Section A

Each question is worth 2 marks.

Summary of answers:

Q1	Q2	Q3	Q4	Q5	Q6
D	**C**	**A**	**C**	**B**	**B**

1. Which statements are true?

   i. Pluto is not considered a planet as it has not cleared the neighbourhood around its orbit.
   ii. Orion is considered as a Winter Constellation.
   iii. Due to the Moon being tidally locked to Earth, there are parts of the Moon there are never see Sunlight. This portion is known as the Dark Side of the Moon.
   iv. Like a terrestrial observer, an observer on the Moon will see Earth go through phases as the Moon orbits around the Earth.

   a) (i) and (ii)
   b) (iii) and (iv)
   c) (i), (ii) and (iii)
   d) (i), (ii) and (iv)

2. The size of Saturn's retrograde loops in the sky are smaller as compared to Jupiter's because:

   a) Saturn moves more slowly in orbit.
   b) Saturn has more pronounced ring systems
   c) Saturn is farther away from Earth than Jupiter.
   d) Saturn is at opposition more frequently.

3. Which planet can never be seen on the meridian at midnight?

   a) Venus
   b) Mars
   c) Jupiter
   d) Saturn

4. Why are we not able to find any black dwarves?

   a) As their name suggests, they are too dim and are hence invisible to our telescopes.
   b) Physically, they are too small for us to see them.
   c) Not enough time has passed in the universe for any to form yet.
   d) This category is a mistake, they are actually black holes.

5. The Hydrogen-Alpha line has a wavelength of 656.28nm as measured in the lab. If the H-alpha line of a star is measured to occur at 660.28nm. The star must be:

a) Moving towards the source, at 1800km/s
b) Moving away from the source, at 1800km/s
c) Moving towards the source at 400km/s
d) Moving away from the source at 400km/s

6. Which concept encompasses the idea that life started when the elements on Earth, accompanied by the varying conditions of heat and pressure of the environment lead to the formation of many essential molecules of life.

a) Panspermia
b) Abiogenesis
c) Spontaneous Generation
d) Primordial Soup

Section B(1): Short Questions

Part	Solution	Mark
1a	Consider an element along the surface of the mirror, a radial distance r from the centre The buoyant force (green arrow) must provide for the centripetal force, and also counter the gravitational force on the element. $\frac{dy}{dr} = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{m\omega^2 r}{mg}$	M1 M2
1b	$\frac{dy}{dr} = \frac{m\omega^2 r}{mg}$ $\int_0^r (\omega^2 r/g)\ dr = \frac{\omega^2 r^2}{2g}$ It is technically not required to integrate; so long the participant has a qualitative understanding of an integral as the area under a graph, the answer can be arrived at through finding the area under the straight line graph with gradient ω^2/g	M1 A1
1c	For a parabola, we have $x^2 = 4fy$ where f is the focal length Rearranging, $y = \frac{1}{4f}r^2$ $\frac{1}{4f} = \frac{\omega^2}{2g} \rightarrow f = \frac{g}{2\omega^2} = 9.0m$ $\frac{f}{D} = \frac{9.0m}{6.0m} = 1.5$ $p = \frac{206265}{9.0m} = 22.9\text{"/mm}$ $= 23\text{"/mm}$ Any appropriate alternative method to deduce focal length will be accepted	M1 M1 M0.5 A0.5 M0.5 A0.5

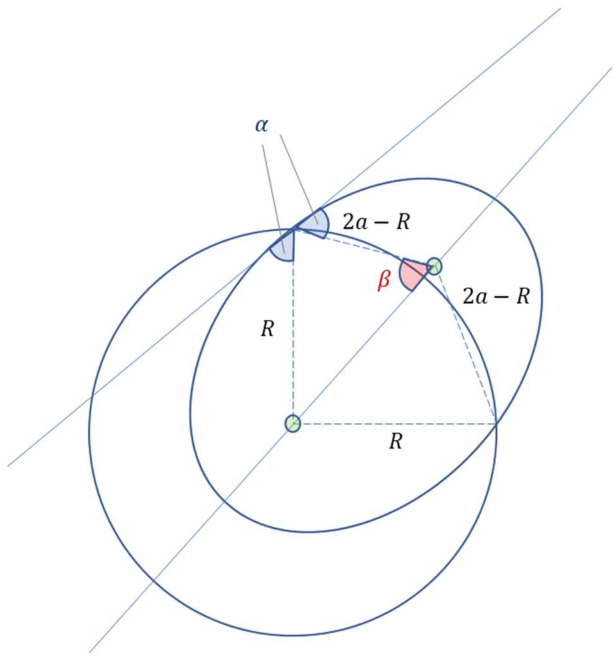
2a	The flux of an object changes by a factor of $\frac{1}{(1+z)^2}$ as the universe expands The first $\frac{1}{1+z}$ arises due to the wavelength of the photons increasing by a factor $1+z$, leading to the enegy of individual photons decreasing by that factor The second $\frac{1}{1+z}$ factor arises from the interval between the detection of photons being $(1+z)$ times larger than that between emission of photons $$f = \frac{L}{4\pi d_L^2} = \frac{L}{4\pi d_c^2 (1+z)^2} \rightarrow d_L = d_c(1+z)$$ The angular diameter distance is $d_A = \frac{D}{\delta\theta}$ where D is the diameter of the object $$d\theta = \frac{D}{d_c a(t)}$$ We note that D is measured at the time of emission of the photon t_1. $$d_A = d_c a(t_1) = \frac{d_C}{1+z}$$ Any reasonable alternative explanation will be accepted	M1 M1 M1
b	$$SB \propto \frac{f}{\delta\theta^2} \propto \frac{1}{(1+z)^2}\frac{1}{(1+z)^2}$$ $$m = -4$$	M2 A1
c	$$d_C = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$ $$= 2\left(\frac{299792458ms^{-1}}{\frac{67.80kms^{-1}}{Mpc}}\right)\left(1 - \frac{1}{\sqrt{2}}\right)$$ $$= 7.993 \times 10^{25} m$$ $$d_A = \frac{d_C}{1+z} = 3.997 \times 10^{25} m$$ $$\theta = \frac{102{,}000ly}{3.997 \times 10^{25} m} = \frac{9.461 \times 10^{20}}{3.997 \times 10^{25}} = 2.41 \times 10^{-5}\text{rad}$$	M1 M1 (awarded for both finding d_A and θ) A1

