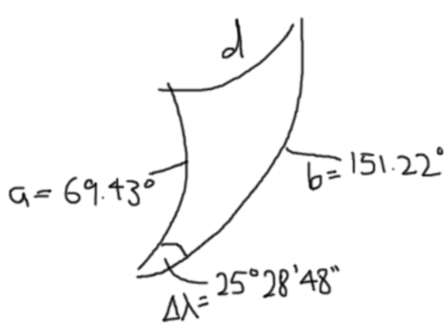
SAO 2022 Marking Scheme

MCQ

Q1.1.1	C
Q1.1.2	A
Q1.2	A
Q1.3	A
Q1.4	C
Q1.5	B

Short Theory (full ECF applied)

<table>
<tr>
<td>Q1a</td>
<td>
Time taken : 7h 34m
- No partial credit for this
Correct spherical triangle drawing/listing angles
- [half credit] If right angle triangle with sides lat/lon diff
- Alternatively, diagram showing wave passing through Earth's crust/listing distances
Cosine rule application
- Full credit if applied correctly regardless of triangle sides
- Alternatively, correctly use coordinates to find distance (8553km)
- [0] (Lat. dist)^2 + (Lon. dist)^2


$$\cos d = \cos a \cos b + \sin a \sin b \cos \Delta\lambda \rightarrow \; d = 131°54'8''$$
$$D = \frac{2\pi R_E}{360°} \times d = \sim 9375 \text{ km}$$
</td>
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</table>

	[half credit for visible effort] - Assume spherical Earth - Pressure wave travels at surface level (or they give the presumed height) **~344.1m/s** (~314.0m/s); +-2m/s
Q1b	Conversion to LMT 175.38deg * 15h/deg -> **11.692h** (civil time is **1.308h ahead**) LMT = **16h 7m 31.2s** Calculate declination of Sun - Wrong attempt to interpolate Sun's dec **25/26 days** from Winter Solstice (21/22 Dec 22) $$\delta_\odot = -23°26' \cos\left(\frac{\text{Days since Winter Equinox}}{365} \times 360°\right)$$ $$\rightarrow \delta_\odot = -21°17'47.86'' \;/\; -21°7'30.68''$$ HA calculation (**-0.5m: not using -34'**) $$\sin(-34') = \sin\delta \sin\phi + \cos\delta \cos\phi \cos HA$$ HA = **5h 28m 58.6s to 5h 29m 16.35s** HA relation to LMT Sunset at LMT **17h 28m 58.6s - 17h 29m 16.35s** Correct answer Time till sunset = **1h 21m (Accept: 1h 19m - 1h 23m)** - Do not accept if difference is due to not accounting for refraction
Q2a	Vacuum index = 1 Critical power is **6.407*10^23 W** Correct answer
Q2b	$\log(\frac{6.408*10^{23}}{3*10^{15}}) * 8 = 66.64 years$ or any other reasonable working Any number between **64-72 years**
Q2c	$Beam\ Energy = 10^{-11}s * 6.408 * 10^{23}W = 6.408 * 10^{12}J$ Photon Momentum (E=pc) / Fully Reflective (factor of 2) $2E = pc\ (fully\ reflective),\ p = 2E/c = 2\,(6.408 * 10^{12})/(3 * 10^{8}) = 4.272 * 10^{4}$ Rearrange equation $v = \sqrt{\frac{p^2}{\frac{p^2}{c^2}+m^2}} = 8.541 * 10^{6} m/s$ Correct Answer $t = d/v = 153 years$
Q3a	$2A\epsilon\sigma T^4 = L\frac{A\epsilon}{4\pi r^2}$; Factor of 2 for only one side absorbing solar flux

