Singapore Astronomy Olympiad 2023

Data Analysis

Instructions

1. The data analysis portion of this Olympiad is worth a total of **43 marks**.
2. When asked to do so, check that you have **8 printed pages**.
3. Submit your answer sheets and all used graph paper together.
4. Fill in these details on each side of your answer sheet and graph paper:
   - Year of competition
   - Your participant code
   - The page number – which should be continuous from 1 to N
   - The section of the paper, and the question number
5. Cross out all workings or answers you do not wish to be evaluated.
6. If you require assistance (e.g. to visit the restroom, enquire about an ambiguity or possible errata, etc.), please get the attention of the invigilators.

Competition Rules and Regulation

1. Only the use of scientific calculators is permitted. No graphing or programmable calculators are allowed.
2. Disruptive behaviour, cheating, collusion to cheat or any integrity-related offences are grounds for immediate disqualification.
3. You may opt to retain the question paper and constants sheet for personal use. Return all unused answer sheets to the Organising Team.

1 Data Analysis [43]

This section contains one data analysis question. Answer **ALL** questions and write all responses on the answer sheets provided.

1.1 Extinction Model [37]

Extinction is the absorption and scattering of electromagnetic radiation by dust and gas between an emitting astronomical object and the observer. Typically in our observations of astronomical objects, we would have to contend with extinction from the Interstellar Medium (ISM).

The ISM is composed of gas and dust and plays a crucial role in the formation and evolution of stars and galaxies. Notably, understanding how to model the ISM and its extinction effects allows us to better understand the formation and evolution of stars and galaxies while also aiding us in calibrating our observations of astronomical objects.

At various galactic latitudes, the ISM varies from a combination of gravitational, radiative and dynamic processes. At high galactic latitudes, the ISM is relatively diffuse and has a low density of gas and dust, while at low galactic latitudes, the ISM is much denser and more complex.

Reddening is the phenomenon associated with extinction where the spectrum of electromagnetic radiation from a radiation source changes characteristics from that which the object initially emitted. It equates to the colour excess ($E_{\lambda-V}$) defined as:

$$\begin{aligned} E_{\lambda-V} &= A_\lambda - A_V \\ &= (\lambda - V)_{\text{observed}} - (\lambda - V)_{\text{intrinsic}} \end{aligned}$$

Where

- A_X refers to the amount of extinction that occurs in the X-band in units of magnitude
- $(\lambda - V)$ refers to the difference in magnitude between two passbands, with λ representing an arbitrary band. For example, $(B - V)$ would refer to the difference between the B- and V-band

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In this question, we will model extinction from diffuse regions of ISM through various parameters, then use this model to estimate the distance to a nearby galaxy.

The UBVRIJHK photometric system will be used for this question, with each band's effective wavelength given in Table 1 alongside photometric data of several G and K-type giant stars.

(a) From Table 1, select **2** stars suitable for our model. They should be affected by extinction from diffuse regions of ISM. **[1]**

(b) Using the data from Table 1 and Table 2, derive a table of the colour excess with respect to the V-band ($E_{\lambda-V}$) for the 2 selected stars in all available UBVRIJHK-bands. **[6]**

(c) Given that in the B and V bands,

$$R_\lambda - R_V = \frac{E_{\lambda-V}}{E_{B-V}}$$

From the data obtained from the 2 stars, plot $R_\lambda - R_V$ against the inverse of λ-band's effective wavelength for all available UBVRIJHK-bands for each of the 2 selected stars. **[18]**

(d) From the 2 plots, estimate their corresponding R_V values in the B and V bands using a best-fit line. **[5]**

We define the total-to-selective extinction ratio for the X-band in the Y and Z bands to be

$$R_X(Y-Z) = \frac{A_X}{E_{Y-Z}}$$

(e) Show that the total-to-selective extinction ratio for the λ-band in the V and I bands is given by the expression **[2]**

$$R_\lambda(V-I) = \frac{E_{\lambda-V} + A_V}{E_{I-V}}$$

(f) Using the above expression and the 2 plots or otherwise, estimate their corresponding R_V values in the V and I bands for each of the 2 selected stars.

Then, with the mean R_V value, express A_V in terms of E_{V-I}. **[5]**

1.2 Distance Measures [6]

NGC 4535 and NGC 4725 are two galaxies close to the North Galactic Pole. The galactic planes of both galaxies are known to be inclined by approximately 43° to 46° to the line of sight from Earth. From photometric analysis of Cepheid Variables in each of the two galaxies, we obtain the mean apparent distance moduli of the Cepheid Variables in both galaxies.

Galaxy	μ_V	μ_I
NGC 4535	31.30	31.19
NGC 4725	31.00	30.80

(g) Using the derived expression for A_V from the earlier parts, estimate the distance to both galaxies. **[6]**

Table 1: UBVRIJHK-band photometric data of 10 G and K-type giant stars

S/N	Name	Parent Constellation	MK Classification	Parallax (mas)	Apparent Magnitude at Specific Bands							
					U (365 nm)	B (445 nm)	V (551 nm)	R (658 nm)	I (806 nm)	J (1220 nm)	H (1630 nm)	K (2190 nm)
1	HP 60197	Pup	K3III	1.03	11.52	9.50	7.76	-	-	4.98	4.21	4.04
2	HD 63323	Pup	K5III	1.42	10.10	8.13	6.40	-	-	3.04	2.21	2.01
3	HD 62576	Pup	K5III	4.14	8.16	6.22	4.59	3.29	2.32	1.75	0.93	0.75
4	HD 111743	Com	G9III	1.67	11.58	10.73	9.71	-	-	7.91	7.38	7.25
5	HD 157354	Ara	K3III	1.76	-	9.63	8.04	-	-	5.23	4.25	4.14
6	HD 162822	Sco	K1III	1.86	-	8.87	7.55	-	-	5.24	4.67	4.46
7	HD 171662	Sgr	K3/4III	1.88	-	8.91	7.02	-	-	3.73	2.78	2.53
8	HD 193092	Cyg	K3IIIa	1.72	-	6.87	5.24	-	-	2.70	1.84	1.56
9	BD+27 2174	Com	K0III	2.48	11.60	10.74	9.68	8.93	8.45	7.89	7.38	7.31
10	BD+27 2183	Com	G8III	1.79	-	11.58	10.55	-	-	8.73	8.21	8.01

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Table 2: Intrinsic colours of G and K-type giant stars in visible and infrared bands

MK Classification	(mag)									
	$(B-V)_0$	$(U-B)_0$	$(V-R)_0$	$(V-I)_0$	$(V-J)_0$	$(V-H)_0$	$(V-K)_0$	$(V-L)_0$	$(V-M)_0$	$(V-N)_0$
G8	0.95	0.68	0.55	0.95	1.11	1.61	1.77	1.79	1.73	1.75
G9	0.98	0.76	0.57	0.99	1.17	1.66	1.85	1.88	1.82	1.83
K0	1.01	0.84	0.6	1.03	1.23	1.72	1.94	1.97	1.9	1.92
K1	1.09	1	0.66	1.12	1.37	1.87	2.12	2.17	2.09	2.11
K2	1.16	1.14	0.74	1.23	1.56	2.08	2.36	2.44	2.35	2.37
K3	1.26	1.33	0.86	1.39	1.84	2.4	2.69	2.82	2.7	2.73
K4	1.43	1.69	0.96	1.61	2.16	2.77	3.05	3.22	3.08	3.02
K5	1.51	1.83	0.99	1.67	2.25	2.87	3.14	3.33	3.18	3.21

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