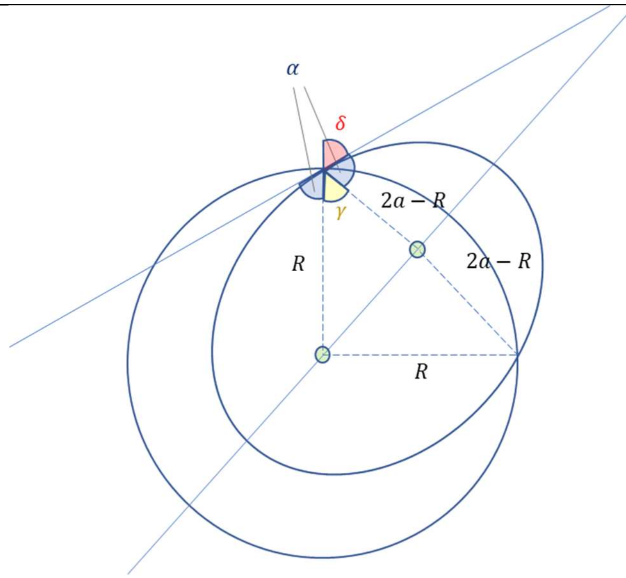
3a	$$n = \frac{1000}{\frac{4}{3}\pi r_h^3} = 7.07 \times 10^{-17}\text{Pc}^{-3}$$ $$= 2.41 \times 10^{-66} m^{-3}$$	M1 A1
3b	The mean free path l is given by $$l = (\sigma n)^{-1}$$ $$= 14.1\text{MPc}$$ $$t = \frac{l}{v} = 4.96 \times 10^{17} s = 15.7 \text{ billion years}$$ This is larger than the age of the universe! Within the half mass of the galaxy, the total average merger rate is $$\frac{N}{t} = 6.37 \times 10^{-8}/\text{year}$$ $$= 6.4 \times 10^{-8}/\text{year}$$	M2 M1 A1
3c	$$\Omega = \frac{\pi \theta_D^2}{4}$$ $$S = m + 2.5\log(\Omega) = 21.8 \text{ mag/arcsec}^2$$	M2 A1

4a	Using the given approximation, we get $B \approx \frac{2hc^2}{\lambda^5}\frac{1}{1+\frac{hc}{\lambda k_B T}-1}$ $= \frac{2hc^2}{\lambda^5}\frac{1}{hc/\lambda k_B T}$ $=\frac{2\lambda c k_B T}{\lambda^4}$	M1 A1
4b	Using Wien's law, $\lambda = \frac{2.898\times10^{-3}}{12100} = 239.5\text{nm}\ \ or\ 240\text{nm}$ (depending on whether 12100 is interpreted as 3s.f. or 5s.f.)	M1 A1
4c	The approximation in (a) is not valid here, because $\frac{hc}{\lambda k_B T} \approx 5$. It's important to use the full planck formula – participants should not be thrown off by the approximation in (a). This is the infamous *Ultraviolet Catastrophe.* $B_{440nm} = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}\right) = 5.194\times10^{14}$ $B_{550nm} = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}\right) = 3.080\times10^{14}$ $B - V = m_B - m_V = -2.5\log\frac{B_{440}}{B_{550}} = 2.5\log\frac{5.194}{3.080}\ = -0.567\ (3.\text{s.f.})$ There is still some deviation from the theoretical value -- stars are not perfect blackbodies.	M2 (choice of eqn) [do not penalize again after] M1 M1 A1
5a	$L = I\omega = (M_1 r_!^2 + M_2 r_2^2)\omega$ $r_1 = \frac{M_2 a}{M_1+M_2}$ $L = \frac{M_1 M_2}{M_1+M_2}a^2\omega$ $M_1 r_1 \omega^2 = \frac{GM_1M_2}{a^2} \rightarrow \omega^2 = \frac{GM_2}{a^2\frac{M_2 a}{M_1+M_2}} = \frac{G(M_1+M_2)}{a^3}$ $L = \frac{M_1M_2}{M_1+M_2}a^2\sqrt{\frac{G(M_1+M_2)}{a^3}} = \sqrt{\frac{G(M_1M_2)^2 a}{(M_1+M_2)}}$	M1 M1 M1

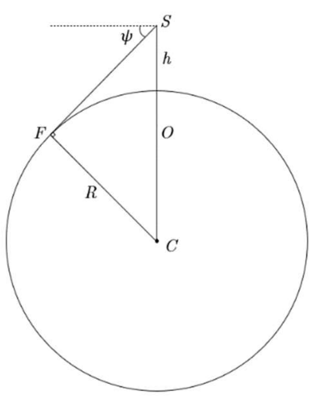
	Any alternative explanation making use of reduced mass will be awarded full credit. For the next step, we note that $M_1 + M_2 = M_T$ is a constant and thus $\frac{dM_1}{dt} = -\frac{dM_2}{dt}$ $\frac{dL}{dt} = \frac{1}{2}\left(\frac{G(M_1M_2)^2 a}{M_T}\right)^{-\frac{1}{2}} \left(\frac{da}{dt}\frac{G(M_1M_2)^2}{M_T} + \frac{Ga}{M_T}2(M_1M_2)(\frac{dM_1}{dt}M_2 - \frac{dM_1}{dt}M_1)\right)$ Since angular momentum is conserved, $\frac{dL}{dt} = 0$ $\frac{da}{dt}\frac{G(M_1M_2)^2}{M_T} = -\frac{2GM_1M_2a}{M_T}\left(\frac{dM_1}{dt}(M_2 - M_1)\right)$ $\frac{da}{dt} = \frac{2a(M_1 - M_2)}{M_1M_2}\frac{dM_1}{dt}$	M2 M1 M1
5b	$\frac{dM_1}{dt} = \frac{da}{dt}\frac{M_1M_2}{M_1 - M_2}\frac{1}{2a}$ $\delta M_1 = \frac{\delta a}{2a}\frac{M_1M_2}{M_1 - M_2} = \frac{-0.0000154}{0.08}\frac{2.51 \times 5.42}{5.42 - 2.51} = -9 \times 10^{-4} M_\odot$ $\delta M_2 = -\delta M_1 = 9 \times 10^{-4} M_\odot$	M1 A1 [-0.5 for wrong sign]