	Rearranging equation $r = \sqrt{\frac{L}{8\pi\sigma T^4}}$ Correct Answer $9.266 * 10^{11} m\ or\ 6.194 AU$
Q3b	$Proton\ Flow\ Rate = 5 * 10^{14} * \frac{1}{6.194^2} = 1.303 * 10^{13}$ $Deflection\ Area = \pi r^2 = 1.257 * 10^9$ $F = Deflection\ Area * Flow\ Rate * velocity * proton\ mass * 1s$ Correct Answer **13.70N**
Q3c	Rearranging equation $\frac{GMm}{r^2} = F,\ m = \frac{Fr^2}{GM}$ Correct Answer **88663kg**
Q4a	Redshift: $z = \frac{\Delta\lambda}{\lambda}$ Hence, $z = \frac{1.5}{21} = 0.0714$
Q4b	Apparent luminosity = 1.7×10^{-8} that of vega: Apparent magnitude = $-2.5 log(1.7 \times 10^{-8}) = 19.42$ Sun's apparent magnitude = -26.74 Hence, flux of supernova = 3.429×10^{-19} L☉ Luminosity distance = 1.3×10^{14} m = 2.057×10^{9} Ly Hence, comoving distance => $d_l = (1 + z)d_c$ => $d_c = 1.920 \times 10^9$ ly = **589 MPc**
Q4c	Recession velocity: $v = zc = 0.0714c$ Hubble's constant = $\frac{v}{d} = 1.179 \times 10^{-18} s^{-1}$ = 36.4 km/s/MPc Hubble time = 26 billion years
Q5a	Average frequency = 1.355 GHz Average photon energy = hv = 8.980×10^{-25} J Flux on telescope = $\pi r^2 f = 1.470 \times 10^{-18}$ W Photon count per second = 1.470×10^{-18} W ÷ $8.980 \times 10^{-25} J$ = 1.636× $10^6 s^{-1}$
Q5b	Resolution = $\frac{\lambda}{D}$ (0 if coefficient of 1.22 used) Resolution = $2.56 \times 10^{-7} rad = 0.0527''$ (with coefficient of 1.22, ans is 0.0643)

Q5c	rearranging, for a n-sigma detection: $t_i = (\Delta v)^{-1}(\frac{2k_B T_{sys}}{NA\sigma})^2$, where $\sigma = \frac{1}{40}(2.5 \times 10^{-29})\ W/m^2 Hz$, and N is the number of radio antennas Substituting, minimum integration time = **4.3 hours**
Q5d	As can be seen from the sunset equation, maximum time object can be seen by the telescope is 12 hours. Substituting, $\sigma = 3.75 \times 10^{-31}\ W/m^2 Hz$ Hence, minimum spectral flux density is $\sigma = 1.142 \times 10^{-29}\ W/m^2 Hz = 1.123\ mJy$

Medium Theory

a	At R_s(stromgren radius), the number of H-atoms undergoing photoionisation and the rate of recombination is balanced. As each H-atom ionises to one proton and one electron, $n_p = n_e = n_h$ $R_{recomb} = n_p n_i \alpha(T_{\text{HII}})(\frac{4\pi R_s{}^3}{3}) = n_h{}^2 \alpha(T_{\text{HII}})(\frac{4\pi R_s{}^3}{3})$ $R_{ionisation} = 10^{49} = Q$ Hence, $R_s = \sqrt[3]{\frac{3Q}{4\pi\alpha(T_{\text{HII}})n_h{}^2}}$ Solving, R_s = **14.13 ly**
b	For an estimation of time, assuming recombination doesn't take place, simply divide the total number of H-atoms within R_s by the rate of ionising photons: T = $\frac{4\pi n_h (R_s)^3}{3Q}$ Calculating, T = 10^{11}s = **3171 yr**
c	As 40% of recombinations will result in further ionisation, $R_{s,new}^3 = (1 + 0.4 + 0.4^2 + \ldots)R_s^3$

	$R^3_{s,new} = \frac{5}{3}R^3_s$ $R_{s,new}$ = **16.75 Ly**
d	With an angular radius of 1.5', The bubble nebula has an angular area ~ 7 $arcsec^2$ Hence, m = M - 2.5 log (7) = **12.83 mag** The bubble nebula has apparent magnitude of 12.83 mag as seen from Earth
e	Nebula's physical radius = $d\theta$ = 4.84 Ly = We can model the nebula as thin shells, of thickness \delta r at a certain distance r from the centre. Volume of each shell = $4\pi r^2 \delta r$ Suppose each unit volume gives off the same amount of flux. Hence, the specific magnitude of each unit volume is: $M_{specific} = m + 2.5log(\frac{4\pi R^3}{3}) - 5log(\frac{3400}{10}) = 126.7$ Magnitude of each shell = $M_{specific} - 2.5log(4\pi r^2 \delta r) + 5log(\frac{R}{32.6})$ $= M_{specific} - 2.5log(4\pi \delta r) - 5log(10Pc)$ Hence, magnitude at the centre can be calculated, replacing \delta r with R: $M_{centre} = M_{specific} - 2.5log(4\pi R) - 5log(10Pc) = -5.15mag$ Hence, the difference is **~18 mag**.

