Singapore Astronomy Olympiad 2022

Data Response

Instructions

1. The data response portion of this Olympiad lasts for **45 minutes** and is worth **a total of 45 marks**.
2. Fill in these details on the Data Response Summary Answer Sheet (Attached as a MS Word document) and each side of an SAO answer sheet:
   - Year of competition
   - Your participant code
   - The page number – which should be continuous from 1 to N
   - The part of the paper, and the question number
3. Cross out all workings or answers you do not wish to be evaluated.
4. If you require assistance (enquire about an ambiguity or possible errata, etc.), please get the attention of the invigilators.
5. Once the Data Response Round is over, you are to combine all documents, **including your graph**, into one singular PDF file to be uploaded. More details on the order can be found in the Cover Page word document.

Competition Rules and Regulation

1. Only the use of scientific calculators is permitted. No graphing or programmable calculators are allowed.
2. Disruptive behaviour, cheating, collusion to cheat or any integrity-related offences are grounds for immediate disqualification.
3. You may opt to retain the question paper, constants sheet and answer script for personal use.
4. Toilet breaks are prohibited for the duration of the paper owing to its decentralized nature. Participants are encouraged to use the washroom before the paper commences.

References for Data Analysis Question

1. Radial velocity data derived from: Winn, Joshua N., et al. 'Measurement of the Spin-Orbit Alignment in the Exoplanetary System HD 189733'. *The Astrophysical Journal*, 3, vol. 653, no. 1, Dec. 2006, pp. L69–72. *arXiv.org*, https://doi.org/10.1086/510528.
2. Graphs obtained from: Boisse, I., et al. 'Stellar Activity of Planetary Host Star HD 189733'. Astronomy & Astrophysics, 2, vol. 495, no. 3, Mar. 2009, pp. 959–66. *arXiv.org*, https://doi.org/10.1051/0004-6361:200810648.
3. Graphs generated using Astropy[1], a community-developed core Python package for Astronomy:
   - The Astropy Collaboration, et al. 'The Astropy Project: Building an Inclusive, Open-Science Project and Status of the v2.0 Core Package'. *The Astronomical Journal*, vol. 156, no. 3, Aug. 2018, p. 123. *arXiv.org*, https://doi.org/10.3847/1538-3881/aabc4f.
   - The Astropy Collaboration, et al. 'Astropy: A Community Python Package for Astronomy'. *Astronomy & Astrophysics*, vol. 558, Oct. 2013, p. A33. *arXiv.org*, https://doi.org/10.1051/0004-6361/201322068.

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[1] http://www.astropy.org

Data Analysis [45]

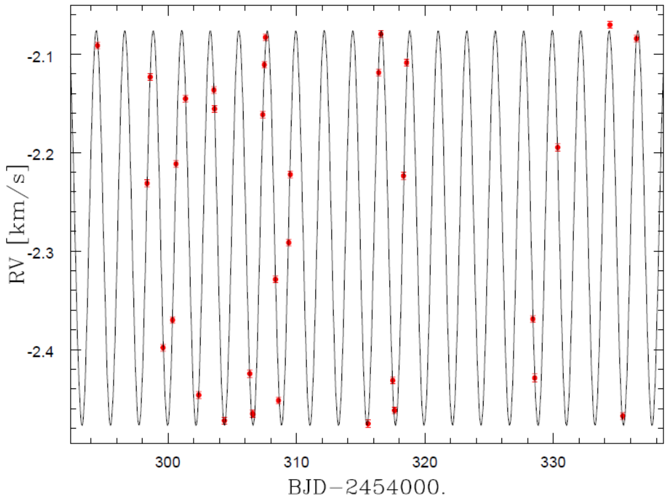
Multiple techniques can be utilized for the detection of exoplanets such as Doppler spectroscopy and transit photometry. Doppler spectroscopy is an indirect detection method looking for variations in radial-velocity measurements corresponding to the host star orbiting around the system's center-of-mass. Depending on instrumentation sensitivity, the observed "wobbling" can be used to constrain the bodies' mass. On the other hand, transit photometry relies on the dropped in observed brightness caused by the exoplanet transiting across the disk of the host star, allowing prediction of the transit depth (ratio of sizes).

HD189733 is a binary star system known in the constellation Vulpecula. One of the stars is a K2V orange dwarf with a known extrasolar planet. The parameters of this exoplanetary system are:

Parameter	**Value**
HD189733 A (Host Star)	
Radius	0.73 ± 0.02 [$R_\odot$]
Mass	0.82 ± 0.03 [$M_\odot$]
Rotational Period	11.953 ± 0.009 [day]
Rotational Velocity[2] ($v \sin i_s$)	2.97 ± 0.22 [km s^{-1}]
HD189733 b (Exoplanet)	
Radius	1.10 ± 0.03 [R_{Jup}]
Mass	1.13 ± 0.03 [M_{Jup}]
Orbital Eccentricity	$\ll 0.008$
Orbital Inclination (i_p)	85.68 ± 0.04 [°]

Part 1: Rossiter-McLaughlin Effect [26]

Using the method of Doppler spectroscopy, Boisse, et al. (2009) characterized the orbital parameters of the exoplanet using the SOPHIE spectrograph. The following graph shows measured radial velocities against observation time[3], with the best fit solution plotted as a solid line.