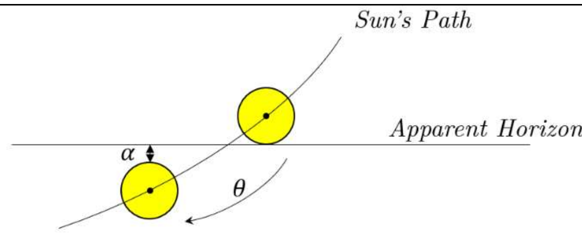
Section B(2): Medium Question

Part	Solution	Mark
1a	Suppose the semi-major axis of the orbit is a. $E = -\frac{GMm}{2a}$, $U = -\frac{GMm}{R}$ The required kinetic energy for launch is then $\frac{1}{2}mv^2 = E - U = GMm\left(\frac{1}{R} - \frac{1}{2a}\right)$, giving $v = \sqrt{GM\left(\frac{2}{R} - \frac{1}{a}\right)}$ From the above expression for v, we observe that the launch velocity is minimized by **minimizing a**, which is where the ellipse property provided will be useful. 	M1 M2 M3

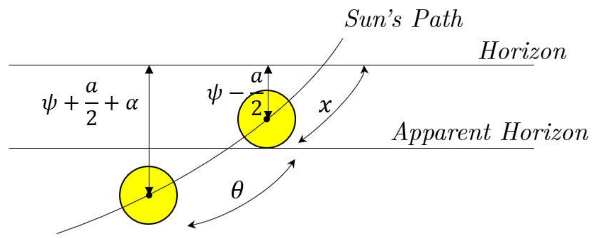
	 In the diagram above, observe that $2a - R$ is minimized by setting it to be the perpendicular distance (i.e. $\beta = 90°$). Then, $\gamma = 45°$ and $2\alpha = 135°$, giving $\alpha = 67.5°$. Observe that $\alpha = \delta$, the angle between the launch and the vertical axis. With respect to the horizon, the angle would be $90° - 67.5° = 22.5°$.	M2 M3 A1
1b	$2a - R = \frac{1}{\sqrt{2}}R \to a = \frac{R}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = 5.438 \times 10^6 m$ $v = \sqrt{GM\left(\frac{2}{R} - \frac{1}{a}\right)} = \sqrt{GM\left(\frac{2}{R} - \frac{2}{R\left(1+\frac{1}{\sqrt{2}}\right)}\right)}$ $= \sqrt{\frac{2GM}{R}\frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$ $= 7.20 \times 10^3 m/s$ Note that this amount of symbolic working is not necessary, and participants are expected to calculate numerical values earlier on instead for efficiency $h = a(1+e) - R = a + c - R$ $= a + \sqrt{R^2 - (2a-R)^2} - R$ $= \frac{R}{2}\left(\frac{1}{\sqrt{2}} - 1\right) + \sqrt{4aR - 4a^2}$	M1 M1 A1 M2 A1

	$= 3.57 \times 10^6 m$	-1 for substitution errors

Q2	We first need to find out the angle of depression due to the height above sea-level. This is important as this will affect the setting angles of the Sun.  Doing a little geometry, we find that the angle of depression, ψ, is given by: $$\psi = \cos^{-1}\left(\frac{R}{R+h}\right)$$ Where h is the height above ground and R is the radius of the Earth. Next, we find the angular size of the Sun's disk, ψ_{Sun}. Simple trigonometry gives us the relation: $$a_{Sun} = 2\tan^{-1}\left(\frac{R_{Sun}}{D_{Sun}}\right)$$ When the sun sets, its motion is not vertically downwards. Hence, we need to find the actual angle the sun has traversed over its arc during sunset. The following diagram will show the relation between the angle traversed, θ.	M1 M1 M1 M1 M1



M2 (Correct Triangles)



M1 (Refraction)
M1 (Appt Horizon.)
M1

We use two triangles to get the two equations:

$$\frac{\sin x}{\sin 90°} = \frac{\sin(\psi - \frac{a}{2})}{\sin \bar{\phi}}$$

M1

$$\frac{\sin(x + \theta)}{\sin 90°} = \frac{\sin(\psi + \frac{a}{2} + \alpha)}{\sin \bar{\phi}}$$

M1

Hence,

$$\theta = \sin^{-1}\left(\frac{\sin(\psi + \frac{a}{2} + \alpha)}{\cos\phi}\right) - \sin^{-1}\left(\frac{\sin(\psi - \frac{a}{2})}{\cos\phi}\right)$$

We last turn to the speed at which the Sun will move below the horizon. Since the plane is travelling due west, the Sun will appear to move slower as the plane is "catching up with" the Sun.

For a stationary observer, the Sun will move with angular velocity equals to: M1

$$\omega_{Sun} = \omega_0$$
$$\omega_0 = \frac{2\pi}{T_{solar}}$$
$$T_{solar} = 24\ hours$$

M1 (Finding ω'_{Sun})

For a moving observer, the new angular velocity of the Sun along its path is now:

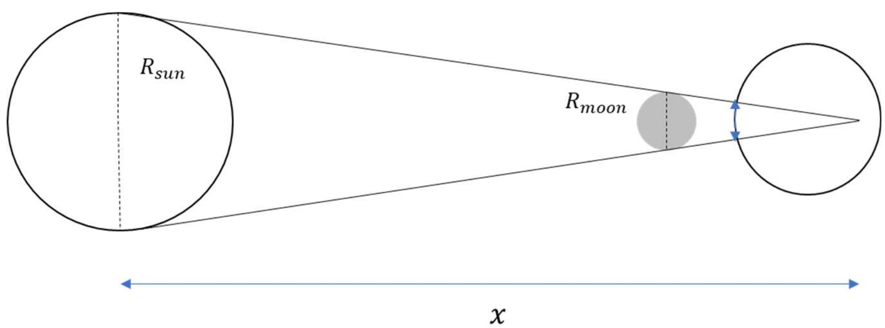
M1 (Using $R\cos\phi$)