Long Theory

a	Diagram or anything showing distance between L2 and Earth/Sun - Let distance between L2 and Sun (resp. Earth) be a (resp. x) Correct angular velocity (based on period of Earth) $$\omega = \sqrt{\frac{GM_1}{a^3}}$$ Equating gravitational to centripetal forces

	$$\frac{GM_1}{(a+x)^2}+\frac{GM_2}{x^2}=(a+x)\omega^2$$ $$=\frac{GM_1}{a^3}(a+x)$$ $$\rightarrow \frac{1}{(a+x)^2}+\frac{M_2}{M_1}\frac{1}{x^2}=\frac{1}{a^3}(a+x)$$ Using L2 distance << a $$(a+x)^{-2}=a^{-2}\left(1+\frac{x}{a}\right)^{-2}$$ $$\approx a^{-2}\left(1-\frac{2x}{a}\right)$$ Some amount of basic algebra later: Final result $$R_{L,2}\approx a\sqrt[3]{\frac{M_2}{3M_1}}$$ Substituting values **1.496 * 10^9 m**
b	Connects both Roche lobes OR Edge of Roche lobe is at L1 Any mention of zero-velocity surface/radial velocities/bound to around primary AND secondary
c	Realize that if R_{L,1} is a function of q, then R_{L,2} is a function of q^-1; if they define a new q that's ok too Correct expression $$\frac{R_{L,1}}{R_{L,2}}\approx\frac{q^{0.33}}{(1+q)^{0.2}}\cdot\frac{(1+q^{-1})^{0.2}}{q^{-0.33}}$$ $$=q^{0.66}\cdot\left(\frac{1+q^{-1}}{1+q}\right)^{0.2}$$ $$=q^{0.46}$$
d	K3L in terms of M_a and M_d Express masses in terms of \rho and q Application of Eggleton's formula

	Final result $$\begin{aligned} T^2 &= \frac{4\pi^2 a^3}{G(M_d + M_a)} \\ &= \frac{4\pi^2 a^3}{GM_d(1+q^{-1})} \\ &= \frac{4\pi^2 a^3}{G(1+q^{-1})}\frac{3}{4\pi\bar{\rho}R_L^3} \\ &= \frac{3\pi}{G\bar{\rho}(1+q^{-1})}\left(\frac{a}{R_L}\right)^3 \\ &= \frac{3\pi}{G\bar{\rho}(1+q^{-1})}\frac{(1+q)^{0.66}}{0.44^3 q^{0.99}} \\ &\approx \frac{3\pi}{0.44^3 G\bar{\rho}}(1+q)^{-0.4} \end{aligned}$$ $$T \approx \sqrt{\frac{10.5}{G\bar{\rho}}}\frac{1}{(1+q)^{0.2}}$$
ei	COAM: J\dot = 0 Setting e=0 because circularized or e\dot = 0 due to COAM
eii	Application of log derivative to AM expression COMass: setting M_a\dot = -M_d\dot COMass: M_d\dot + M_a\dot = 0 Using part (i) to simplify Correct expression $$\frac{\dot{a}}{a} = (2q-2)\frac{\dot{M}_d}{M_d}$$
f	Application of log derivative to Eggleton's formula COMass: M_d\dot + M_a\dot = 0 Correct values for a and b $$R_L \approx \frac{0.44 M_d^{0.33}}{M_a^{0.13}(M_d+M_a)^{0.2}}a$$ $$\frac{\dot{R}_L}{R_L} \approx 0.33\frac{\dot{M}_d}{M_d} - 0.13\frac{\dot{M}_a}{M_a} - 0.2\frac{\dot{M}_d+\dot{M}_a}{M_d+M_a} + \frac{\dot{a}}{a}$$ $$\begin{aligned} \frac{\dot{R}_L}{R_L} &\approx 0.33\frac{\dot{M}_d}{M_d} + 0.13\frac{\dot{M}_d}{M_a} + \frac{\dot{a}}{a} \\ &= 0.33\frac{\dot{M}_d}{M_d} + 0.13q\frac{\dot{M}_d}{M_d} + (2q-2)\frac{\dot{M}_d}{M_d} \\ &= \underbrace{(2.13q - 1.67)}_{\zeta_L}\frac{\dot{M}_d}{M_d} \end{aligned}$$

g	Use of part (d) Density of donor $$\bar{\rho} = \frac{3\pi}{0.44^3 G T^2} = \sim 235 \text{ kg/m}^3$$ - **MS star**; (Sun is ~1410 kg/m^3) - Density is too low to be white dwarf (10^9 kg/m^3) or neutron star (10^17 kg/m^3) - Giant stars are usually lower density; some have densities <10 kg/m^3 but without extra information, MS star is a safer guess

