (i), (ii), and (iii)

E) None of the above

1.5 p-p Chain [2]

Nuclear fusion is the primary process powering main-sequence stars, whereby multiple atomic nuclei combine to form different nuclei and subatomic particles. Given below are the equations for the proton-proton III chain:

$$\begin{array}{lcllcllcl}
{}^{1}\mathrm{H} & + & {}^{1}\mathrm{H} & \rightarrow & {}^{2}\mathrm{H} & + & e^{+} & + & \nu \\
{}^{2}\mathrm{H} & + & {}^{1}\mathrm{H} & \rightarrow & {}^{3}\mathrm{He} & + & \gamma & & \\
{}^{3}\mathrm{He} & + & {}^{4}\mathrm{He} & \rightarrow & {}^{7}\mathrm{Be} & + & \gamma & & \\
{}^{7}\mathrm{Be} & + & {}^{1}\mathrm{H} & \rightarrow & {}^{8}\mathrm{B} & + & \gamma & & \\
 & & {}^{8}\mathrm{B} & \rightarrow & {}^{8}\mathrm{Be} & + & e^{+} & + & \nu \\
 & & {}^{8}\mathrm{Be} & \rightarrow & {}^{4}\mathrm{He} & + & {}^{4}\mathrm{He} & &
\end{array}$$

Where e^{+} denotes a positron, ν denotes a neutrino, and γ denotes a gamma ray. What is the total energy released for the overall reaction?

The following table contains the masses of the isotopes (1 amu = 931.5 MeV/c^2).

Isotope	**Mass (amu)**
^{1}H	1.0078250
^{2}H	2.0141017
^{3}He	3.0160293
^{4}He	4.0026032
^{7}Be	7.0169287
^{8}Be	8.0053051
^{8}B	8.0246073

A) 26.73 MeV

B) 1.59 MeV

C) 23.52 MeV

D) 18.21 MeV

1.6 Thorne-Zytkow Object [2]

A Thorne–Żytkow object is a hypothetical type of star where a neutron star is contained by a red giant or red supergiant. It is formed by a collision between the neutron star and the giant.

Which of the following is **NOT** characteristic of a Thorne–Żytkow object:

A) Emits gravitational waves immediately after formation

B) Exhibits multiple spectral absorption lines

C) Low variability in luminosity over cosmological timescales

D) Possible collapse into a black hole

1.7 Rocky Bodies [4]

A socially anxious guitarist goes to her local astronomy club to observe the recent opposition of Mars on 8 December 2022. She noted that the moon occulted (passed in front of) Mars this night.

What will be the phase of the moon during the next opposition of Mars? Assume all orbits are circular, and take the synodic period of the moon to be 29.53 days.

A) Waning Gibbous

B) Waning Crescent

C) Waxing Crescent

D) Waxing Gibbous

E) Full Moon

1.8 Exoplanet Transit [4]

Exoplanets can be detected via transit photometry, where a planet crosses in front of its parent star (or *transits*), causing the apparent brightness of the star to temporarily decrease. A Sun-like star is observed to dim to 99.56% of its typical brightness due to a transiting planet.

Which of the following, to the nearest order of magnitude, is a plausible mass of the planet? Assume uniform stellar brightness and a rocky, atmosphere-free planet.

A) 10^6 kg

B) 10^{30} kg

C) 10^{24} kg

D) 10^{21} kg

E) 10^{13} kg

2 Short Questions [30]

This section contains a total of 3 short-length questions. Answer **ALL** questions and write all responses on the answer sheets provided.

2.1 Venus Observation [10]

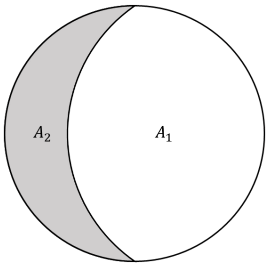
The discovery of the phases of Venus by Galileo was one of the factors contributing to the downfall of the geocentric worldview. However, even the outer planets exhibit phases as observed from Earth.

Using values from the constants sheet,

(a) Find the maximum value of the Sun-Jupiter-Earth angle ∠SJE, assuming circular orbits **[3]**

(b) The phase of Jupiter is at its minimum for the value of ∠SJE found in part (a). Hence, find the minimum phase of Jupiter. **[3]**

Note: *The phase is the percentage of the* ***planetary disk*** *that is illuminated by the Sun when observed from Earth. In the following diagram, the phase of the disk is given by* $\frac{A_1}{A_1+A_2}$ *where* A_1 *and* A_2 *refer to the illuminated and unilluminated region of the disk respectively.*



The moon exhibits a phenomenon known as "Earthshine" where light from Earth reflects off the unilluminated portion of the moon and back to Earth.

(c) Calculate the (very small!) flux density (as observed from Earth) from the unilluminated portion of Jupiter due to such reflection. You may assume this portion of Jupiter to behave like a Lambertian reflector (i.e. it reflects diffusely with an equal amount of light reflected in all directions), and you may make the simplifying assumption that the Earth is a blackbody. **[4]**

2.2 Auriga's Binary [10]

Menkalinan (β Aur) is a spectroscopic binary located $d_0 = 24.9$pc from Earth. The binary system has orbital period $T = 3.9600$ days with apparent magnitude varying from $m = 1.900$ to $m = 1.980$.