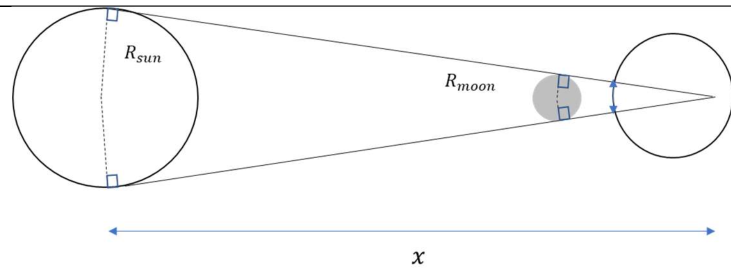
$$\omega'_{Sun} = \omega_{Sun} - \frac{v_{plane}}{R\cos\phi}$$

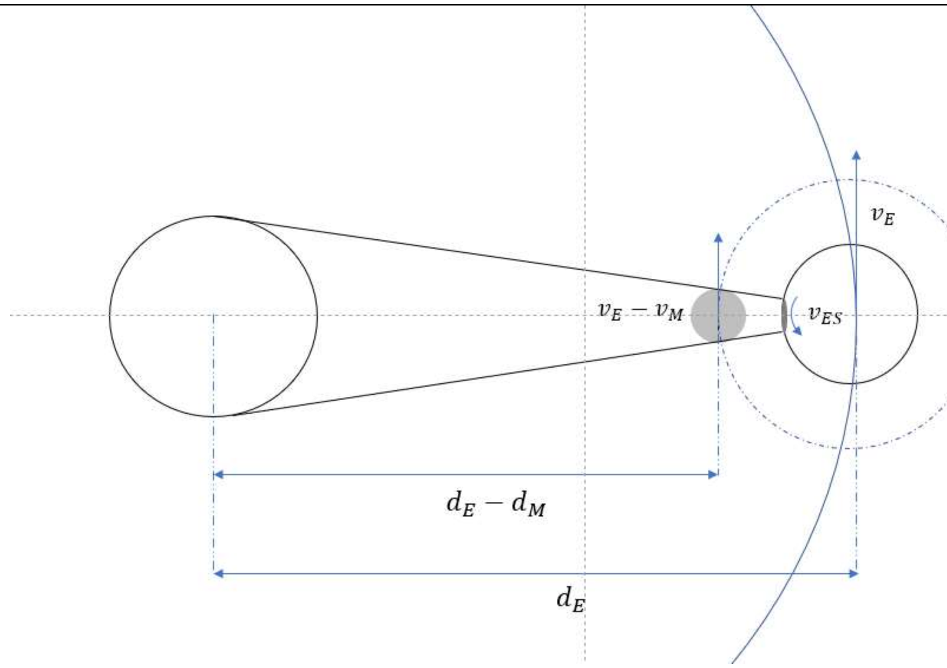
With that, we can now find the duration of sunset: M1

	$$t = \frac{\theta}{\omega'_{Sun}} = \left[\sin^{-1}\left(\frac{\sin(\psi + \frac{a}{2} + \alpha)}{\cos\phi}\right) - \sin^{-1}\left(\frac{\sin(\psi - \frac{a}{2})}{\cos\phi}\right)\right]\left(\frac{1}{\omega_{Sun} - \frac{v_{plane}}{R\cos\phi}}\right)$$ Plugging in the values given in the question, we get the duration of the sunset is: $$t = 3510s = 58^m 30^s$$	M1 A1	

Section C Long Question

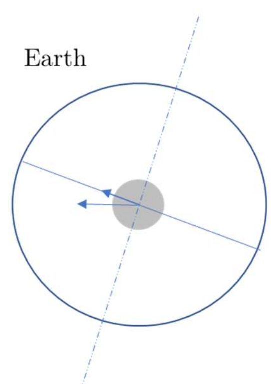
1a	Small angle approximation will also be accepted ($\theta_M = \frac{R_m}{d_E}$) $\theta_M = 2\tan^{-1}\left(\frac{R_M}{d_E}\right) = 0.556°$ $\theta_S = 2\tan^{-1}\left(\frac{R_S}{d_S}\right) = 0.536°$ (accepted range $0.535° - 0.536°$)	M1 Expression for θ A1 A1
1b	 $\frac{x}{R_{sun}} = \frac{x - d_{sun} + d_{moon}}{R_{moon}} \rightarrow x = \frac{(d_{sun} - d_{moon})}{(1 - \frac{R_{moon}}{R_{sun}})} = 1.4898 \times 10^{11} m$ $r_{umbra} = (x - d_{sun} + R_{earth}) \times \frac{R_{sun}}{x}$ $= 95.397\text{km}$ $= 95.4\text{km}$ A more correct diagram would in fact be the following, but we note that the similar triangles approach yields the same answer either way; as such, both diagrams (or results of) would be accepted and given full credit	M2 (correct geometry) M1 (similar triangles) M2 (correct manipulation) M1 A1

		
1c	We may invoke a form of kepler's second law: $$\frac{dA}{dt}=\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{\pi ab}{T}$$ $$\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{\pi a^2\sqrt{1-e^2}}{T}$$ $$\left(\frac{d\theta}{dt}\right)_{moon}=3.065\times10^{-6}\text{rad/s}$$ $$\left(\frac{d\theta}{dt}\right)_{Sun}=2.008\times10^{-7}\text{rad/s}$$	M3 M1 A1 A1
1d	Essentially, what is important here is to calculate the velocity of the moon's shadow on the surface of the Earth. The previous parts of the question were meant to guide the readers to the following formulation: $$v_{E,t}=d_{sun}\times\left(\frac{d\theta}{dt}\right)_{sun}=2.991\times10^4 m/s$$ $$v_{M,t}=d_M\times\left(\frac{d\theta}{dt}\right)_{moon}=1097m/s$$ $$v_{shadow}=\left(v_{E,t}-v_{M,t}\right)\frac{d_E}{d_E-d_M}-v_{E,t}=-1027m/s$$ Another solution that students may provide could be the following, which is a valid approximation – however, it is important to be careful when considering the direction of the shadow's velocity. $$v_{shadow}=\left(-\left(\frac{d\theta}{dt}\right)_{sun}+\left(\frac{d\theta}{dt}\right)_{moon}\right)\times d_{moon}=1025m/s$$	M1 M1 M3 M3



It is also important to take into the account the Earth's rotational velocity at the surface $v_{ES} = \omega_E R_E = 463m/s$

