

Singapore Astronomy Olympiad 2019

Instructions

1. The entire Olympiad lasts for **3.5 hours** and is worth a **total of 200 marks**.
2. Fill in these details on the cover page (at the back of this page) and each side of an SAO answer sheet:
 - Year of competition
 - Your participant code
 - The page number – which should be continuous from 1 to N
 - The part of the paper, and the question number
3. Cross out all workings or answers you do not wish to be evaluated.
4. You are allowed to leave the venue early upon submission of your answer script (**see Instructions on Cover Page**), but only **after one hour after the start** of the paper and **not within 30 minutes to completion** of the paper.
5. Please raise your hand to seek assistance from the invigilators (e.g. to visit the restroom, request for more answer sheets, enquire about an ambiguity, etc.)

Competition Rules and Regulation

1. Only the use of scientific calculators is permitted. No graphing or programmable calculators are allowed.
2. Disruptive behaviour, cheating, collusion to cheat or any integrity-related offences are grounds for immediate disqualification.
3. You may opt to retain the question paper and constants sheet for personal use. Return all unused answer sheets to the Organising Team.

References for Data Analysis Question

1. Light curve data is derived from Tümer (1984) and downloaded from CALEB, http://caleb.eastern.edu/star_summary.php?star_id=264
2. Danjon A. Recherches de Photométrie Astronomique. Annales de l'Observatoire de Strasbourg. 1928;2: 1–185
3. Evren S, Ertan AY, Tunca Z, Ibanoglu C, Kurutac M, Tümer O. Photoelectric photometry of Z Herculis.
4. Astrophysics and Space Science. 1982 Oct;87(1-2): 51–9. Popper DM. Orbits of close binaries with CA II H and K in emission. I - Z Herculis and RS Canum Venaticorum. Astronomical Journal. 1988 Apr;95: 1242–50.
5. Popper DM. Rediscussion of Eclipsing Binaries. I. Z Herculis. Astrophysical Journal. 1956 Jul;124: 196–207.
6. Tümer O, Ibanoglu C, Tunca Z, Evren S. Light-curve analysis of Z Herculis. Astrophysics and Space Science. 1984 Sep;104(2): 225–44.

Astronomy.SG would like to thank the organizing committee and the question setters. This document is typesetted in L^AT_EX.



Cover Page

1. Fill in all required details below, before time is up.
2. Ensure that all materials (answer sheets, graphing paper, practical question paper) you intend to submit for grading have been attached. Amendments to an answer script after it has been collected will not be entertained.

3. Details on each side of answer sheet:

- Your participant code
- Question number
- Page number – continuous from 1 to N throughout your answer script
- **DO NOT WRITE YOUR NAME ON YOUR ANSWER SCRIPT**

4. Order of answer script:

- This cover page
- Answer sheets from Part 1-7, in continuous order from page 1 to N
- Tables and graphs from Part 4
- Practical component from Part 6

Question	Marks	Question	Marks
1.1		4	
1.2		Total	
1.3		5.1	
1.4		5.2	
1.5		5.3	
1.6		5.4	
Total		5.5	
2.1		Total	
2.2		6.1	
2.3		6.2	
2.4		6.3	
2.5		Total	
2.6		7.1	
2.7		7.2	
Total		7.3	
3		Total	

Participant Code	
Total Number of Pages of SAO Answer Sheets	

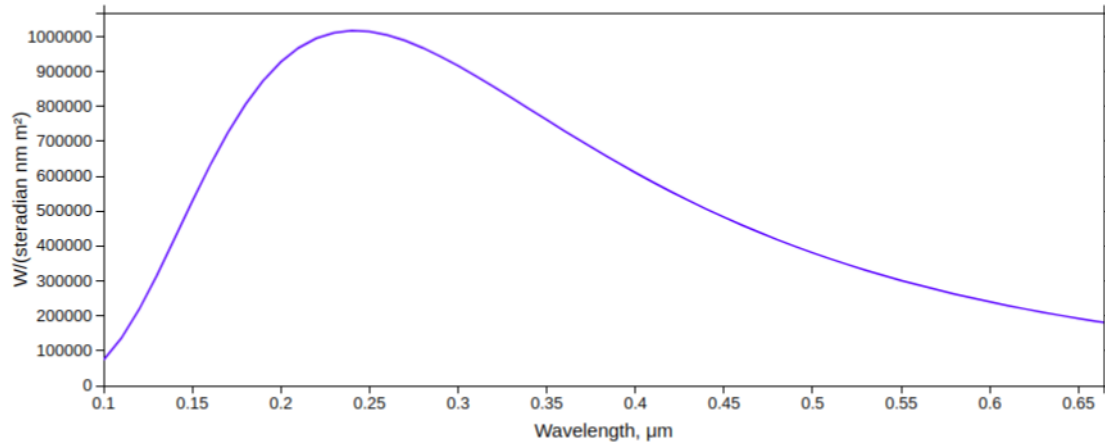
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1 Short Questions [12]

1.1 Rigel [2]

A student was asked to plot the blackbody spectrum of Rigel, based on the data she has collected. Given that Rigel has a radius $79 R_{\odot}$, find its luminosity in terms of L_{\odot} .