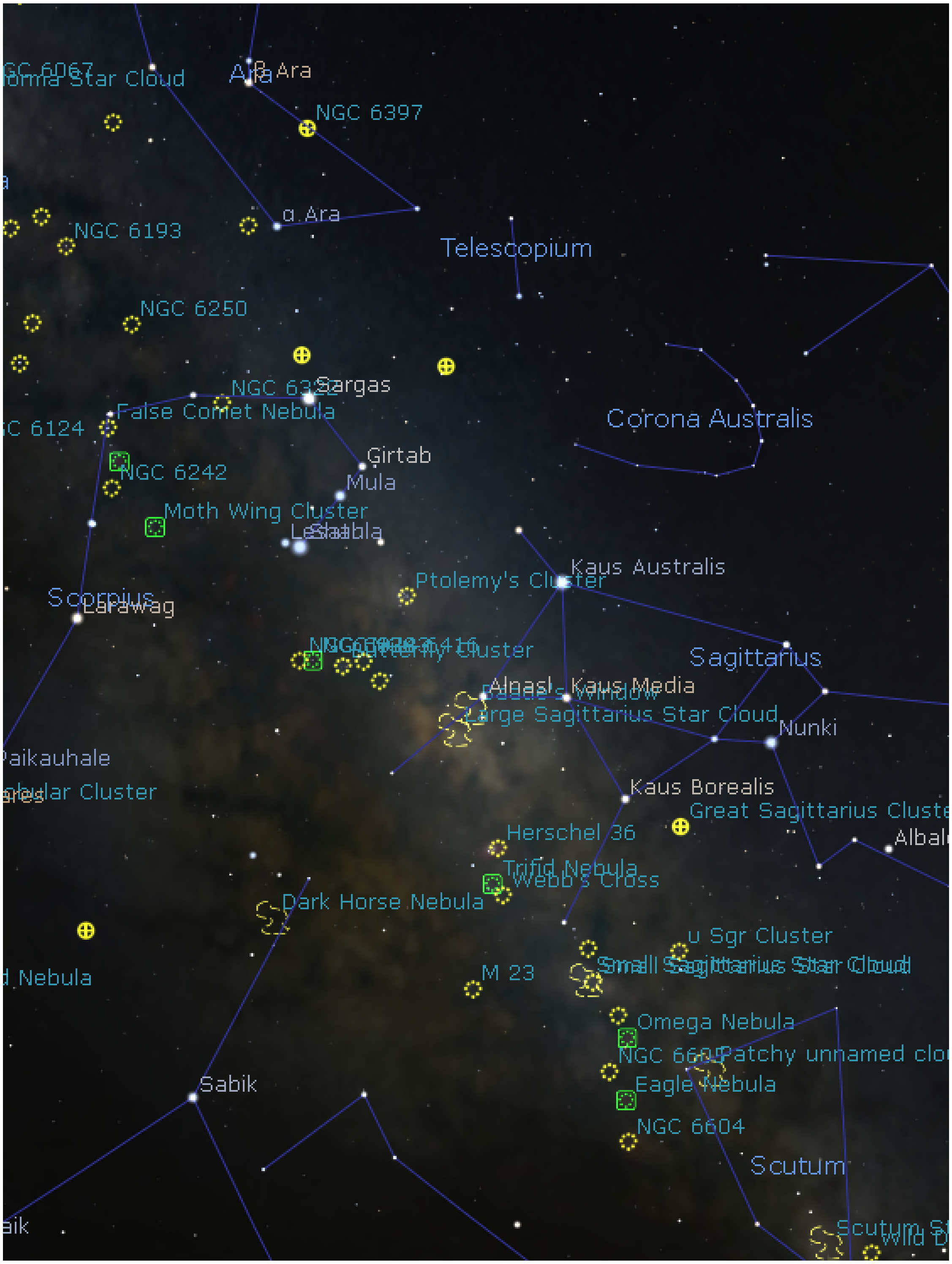
Practical MCQ