Note that this velocity is at angle of $23°26'$ to the velocity of the shadow (the ecliptic makes that angle with the equator), and their relative velocities summed through use of the cosine rule — M1



$$v = \sqrt{v_S^2 + v_{ES}^2 - 2v_S v_{ES}\cos 23°26'} = 630\text{m/s}$$

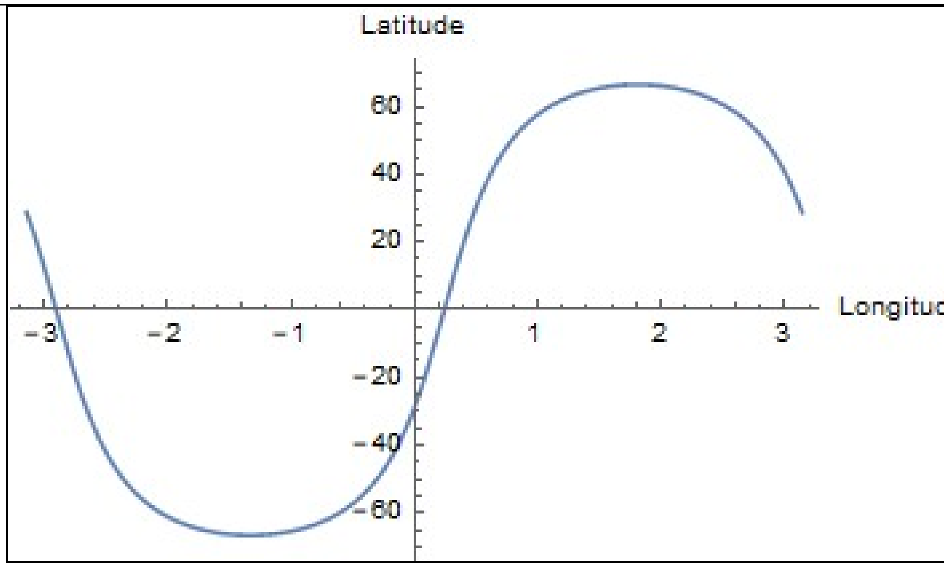
$$T = \frac{2r}{v} = 303s$$

(accepted range: $301 - 305$s)

M1
M1
A1

1e	The eclipse year is in fact the beat period/synodic period of the draconic month and the synodic month – the conditions repeat when the moon is at the same position with respect to the nodes again and along the lunar cycle $$T_{syn} = \frac{1}{27.2122} - \frac{1}{29.5306} = 346.6 \text{ days}$$	M2 M1 (Sign) A1
1f	Given the set-up in the question, we are hinted to plot a latitude vs longitude graph, where the line in the graph symbolises the **points on Earth that experience Sunset/Sunrise**. Hence, we should turn our attention to finding an expression that gives us every pair of latitude and longitude points that experience sunset/sunrise. To do that, we need to employ more celestial geometry. We will want to find out how many hours after noon will sunset/sunrise occur, given a specific latitude. (Figure) From the figure, we can find a relation between the Hour Angle (HA), latitude ϕ and the declination of the sun δ. Remember that δ is negative, and ϕ is negative for the southern hemisphere.	 M1 M1 M1

	We can conjure up a spherical cosine law to get the relation. $$\cos 90^\circ = \cos\bar{\delta}\cos\bar{\phi} + \sin\bar{\delta}\sin\bar{\phi}\cos HA$$	M1
	We can simplify it to: $$0 = \sin\delta\sin\phi + \cos\delta\cos\phi\cos HA$$ Hence, we get: $$HA = \cos^{-1}(-\tan\phi\tan\delta)$$	M1
	Looking at this, we may get stuck, but we need to remind ourselves the importance of the hour angle expression we have gotten. The HA in question represents the number of hours that will have pass before sunset occurs.	M1
	Putting a spin to that by bringing in the idea of time zones, it tells us which place that is an $\frac{HA}{15}$ number of hours ahead of Greenwich will be experiencing sunset.	M1
	So, for example at the Equator, $HA = 90^\circ$ which translates to 6 hours, and so a place that is 6 hours ahead of Greenwich at the Equator will be experiencing Sunset! This is basically how time zones work.	
	Using time zones, we can then realise that the HA we calculated is just the longitude coordinate we are looking for. Coincidentally, we have found the expression, we just need to put ϕ in terms of HA. $$\phi = \tan^{-1}\left[-\frac{\cos HA}{\tan\delta}\right]$$	M1
	Finally, we need to translate the graph to the right, since the expression above is for local noon at Greenwich, but we require it when it is local noon in Singapore. Luckily, this tweak is a simple one. $$\phi = \tan^{-1}\left[-\frac{\cos(HA-\theta)}{\tan\delta}\right]$$	M1
	Where $\theta = 103.82^\circ$, the longitude of Singapore.	A1
	When plotted, we see that the graph looks neat:	

Section D: Data Analysis

Part	Solution	Mark
a	We can find the temperature easily by using *Wien's Displacement Law.*	
	$$T = \frac{b}{\lambda_{peak}}$$ $$= \frac{2.898 \times 10^{-3}}{520 \times 10^{-9}}$$	M1
	$$T = 5570\ K$$	
	With the error being: $$\frac{\Delta T}{T} = \frac{\Delta\left(\frac{1}{\lambda}\right)}{\frac{1}{\lambda}}$$	M1
	Hence, $$\Delta T = \frac{(73990)}{1923100}(5570) = 214\ K$$	M1
	Thus, $$T = (5570 \pm 214)\ K$$	A1