1.2 Galilean Moons [2]

Verify that Ganymede, Europa and Io have resonant orbits. Write down the ratio. You are given the following data, together with the mass of Jupiter (1.898×10^{27} kg):

Name	Mass/kg	Semi-Major Axis/km
Io	8.9×10^{22}	421700
Europa	4800000×10^{16}	671034
Ganymede	14819000×10^{16}	1070412
Callisto	10759000×10^{16}	1882709

1.3 Achromatic Lens [2]

An achromatic lens is used to ensure lights of different wavelengths have the same focal lengths, i.e. to correct chromatic aberration. An achromatic lens was made by combining plano-convex (one side flat and one side convex) and plano-concave (one side flat and one side concave) lenses, made from two different types of glasses, A and B. These two glasses have the following refractive indices:

	A	B
red	1.48	1.66
blue	1.50	1.70

Find the radii of curvature, R_A and R_B , necessary to produce a combination, equivalent to a converging lens with a focal length of 600mm.

You are given the following equation for two lenses of radii of curvature R_A and R_B respectively, and refractive indices n_A and n_B respectively:

$$\frac{1}{f} = (n_A - 1) \frac{1}{R_A} + (n_B - 1) \frac{1}{R_B}$$

1.4 The Flying Eagle [2]

Altair is a bright, nearby star in the night sky that has an apparent magnitude of +0.77. It has a parallax angle of 0.195".

Calculate the absolute magnitude of Altair, giving your answer to 2 decimal places.

1.5 Perseverance [2]

The Opportunity rover measures 2.3m in largest dimension.

What is the minimum possible diameter of a telescope located in an areosynchronous orbit 17300km above the Martian surface required to resolve the Opportunity rover at visible light wavelengths ($\lambda = 500$ nm)?

1.6 Cosmological Scale Factor [2]

In this question, we will like to determine the epoch where the Universe transitioned from a radiation-dominated to a matter-dominated Universe.

You are given the following parameters for the fraction of critical density contributed by:

- photons: $\Omega_{\gamma,0} = 5.35 \times 10^{-5}$
- neutrinos: $\Omega_{\nu,0} = 3.65 \times 10^{-5}$
- baryons: $\Omega_{bary,0} = 0.048$
- dark matter: $\Omega_{DM,0} = 0.262$

2 Medium Questions [54]

Choose **six** out of seven questions. Total marks is 54, with each question worth 9 marks.

2.1 Spacefill [9]

According to the World Bank, the Earth produced about 2 billion tonnes (1 ton = 1000 kg) of solid waste in 2016. To solve the problem of handling all this waste, an undergraduate physics student proposes dumping the waste at a constant rate into a Schwarzschild black hole which loses mass at the same rate due to Hawking radiation. Calculate the following, in SI units:

- Its mass;
- Its Schwarzschild radius;
- Its temperature.

You may use

$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

for the Hawking temperature of a black hole, and the Stefan-Boltzmann constant

$$\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}$$

2.2 Sunrise at Le Havre [9]

Impression, Sunrise is a depiction of Le Havre (49°29' N, 0°06' E) by Claude Monet. A recent analysis based on the depicted landscape, the tide, and historical weather records has argued that it was painted at 7.35 am local mean solar time, on the 13th of November 1872. Looking up an astronomical almanac, on the 13th of November 1872, the Sun's right ascension is 15h 15m 23.00s, and its declination is -18°05' 38.20". The equation of time (apparent solar time - mean solar time) on that day is +15 m 33 s. Defining sunrise as the point where the upper limb of the Sun contacts the horizon, calculate the following:

- The local mean solar time of sunrise on that day to the nearest minute;
- The azimuth of the sun at the point of sunrise to the nearest degree, measured from the North (0°) towards the East (90°).

The effect of atmospheric refraction causes objects with an altitude 34' below the horizon to appear at the horizon. You may neglect the effects of Earth's precession.

2.3 Space City [9]

In a space city, there is an art installation where two Osmium spheres of $m_1 = 1\text{kg}$, $m_2 = 1.5\text{kg}$ rotate about each other due to gravity at distance $r = 0.10\text{m}$ apart. Find the period of their rotation.