Data was obtained over two months and hence spread over multiple orbital periods of HD189733 b. From the best fit solution, the following graph shows the rearranged data plotted as a function of orbital phase.

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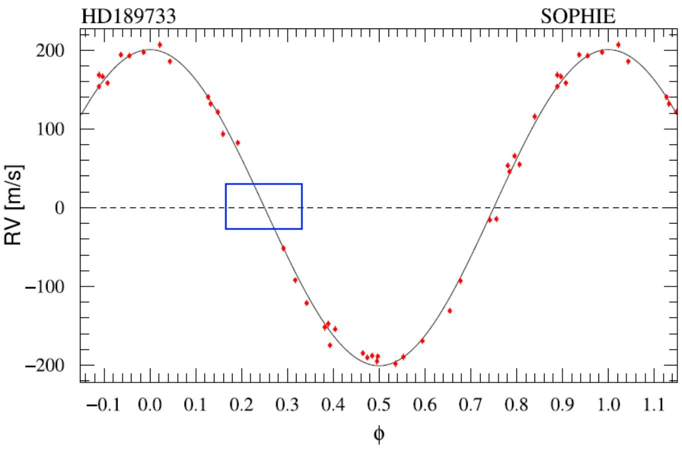
[3] BJD here is the Barycentric Julian Date



Not explored in this paper is the quantification of the Rossiter-McLaughlin (RM) effect, observed as an anomaly in the measured radial velocity of the star during a transit of the exoplanet. The region marked in blue (our emphasis) corresponds to one such instance of a transit.

As the star rotates on its axis, half of the stellar disk is moving towards the observer and will be observed to be blueshifted. Along our line-of-sight, the varying radial component of stellar rotation forms a Doppler shift **gradient** across the stellar disk – the magnitude of the shift will be more intense near the disk's edge compared to the center of the disk. A transiting body blocks some of this Doppler shifted light, causing the mean redshift of the star to vary. By analyzing this resultant anomaly in the measured redshift, the impact parameter b and the sky-projected spin-orbit angle λ can be indirectly determined.

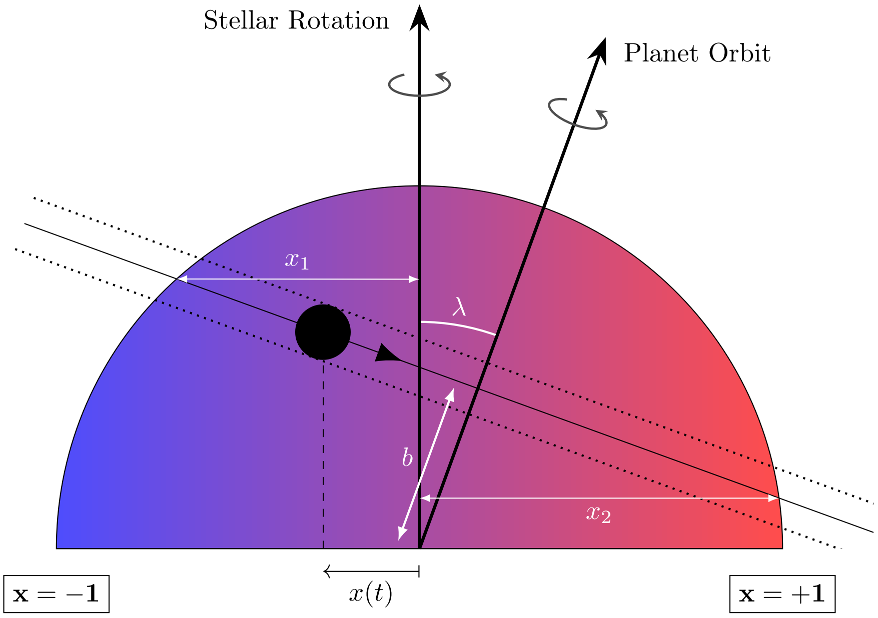


Figure 1: Relevant parameters for the RM effect[4].

The impact parameter ($0 \leq b \leq 1$) is defined as the ratio of closest approach to the disk center to the disk radius. The sky-projected spin-orbit angle λ is the **observed** angle between the host star's equatorial plane and the planet's orbital plane, whereas the **actual** angle between the planes is the true spin-orbit angle ψ. These two angles are related by

$$\cos\psi = \cos i_s \cos i_p + \sin i_s \sin i_p \cos\lambda$$

Where i_s and i_p are the inclinations of the host star and the planet's orbit respectively.