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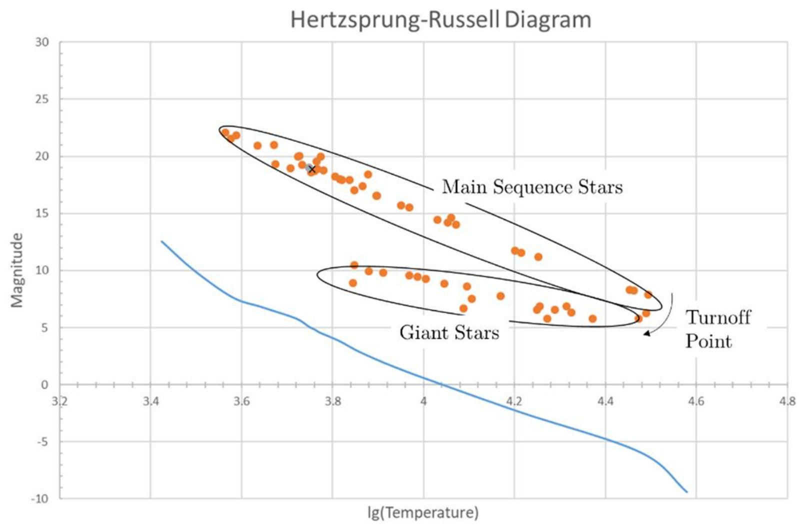
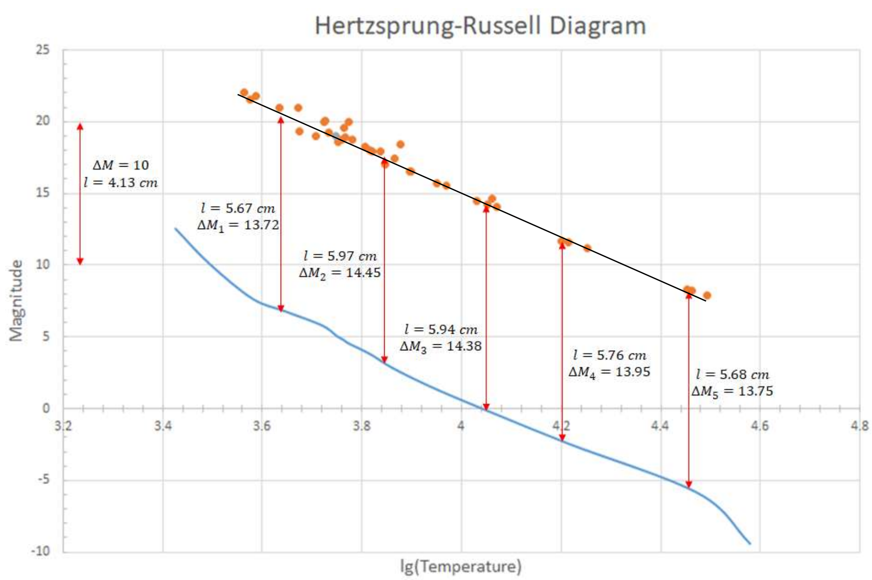
Before we begin measuring the distances, we need to do some filtering. Refer to the labelled diagram below.



With it labelled, we can then proceed. Referring to the diagram given, we then follow the instructions given in the question. We start by



drawing a rough best fit and then measure accordingly.

We now have five readings for μ, hence we can now find $\bar{\mu}$.

$$\bar{\mu} = \frac{\sum_1^n \Delta M_i}{n} = \frac{13.72 + 14.45 + 14.38 + 13.95 + 13.75}{5}$$

Thus:

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M1 Correct identification of MS Stars

M1 – Scale

M1 – ≥ 3 readings

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	$\bar{\mu} = 14.05$ Tolerance of 1%, $13.91 \leq \bar{\mu} \leq 14.19$ will be given full score. A $\bar{\mu}$ that is outside the 1% but within 2% tolerance will be given half credit: $13.77 \leq \bar{\mu} < 13.91$ and $14.19 < \bar{\mu} \leq 14.33$ Any other value of $\bar{\mu}$ will not be accepted. If the student measures the scale wrongly, but everything else is consistent, then only the marks allocated to measuring scale is deducted. Extra FYI: This process is known as *Main Sequence line fitting.*	M1 A1
c	Notice that the "distance factor" μ is actually the *distance modulus* of the globular cluster. In other words: $$\mu = 5(\log_{10} d - 1)$$ Where d is the distance to the globular cluster. Hence, to find d, $$d = 10^{\frac{\bar{\mu}}{5}+1}$$ Subbing in the values from part (b), we get: $$d = 10^{\frac{14.05}{5}+1}$$ $$d = 6500\ pc$$ Note: The value d that participants get will be dependent on their answer in part (b). If the values are consistent, full credit will be given. There will not be double penalty.	M1 M1 A1
d	Here, technically students should use the transit depth of event (C), due to the fact that SAO-2021c has a moon which causes the double dip. Hence, this part presents two acceptable solutions. Solution 1: Assume that r_{moon} is negligible for now. Focusing on event (B), we get that the transit depth (T.D.) is: $$T.D. = 1 - 0.999896 = 0.000104$$ Since this system is incredibly far away, we can approximate that the planet and star are on top one another, hence the brightness drop during maximum transit is: $$\frac{b_{transit}}{b_{normal}} = \frac{(r_s^2 - r_p^2)\pi I}{r_s^2 \pi I} = \frac{r_s^2 - r_p^2}{r_s^2} = \frac{1 - T.D.}{1} = 1 - T.D.$$ Rearranging: $$r_s^2 - r_p^2 = r_s^2(1 - T.D.)$$	M1 M1

	Hence: $$r_s^2(T.D.) = r_p^2$$ Thus: $$\frac{r_p}{r_s} = (T.D.)^{\frac{1}{2}} = 1.02 \times 10^{-2}$$	A1
	Solution 2: Assume that r_{moon} is significant. For this, we focus on event (C), where the moon has moved past and is no longer blocking SAO-2021. Then, we get the transit depth again and repeat the steps in solution 1: $$T.D. = 1 - 0.999903 = 0.000097$$ Since this system is incredibly far away, we can approximate that the planet and star are on top one another, hence the brightness drop during maximum transit is: $$\frac{b_{transit}}{b_{normal}} = \frac{(r_s^2 - r_p^2)\pi I}{r_s^2 \pi I} = \frac{r_s^2 - r_p^2}{r_s^2} = \frac{1 - T.D.}{1} = 1 - T.D.$$ Rearranging: $$r_s^2 - r_p^2 = r_s^2(1 - T.D.)$$ Hence: $$r_s^2(T.D.) = r_p^2$$ Thus: $$\frac{r_p}{r_s} = (T.D.)^{\frac{1}{2}} = 9.85 \times 10^{-3}$$ Notice that there is a slight difference in the answers from solution 1 and 2. However the difference is 3.5% and hence both solutions are acceptable. Note: One can also look at the transit depths of events (A) and (B) and then subtract accordingly. Some people may say that we cannot do this because we are not sure if we can treat the planet and its moon to be separate discs. We can be certain that the planet and moon's discs are separate because of the transition between events (A) and (B), and that (A) has a plateau, which we will not see if the two discs are joined.	M1 M1 A1
e	The most likely candidate that will cause such a transit curve is that the planet <u>SAO-2021c has a moon</u>.	A1 A1