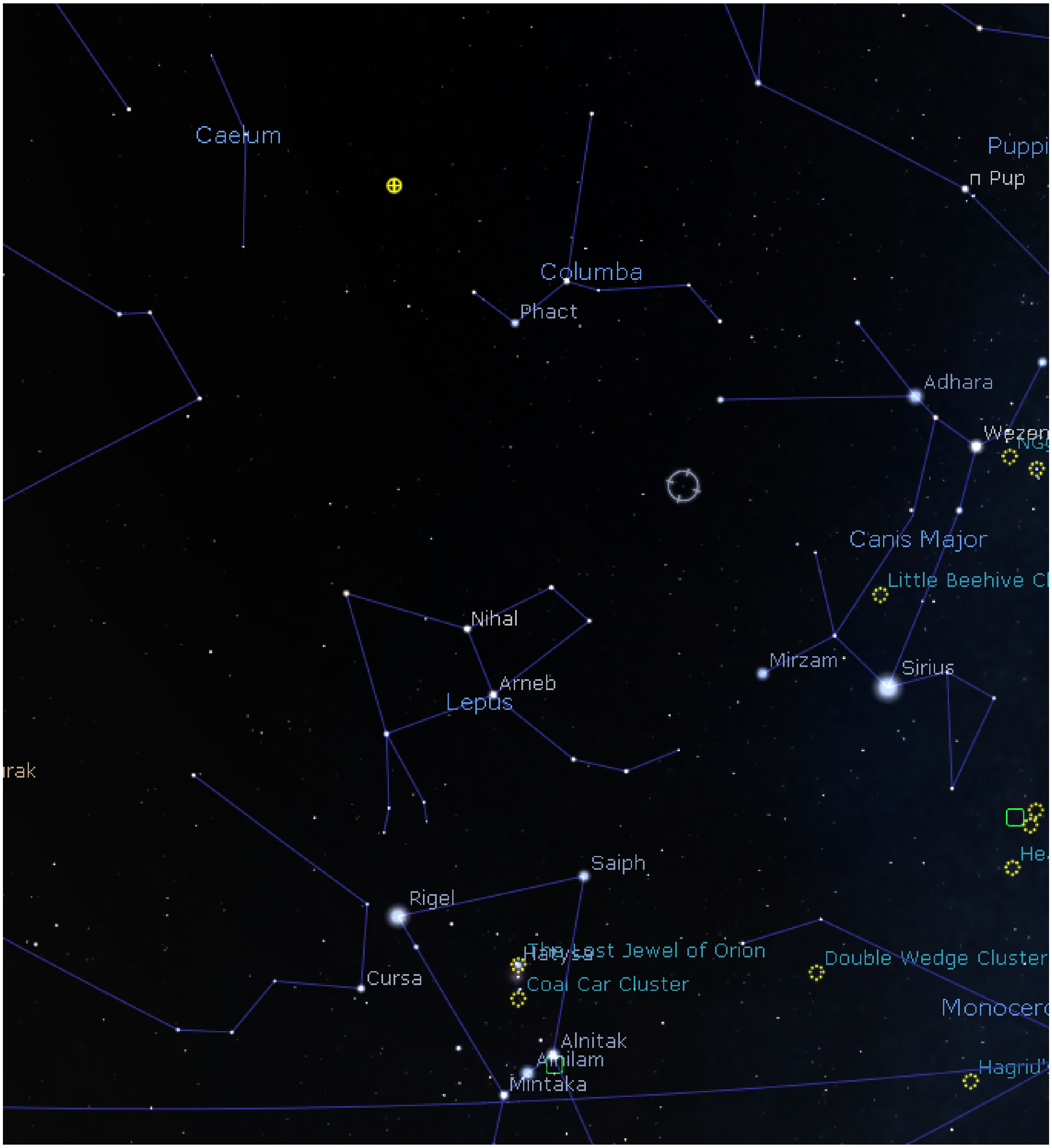
Q1	B
Q2	D
Q3	C
Q4	B
Q5	D
Q6	A
Q7	B
Q8	A
Q9	C
Q10	D