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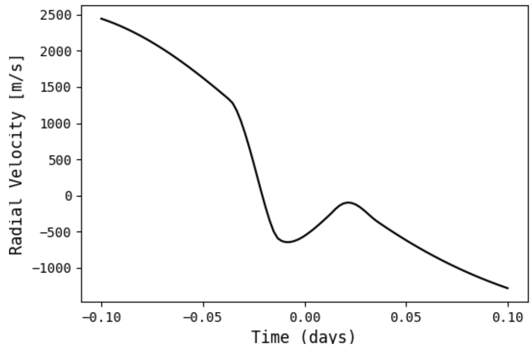
[4] Colour of diagram represents direction and intensity of Doppler shift

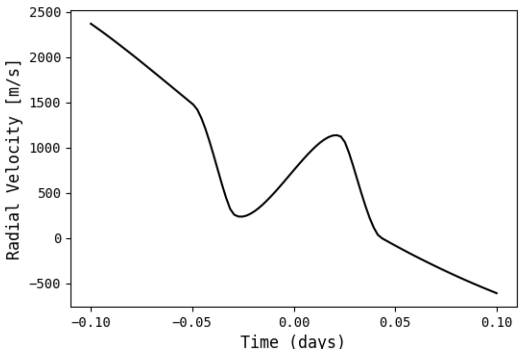
Winn, et al. (2006), modeled the RM effect for the exoplanet HD189733 b based on their radial velocity measurements (Table 1).

The following graphs illustrate the Rossiter-McLaughlin effect for two different binary systems. Qualitatively, the shape of the RM effect is directly affected by the two parameters b and λ.

(a) Sketch the geometry (like Fig. 1) of each system's transit, indicating the direction of stellar rotation and orbital path of the transiting body. **[4]**



Binary System A



Binary System B

(b) From Table 1, using the relevant data points, plot a graph of radial velocity against time **[12]**

(c) From any of the graphs, determine:

- **(i)** The orbital period of HD189733 b **[2]**
- **(ii)** The recession velocity of the system **[2]**
- **(iii)** Duration of transit **[2]**

(d) On your graph in part (a), mark and label the point of first contact (exterior ingress) and third contact (interior egress) **[4]**

Part 2: Hot Jupiter [19]

The anomalous radial velocity due to the RM effect (ΔV_{RM}) is approximately

$$\Delta V_{RM}(t) \approx -\left(\frac{R_p}{R_s}\right)^2 v_p(t)$$

Here, R_p and R_s are the radii of the exoplanet and star respectively. The quantity $v_p(t)$ is the radial velocity of the patch of star obscured by the planet at time t. Ignoring differential rotation, this is expressed in terms of the equatorial radial velocity

$$v_p(t) = v \sin i_s \cdot x(t)$$

Note that $x(t)$ is in terms of the stellar radius R_s (i.e. $-1 \leq x(t) \leq 1$) and is defined to be positive on the **redshifted** side. Let the peaks of ΔV_{RM} correspond to distances x_1 and x_2 (Fig. 1).

(e) For simplicity, the oblateness of the star can be ignored and the stellar disk taken to be circular. Since $R_p \ll R_s$, you may also treat the transiting exoplanet as a point in this part. Show that: **[6]**

$$\frac{1}{2} v \sin i_s (x_1 + x_2) = \sqrt{1 - b^2}\; v \sin i_s \cos \lambda$$

$$\frac{1}{2} v \sin i_s (x_2 - x_1) = b\; v \sin i_s \sin \lambda$$

(f) These two quantities are related to the mean and asymmetry of the peaks of the RM effect. From your graph in part (a), calculate the different possible **values** for b and λ. **[9]**

A more accurate method is to use iterative methods on a theoretical model and compare predicted against observed data to successively approximate the system's parameters. Through this process, HD189733 b is estimated to have λ very close to zero. Measurement of the stellar inclination is is much harder to determine accurately, so let us assume that the stellar inclination is edge-on ($i_s = 90°$).

(g) Under this assumption for i_s, would the stellar rotation be determined to be faster or slower than the actual rotation of HD189733 A? **[1]**

(h) Using the possible values found in part (e) and the additional constraint on λ, find the possible value(s) for the true spin-orbit angle ψ of the system. **[3]**

Julian Date	**Radial Velocity (m/s)**
2453693.688738	-2261.257
2453694.690856	-2360.660
2453696.700914	-2483.494
2453723.712604	-2279.024
2453927.929745	-2223.200
2453934.845208	-2096.698
2453968.728750	-2220.455
2453968.733611	-2223.589
2453968.757928	-2233.728
2453968.777245	-2246.532
2453968.789433	-2251.471
2453968.794433	-2256.225
2453968.796944	-2254.833
2453968.802049	-2244.528
2453968.807373	-2230.567
2453968.812650	-2227.370
2453968.815289	-2230.845
2453968.823391	-2248.508
2453968.828912	-2269.282
2453968.839641	-2301.113
2453968.845845	-2318.073
2453968.851991	-2324.086
2453968.855220	-2324.246
2453968.861354	-2314.456
2453968.864352	-2306.535
2453968.870301	-2301.715
2453968.884583	-2305.694
2453968.915637	-2323.832
2453968.933565	-2335.050

Table 1: Radial Velocity Measurements of HD189733 A

End of Data Response Paper