	Event (A) is caused by the transit of the moon as it first passes across SAO-2021. Event (C) is caused by moon exiting its transit across SAO-2021. Note: Some students may raise the possibility that it is due to the ***Rossiter-McLaughin*** effect. This effect is a valid explanation for the presence of event (C) but it cannot explain event (A). Hence, it will not be a valid response and thus will not be accepted. Only 1 mark will be given should the explanation be correct in the account for event (C).	A1
f	This artefact on a transit curve is a tell-tale sign of **limb darkening**. When looking face-on towards a star, the edges will look darker than the centre. So, as SAO-2021c is blocking the star, it will block more light when it is covering the centre of the star than when it is near the edge of the star. Accept any other plausible sounding answers. Note: Limb brightening will **not** be accepted since this transit is observed in the visible light range and the edges of the star (the limbs of the star) is darker. FYI: The word *Limb* comes from the Latin *Limbus* meaning *edge.*	A1 A1 A1
g	The value of **+0.08** shows that SAO-2021 has a very similar metallicity as compared to the Sun. From this we can infer that SAO-2021 is highly likely a Population III star, which formed at around the same period as our Sun did. The differences could be due to the local variations in stellar material.	A1 A1
h	Considering the hint that the question gives, we should be trying to find the mass via the luminosity of SAO-2021, using the mass-luminosity relationship. We are given SAO-2021's apparent magnitude and we have found the distance to the globular cluster in part (c). Hence, we can find SAO-2021's brightness and then use the distance found in (c) to find the luminosity. We will compare it to the Sun in this case: $$m_{SAO} - m_{Sun} = -2.5 \lg \left(\frac{b_{SAO}}{b_{Sun}} \right)$$ Hence:	M1

$$b_{SAO} = b_{Sun} \times 10^{\frac{m_{Sun} - m_{SAO}}{2.5}}$$
$$= 1366 \times 10^{\frac{-26.7-18.61}{2.5}}$$
$$b_{SAO} = 1.027 \times 10^{-16}\ Wm^{-2}$$

M1

Thus:

$$L_{SAO} = 4\pi d^2 b_{SAO}$$
$$L_{SAO} = 5.19 \times 10^{26}\ W = 1.36 L_{Sun}$$

M1

Now, we plot the graph to get the exponent α for the Mass-Luminosity relationship. There are certain restrictions before we plot. Mainly:

1. Star is in Main Sequence
2. Star's mass must be between 0.1 and 50 times the mass of the Sun.

Filtering, we get the following list:

Star	Spectral Type	Mass/ M_{Sun}	Luminosity/ L_{Sun}
Sirius	A1 V	2.06	25.4
Altair	A7 V	1.79	10.6
Spica	B1 V	11.4	20500
Alkaid	B3 V	6.10	594
Alphecca	A0 V	2.58	74
Achernar	B6 V	6.70	3150
Albireo B	B8 V	3.70	230
Segin	B3 V	9.20	2500

M3 – Filtering the correct stars. (0.5M deducted for each star wrongly filtered.)

Since we are trying to find the exponent, we do a lg(L) vs lg(M) graph, and the gradient we get will be the exponent α.

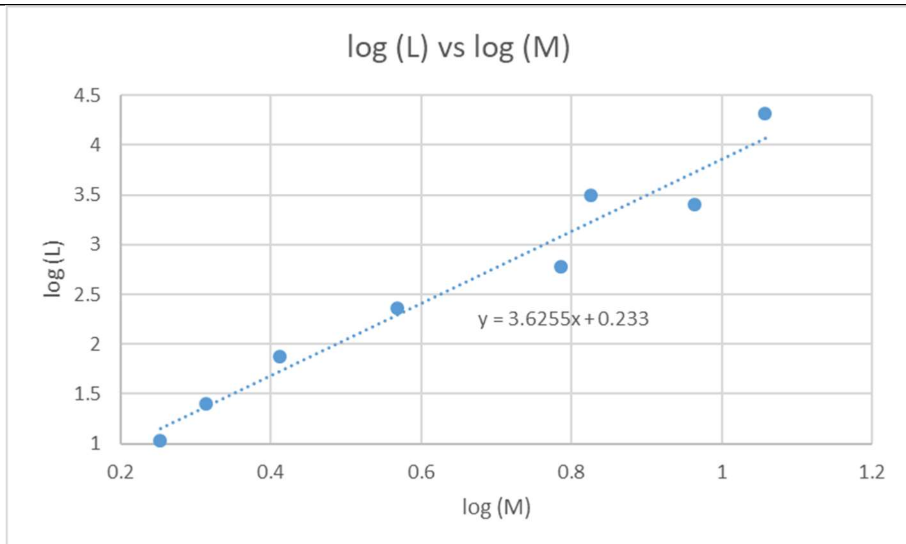
M1 – Correct type of plot.

Note that there is no need to convert the units to kg and watts as scaling either one will not affect the gradient.

M4 – correctly getting the lg values.

Plotting, we get:

M6 – Plot:
1- Title
2- Axes
3- Good Scale



4- Plot correctly
5- Acpt BF line
6- Gradient triangle or Regression.

Hence,

$$\alpha = gradient = 3.63$$

M1
A1

The acceptable range of α is: $3.45 \leq \alpha \leq 3.81$. (5% Tolerance)

Thus:

$$L \propto M^{3.63}$$

Finally, we can find the mass of SAO-2021.

$$M_s = M_{Sirius}\left(\frac{L_{SAO}}{L_{Sirius}}\right)^{\frac{1}{3.63}}$$

$$M_S = 0.919\ M_{Sun}$$

Note: It is of importance that one uses the mass and luminosity of a star in the given list. Usage of the values by different stars in the table results in slightly different answers, of which all are acceptable.

Note: Some participants may choose to skip the plotting of the graph and use the usual value of $\alpha = 3.5$ for MS stars.
In these cases, marks will be awarded for the correctness of ideas while marks allocated for graph drawing will not be awarded since participants have been explicitly told to plot one.