Prove that for a spherical satellite of radius r and mass m , that its Roche Limit is as below.

$$r_T = r \left(2 \frac{M_E}{m} \right)^{1/3}$$

r_T is the distance of the center of the Earth (which has mass M_E) to the center of the satellite. Hint: approximation should be used where appropriate.

Hence, find the minimum orbital altitude for the space city, assuming it is spherical, its mass is $1.0 \times 10^{10}\text{kg}$ and its radius is 1km.

2.4 Geminids [9]

3200 Phaethon is a small asteroid named after the son of Helios. Just 5.80km in diameter, its orbit has a semi-major axis of 1.271AU and eccentricity 0.8899, bringing it closer to the sun than any other named asteroid. Interestingly, it is also the parent body of a stream of meteoroids. These meteoroids, following orbits nearly identical to their source, are responsible for the Geminids meteor shower, which peaks around 14 December every year.

For this question, you may assume the orbit of 3200 Phaethon to be in the plane of the ecliptic, and that the Earth's gravitational effects are negligible.

a) Find the relative velocity, with respect to an observer on the Earth's surface, of the Geminid meteors.

b) Find, to the nearest 5 minutes, the Right Ascension of the Geminids' radiant point. (This is the point in the sky from which, to an observer on the Earth's surface, the meteors appear to originate)

2.5 Gravitational Mapping [9]

Suppose two satellites, Tom and Jerry, are in identical orbits around a neutron star, with Jerry leading Tom by a phase of 67.210° , as measured from the center of the neutron star. The satellites aim to map the gravitational field of the neutron star, not unlike the Earth-based GRACE mission, by sending 1575.25MHz microwave pulses from Tom to Jerry. The orbital parameters of the system are as follows:

Semi-major axis	$1.7880 \times 10^4 \text{m}$
Eccentricity	0
Mass of Neutron Star	$2.7839 \times 10^{30} \text{kg}$

You may neglect the bending of light due to general relativity. Taking into account all other relativistic effects, calculate, to 6 significant figures, the frequency of the pulse as it is received by Jerry.

2.6 Interstellar Proton [9]

Suppose we have an interstellar proton with total energy of 1.0 TeV travelling towards the Earth. If $\delta = 1 - \frac{v}{c}$, where v is the speed of the proton. Evaluate δ .

Suppose the proton flies into the vicinity of a satellite and experiences a constant uniform magnetic field. Obtain the radius of curvature of the proton's trajectory in terms of the force it experiences, F , (where $F = \gamma ma$) and total energy, E .

2.7 Age of the Universe [9]

In this section, you are required to execute differentiation. You are thus given the result for differentiating a polynomial function:

$$\frac{d}{dx}x^n = nx^{n-1}$$

where n is a real number and x is a variable.

In a quiz show, Tommy is challenged to estimate the age of the Universe. Tommy is not allowed to quote the value from Wikipedia and is required to work it out.

It is your job to help him out. You are given the Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

where

- ρ is the density of the Universe as a function of time;
- a is the scale factor of the Universe as a function of time;
- H is the Hubble constant as a function of time;

$$H = \frac{1}{a} \frac{da}{dt}$$

The most recent measured Hubble constant is $67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

- k is the curvature of the Universe.

Moreover, you are also given the First Law of Thermodynamics:

$$\Delta E + p\Delta V = T\Delta S$$

where ΔE is change in internal energy, p is pressure, ΔV is change in volume, T is temperature and ΔS is change in entropy.

Assuming a reversible expansion where $\Delta S = 0$, i.e. no change in entropy, and using the First Law of Thermodynamics, derive the fluid equation for our Universe.

$$\frac{d\rho}{dt} + 3\frac{1}{a}\frac{da}{dt}\left(\rho + \frac{p}{c^2}\right) = 0$$

and hence derive the acceleration equation:

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

Under certain circumstances, one can obtain a solution for the scale factor a , as a function of time, to be

$$a = \left(\frac{t}{t_0}\right)^{2/3}$$

where a at $t = t_0$ has been set to 1. State the necessary conditions to obtain such a solution.

Suppose that the above conditions are indeed satisfied, estimate the age of the Universe.