Practical True & False

i	F
ii	T
iii	T
iv	F

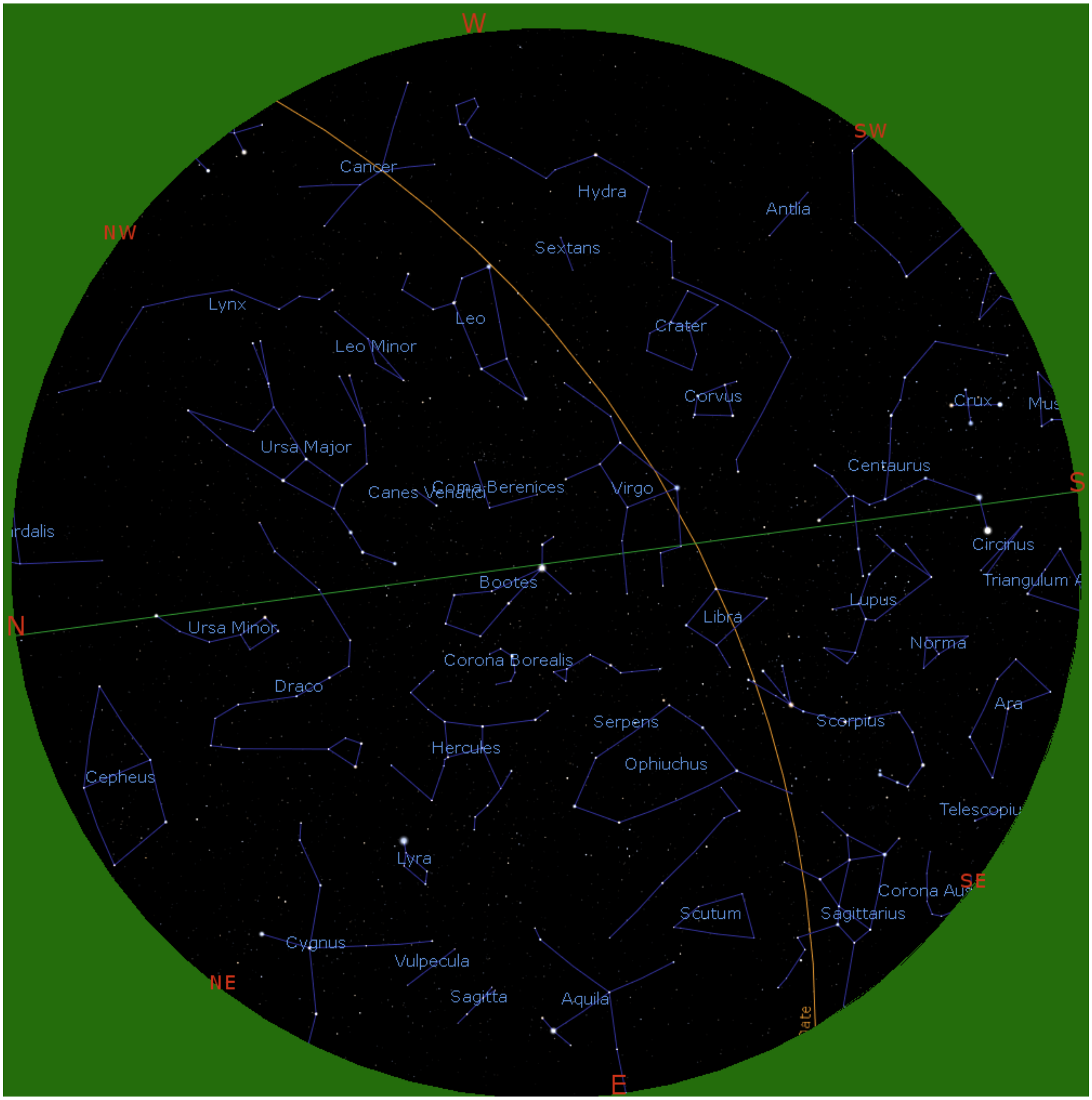
Practical Star Maps





Caelum
Columba
Phact
Puppi
π Pup
Adhara
Wezen
Canis Major
Little Beehive Cl
Nihal
Arneb
Lepus
Mirzam
Sirius
rak
Saiph
Rigel
Hatysa
The Lost Jewel of Orion
Coal Car Cluster
Double Wedge Cluster
Cursa
Monocero
Alnitak
Alnilam
Mintaka
Hagrid's

Star Chart:



1	In diagram
2	In diagram
3	Cancer, Leo, virgo, Libra, Scorpius or Sagittarius
4a	Deneb, Lyra and Vega

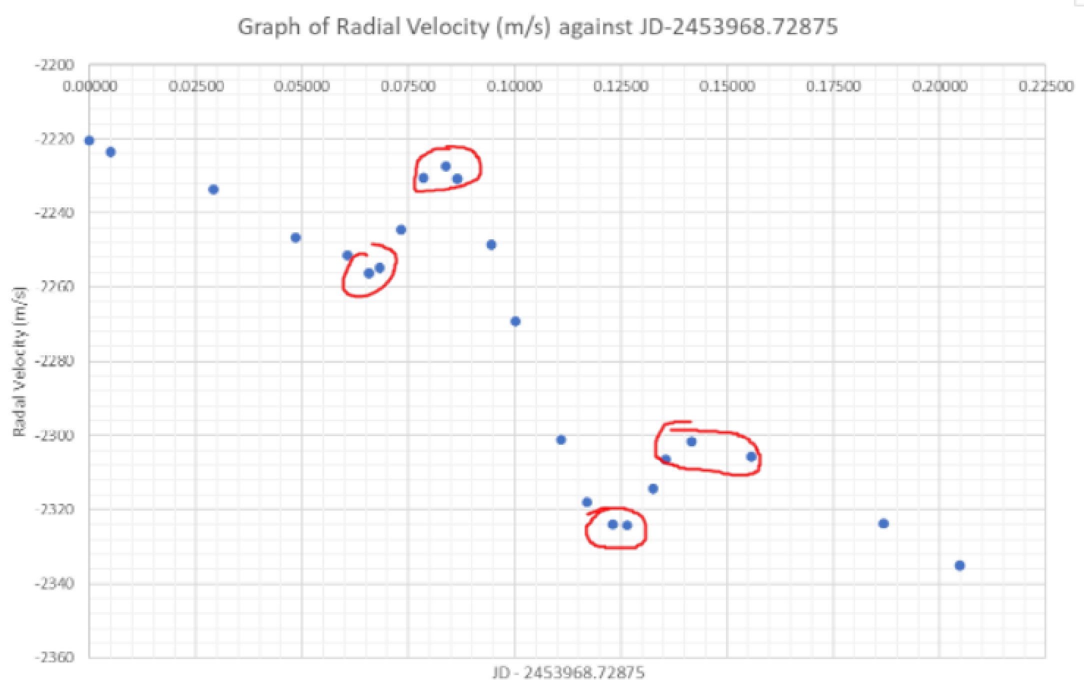
4b	Rigel Kent and Polaris are missing
4c	Any other three visible constellation
4d	Any other three visible deep sky constellations, half credit if cross is within the general area, but not on the DSO itself
5a	Arcturus is at the zenith
5b	18 deg N, 164.5 deg E (values from when star chart is made) above answer is based from star chart screen shot, actual workings: Current time in UTC +8 is 12am, hence time at prime meridian is 4pm. Current date is 11th of March, which is close to the vernal equinox on the 20th of March. The vernal equinox is thus approximately located at GST noon. Since it is 4pm at Greenwich, it is 4 hours past local noon and the GST is correspondingly 0400h. Given the RA of arcturus, which is currently at the zenith, the location is at GST +10h16m (14h16m - 4h), corresponding to a longitude of 154 deg Arcturus is at the zenith, hence the local latitude is roughly 19.17deg any reasonable close answer is acceptable, $\pm\ 10deg$