Spectroscopic observations on the Hα line ($\lambda_0 = 656.28$nm) revealed a splitting into two lines with wavelengths $\lambda_a = 656.08$nm and $\lambda_b = 656.55$nm at their largest separation.

(a) Assume the orbital plane of the binary is edge-on and both stars have a circular orbit, and ignore relativistic effects and the size of the stars. Find:

i) The sum of the components' orbital velocities relative to the system's center of mass **[2]**

ii) The distance between both components **[2]**

iii) The total mass of the system **[2]**

If there was no answer obtained for the components' separation in part **(a)(ii)**, you may take its value as 0.0800 AU for subsequent parts.

(b) Determine with relevant calculations, if an amateur astronomer can visually distinguish β Aur Aa and Ab at $\lambda = 550$nm using a telescope with diameter of its aperture $D = 15$cm. **[2]**

Menkalinan will become one of the brightest stars in the sky in about 900,000 years when it reaches its nearest approach of $d' = 28.5$ly.

(c) Calculate its brightest possible apparent magnitude as observed from Earth at this time. Assume the luminosity and light curve of the system to be unchanged. **[2]**

2.3 Cassegrain Telescope [10]

Consider a Schmidt-Cassegrain reflector with an effective focal length of 1200mm and a physical length of 30cm.

(a) Suppose we place a sensor with dimensions 24mm $\times$ 16mm at the focal plane. What is the longest time a star with declination $\delta = 0°$ can stay within the sensor? **[4]**

The telescope is modified to have the distance between mirrors changed to 265mm. The focal length of the primary mirror is 300mm and that of the secondary mirror is -50mm.

(b) Calculate the effective focal length of the modified telescope. **[6]**

If necessary, you are given the thin lens equation: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

3 Medium Questions [32]

This section contains a total of 2 medium-length questions. Answer **ALL** questions and write all responses on the answer sheets provided.

3.1 Sunsets and Aliens [16]

To celebrate the summer solstice, an observer decides to watch the sunset from the 6300m summit of Chimborazo ($\phi = 1.47°\text{S}$, $\lambda = 78.8°\text{W}$). At this particular moment, she sees Venus on the meridian ($\text{RA} = 9^{\text{h}}5^{\text{m}}33.34^{\text{s}}$, $\delta = 18°6'9''$).

(a) Show that the sunset equation is given by: **[4]**

$$\cos \text{HA}_\odot = \frac{\sin a - \sin\phi \sin\delta_\odot}{\cos\phi\cos\delta_\odot}$$

Where $\text{RA}_\odot$ and $\delta_\odot$ denotes the right ascension and declination of the Sun respectively, and a denotes the altitude of the Sun relative to an observer at latitude ϕ.

(b) Hence or otherwise, calculate the following, considering all relevant effects and taking the effect of atmospheric refraction to be $\theta_{\text{atm}} = 34'$:

i) How long more must she wait before sunset (i.e. the entire solar disk disappears below the horizon) **[7]**

ii) The time elapsed between the solar disk first touching the horizon and sunset **[2]**

Unfortunately, her watch and all time-keeping devices have coincidentally stopped. Fortunately, she remembers receiving a warning of an alien invasion from a friend in Casablanca ($\phi = 33.57°\text{N}$, $\lambda = 7.59°\text{W}$) earlier that day. Provided as proof was a photo of a supposed UFO seen directly above cardinal South, timestamped 4:40pm (GMT+1). The UFO was, in fact, Venus.

(c) What time is sunset, according to her watch (following GMT-5)? **[3]**

3.2 Modelling the Universe [16]

The Friedmann–Lemaître–Robertson–Walker (FLRW) metric describes a homogeneous and isotropic model of the universe. Such a universe may either be expanding or contracting, depending on the evolution of the scale factor $a(t)$. The scale factor in the present day $a(t_{\text{now}}) = a_0 = 1$.

The Friedmann equation can be obtained from the FLRW metric, and in its simplified form, is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{r^2}$$

Here, H is the Hubble parameter, $\dot{a}$ is the rate of change of a, and k is the curvature parameter. In our simplified derivation, we shall assume the Universe is flat and so the curvature term vanishes ($k = 0$).

(a) Consider a test mass at a distance r from the center of a spherical universe. By applying energy considerations, derive the Friedmann equation for a flat universe. **[4]**

(b) Measurements from the Wilkinson Microwave Anisotropy Probe (WMAP) have shown that the universe is flat (to within a 0.4% margin of error). Hence, approximate the density of our Universe. **[2]**

(c) A matter-dominated universe is a universe in which all the energy density is in the form of matter. In such a universe, $H \propto a^n$. Determine n. Likewise, determine n for a radiation-dominated universe. **[5]**

(d) At the present moment, the universe is dark-energy dominated. In this case, $a \propto e^{kt}$. By considering a small Δt and given that $e^x \approx 1 + x$ for small x, derive k in terms of H. **[2]**

(e) From the present value of H, deduce the *Hubble time*. Why might this be different from the age of the universe? **[3]**