When the participant forgets to filter out the non-MS stars, they will get a gradient and a value of α to be in the range: $2.54 \leq \alpha \leq 2.74$.
Here, only the marks allocated to filtering for the correct stars to plot

	will be penalised. Points for graph drawing and the subsequent workings will still be considered and awarded accordingly.	
i	We have the luminosity of SAO-2021, we can find the radius, since we also know the temperature. $$R_S = \left(\frac{L_{SAO}}{4\pi\epsilon T^4}\right)^{\frac{1}{2}} = 8.69 \times 10^8\ m = 1.26 R_{Sun}$$ Hence, using our answer in (d) $$R_P = 8.87 \times 10^6\ m = 1.39 R_{Earth} \quad (Solution\ d1)$$ $$R_P = 8.56 \times 10^6\ m = 1.34 R_{Earth} \quad (Solution\ d2)$$	M1, A1 A1

Section E: Practical

P(A) Multiple Choice Questions

1. C
2. A
3. B
4. D
5. A
6. C
7. C
8. B

9.

You can see Ursa Minor from Antarctica.	
To deem if an object can be more easily spotted as compared to another, one should only compare the objects' apparent magnitude and declination.	
To improve the Signal-to-Noise Ratio (SNR) of any photo, you should choose increasing the number of shots taken over increasing the exposure time, assuming the total exposure time is kept constant.	X
For the best tracking of celestial objects, one should perform a Drift Alignment over visual alignment to Polaris.	X

10.

Fix on the counterweight bar and counterweights.	4
Set up the tripod, remembering to level it.	2
Fix the mount onto the tripod. Aligning the tripod and the mount to the North Celestial Pole.	3
Find a place with flat ground and with no blockages overhead.	1
Fix on the main body of the telescope.	5
Balance the RA axis.	8
Balance the Dec axis.	7
Attach the required accessories like the eyepieces and finder scope.	6

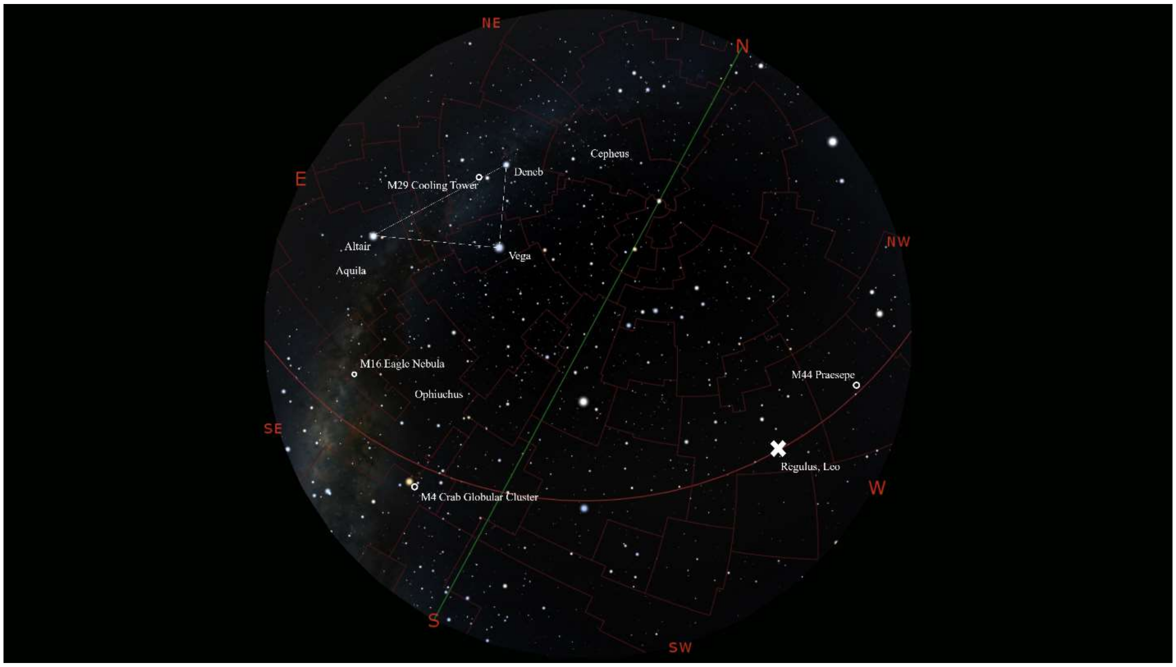
P(B) Star Chart

1.
   a. Vulpecula, Sagitta, Lyra, Lacerta, Cygnus
   b. Any visible stars within the frame
   c. Any visible DSOs within the frame
2.
   a. Carina Vela
   b. Any visible stars within the frame
   c. Any visible DSOs within the frame

0 Credit for misspellings; For part (a), if any traces lie outside of constellation boundary, 0 credit; 0 credit if most of the DSO identified does not fall within their respective circles drawn in part (c)

P(C) All Sky

3.	(a)	'X' should be marked on East.	A1
	(b)	LST = 14h 57m 19.2s ± 5m Given that the RA of Arcturus is 14h 15m 38.22s, we can estimate the LST of the observer by considering the distance between Arcturus and the meridian.	A1 M1
4.	(a)	*Yes, refer to star chart.* - Labelling - Tracing Full credit will only be given if the asterism was correctly traced and identified; Else, 0 credit. Altair, Deneb Vega 0 Credit if any of the 3 stars were misidentified.	A1 A1
	(b)	*Yes, refer to star chart.* - Labelling - Tracing Full credit will only be given if the asterism was correctly traced and identified; Else, 0 credit	A1
	(c)	Accepted answers: Aquila. Ophiuchus, Cepheus If the constellation lines lack labels, deduct 0.5 points; If the constellation was misidentified, 0 credit; 0 credit if any traces lie outside of constellation boundary.	A2
	(d)	Accepted answers: M4 Crab Globular Cluster, M16 Eagle Nebula, M29 Cooling Tower, M44 Praesepe If the DSO lacks labels, deduct 0.5 points; If the DSO was misidentified, 0 credit; 0 credit if most of the DSO identified does not fall within their respective circles drawn in part (c)	A2
	(e)	*Refer to star chart.* Regulus, Leo	A1 A1


NE
N
E
Cepheus
Deneb
M29 Cooling Tower
Altair
Vega
Aquila
NW
M16 Eagle Nebula
M44 Praesepe
Ophiuchus
SE
Regulus, Leo
W
M4 Crab Globular Cluster
S
SW