Data Analysis

<table>
<tr><td>a</td><td>Binary System A:

Orbital path along redshifted hemisphere only
- Only 1m if some parts are in blueshifted hemisphere
[-1m] If rotation of star not specified/wrong rotation

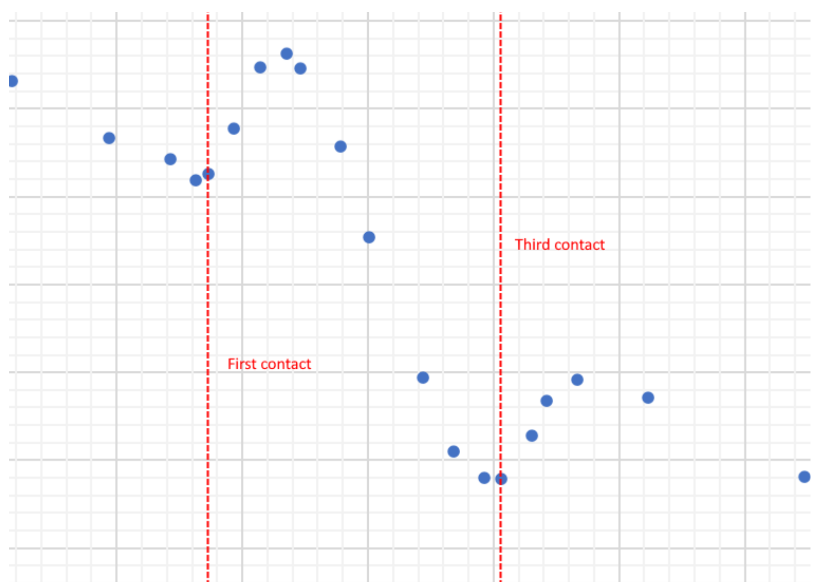
Binary System B:

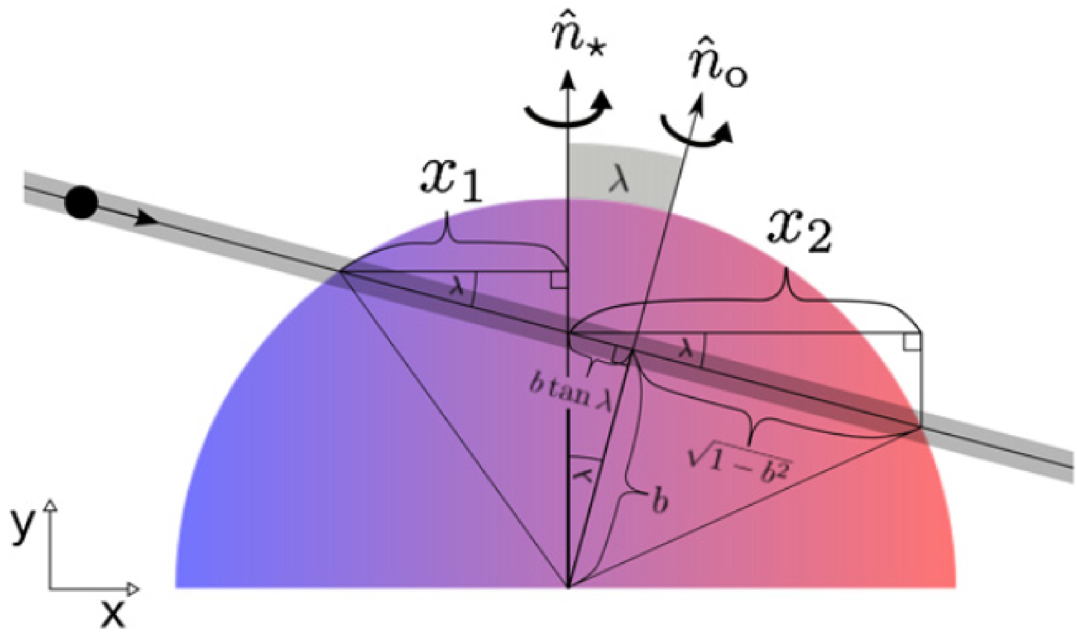
Retrograde motion (0m if rotation dir. of star/orbital dir. not specified)
Impact parameter close to 0 / no angle (<⅓ of radius) (symmetricity)</td></tr>
<tr><td>b</td><td>

Negative marking:

(-1m) No Title
(-1m) Axes not labelled
- (-0.5m) If y-axis units is not given
(-1m) x-axis does not specify offset from JD
- Also -1m if no processing is done and raw JD is plotted
(-1m) Bad scale (<½ of paper)
- Deduct for not on graph paper too
(-1m) Not on graph paper
Don't deduct marks for flipped y-axis

Points plotted incorrectly
- (-2m) wrong shape</td></tr>
</table>

	- (-1m) if irrelevant points at the start are not removed - (-1m each up to 4m) if points at start/end/peaks of RM curve are not plotted - No penalty for missing points before/after RM curve
c(i)	Actual value: ~**2.2 days** 2.1 - 2.3 = full credit 2.0 - 2.4 = partialcredit Give working marks if they show some evidence of averaging the period: e.g. 9 periods occurred from BJD* 300 - 320
c(ii)	More accurate result is reading from their graph in (a) **RV = -2277 m/s** -2275 to -2279 = full credit -2273 to -2281 = partial credit (estimate from Fig. 1 will be pretty off)
c(iii)	**0.076 day** OR **1.824 hr** OR **6566.4s** 0.071 - 0.081 day any evidence of measurement of start/end of RM curve
d	 deduction for unlabelled contact
e	

	Any diagram showing geometry of the system  From geometry: $$x_1 = \left(\sqrt{1-b^2} - b\tan\lambda\right)\cos\lambda$$ $$x_2 = \left(\sqrt{1-b^2} + b\tan\lambda\right)\cos\lambda$$ Simple algebra yields the expected expressions (v sin i_s is an extra constant on both sides)
f	Recognize that the reading from the graph is $$\text{Graph reading} = \Delta V_{RM} + RV_{sys} = -\left(\frac{R_p}{R_s}\right)^2 x \cdot v \sin i_s + RV_{sys}$$ - Remember that x_1 and x_2 in part (e) correspond to the **distance** so the signs of x(t) will be dropped - i.e., (x v sin i_s)_{1,2} = | Graph reading - RV_{sys} | * (R_s/R_p)^2

	Solve for parameters Let $(x\, v \sin i_s)_{\{1,2\}} = \{\alpha, \beta\}$ and $V = v \sin i_s$ $$\left[\frac{1}{2}(\alpha+\beta)\right]^2 - \left[\frac{1}{2}(\beta-\alpha)\right]^2 = (1-b^2)V^2\cos^2\lambda - b^2V^2\sin^2\lambda$$ $$= V^2\cos^2\lambda - b^2V^2$$ Express in terms of b $$\alpha\beta = V^2\left(\frac{\frac{1}{4}(\alpha+\beta)^2}{(1-b^2)V^2}\right) - b^2V^2$$ $$\alpha\beta(1-b^2) = \frac{1}{4}(\alpha+\beta)^2 - (1-b^2)b^2V^2$$ This is a quadratic in b^2: $$b^4V^2 + b^2(\alpha\beta - V^2) + \left(\frac{1}{4}(\alpha+\beta)^2 - \alpha\beta\right) = 0$$ Solving for b^2: b = **0.0323493** OR **0.833366** (-1m if negative b are not rejected) Correspondingly, \lambda = —**56.4461°** OR —**1.8538°**
g	Slower
h	The parameters of the system are (\lambda = -1.8538°, b= 0.833366) Using i_s = 90°, the true-spin orbit angle equation is simplified to $$\cos\psi = \sin i_p \cos\lambda$$ The true spin-orbit angle is ~**4.7°**