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3 Long Question [34]

While studying the exoplanets revolving around the star HD85512, an astronomer, Tommy, has possibly made contact with intelligent life on one of the planets, HD85512b. This exoplanet has a mass of roughly $3.6 M_{\oplus}$ and its orbit has a semi-major axis of 0.26 AU with orbital period of 58.43 days. To further understand the exoplanetary system, Tommy collected a few data of the parent star HD85512 and compares it with our Sun:

	Sun	HD85512
Mass	$1 M_{\odot}$	$0.69M_{\odot}$
Radius	$1 R_{\odot}$	$0.533 R_{\odot}$
Surface Temperature	5770K	4715K
Mean Molecular Weight, μ	0.62	Unknown
Spectral Type	G	K

To better understand the history of HD85512 exoplanetary system, a good understanding of stellar evolution and processes is required. In this question, we will study star birth, star death and heat transport in stars at equilibrium.

3.1 Part 1: Star Birth [9]

It is commonly said that stars are born from diffuse molecular clouds. In this section, we will understand the conditions required for these molecular clouds to collapse to form stars.

For a gravitationally bound system, the Virial theorem states that

$$2K + U = 0$$

where K and U are the kinetic and potential energies of the system respectively.

a) Consider a spherical cloud of constant density with radius R and mass M . Show that when we increase the radius of the shell by an amount Δr , the change in gravitational potential energy ΔU is subjected to the following constraint: **[6]**

$$\Delta U = -\frac{3GM^2}{R^6} r^4 \Delta r$$

From this, we may integrate this result to obtain the gravitational potential energy of the spherical star:

$$U = -\frac{3GM^2}{R^6} \int_0^R r^4 dr = -\frac{3GM^2}{5R}$$

b) Assuming the molecular cloud is an ideal gas of temperature T , such that the kinetic energy (due to random thermal motions of molecules, with mean molecular weight μ , in the cloud) of the cloud is

$$K = \frac{3}{2} N k_B T$$

where N is the number of particles and k_B is the Boltzmann constant. Find the minimum radius and mass for the spherical cloud to collapse, labelled R_J and M_J respectively (otherwise known as Jeans Length and Jeans Mass respectively). **[3]**

Unfortunately, not much information, regarding the formation of HD85512, can be deduced here. But, we now have a rough idea of how stars are formed. A small perturbation can initiate the contraction of a molecular cloud and trigger gravitational collapse, hence promoting star formation.

3.2 Part 2: Star Mid-Life [15]

The sequence of events between the collapse of a molecular cloud and the formation of a main sequence star is complex and will not be discussed. In this section, we aim to study the heat transport processes for a star in hydrostatic equilibrium. In particular, we will like to study main sequence stars. Note that both our Sun and HD85512 are main sequence stars.

a) Gravity and pressure gradients are key driving forces behind stellar evolution. We will treat the star as a spherical mass. By spherical symmetry, we will only need to consider forces along the radial direction.

Let's consider a small mass $\Delta m = \rho(r)\Delta A\Delta r$ with density $\rho(r)$, cross-sectional area ΔA and length Δr . Using Newton's Second Law, show that the equation of motion of the small mass is

$$-a_r = g(r) + \frac{1}{\rho(r)} \frac{\Delta P}{\Delta r}$$

where a_r is the acceleration of the mass in the radial direction (outward). From this, state the condition for hydrostatic equilibrium. [3]

b) The first heat transfer process we will like to discuss is convection. Gravity is an important driving force, as outlined below:

A rising packet of stellar gas may sometimes find itself in a cooler and more dense environment, and it will continue to rise because of its buoyancy. A similar argument holds for a falling packet of gas. Effectively, heat is transferred by the collective motion of the constituent particles.

During this convective process, this gas packet is said to expand and contract adiabatically. This is because there is insufficient time for heat conduction between this gas packet and the surroundings. We outline the adiabatic process below:

An adiabatic process occurs when there is no heat transfer between the environment and the system (in our case, it's the displaced packet of gas). For adiabatic processes, the pressure of the ideal gas is directly proportional to the density ρ^γ , where γ is the adiabatic index.

Assuming the star is made up of only ideal gas, show that the critical temperature gradient for convection is

$$\frac{\Delta T}{\Delta r} < \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{\Delta P}{\Delta r}$$

For hydrostatic equilibrium, you are given that the pressure gradient is

$$\frac{\Delta P}{\Delta r} = -\frac{Gm(r)\rho(r)}{r^2}$$

Hence, find the expression for the critical temperature gradient for convection for a star achieving hydrostatic equilibrium. [4]

c) Next, we study the transfer of energy by radiation and hence we wish to find a similar expression for the thermal gradient profile for radiation.

Let $L(r)$ denote the rate at which energy flows outwards through a spherical surface of radius r within the star. The release of the nuclear energy in the hot centre of the star implies that $L(r)$ increases with r until a region is reached in which no energy is being released. If $\epsilon(r)$ denotes the nuclear power generated per unit volume at r , then evidently,

$$\frac{\Delta L}{\Delta r} = 4\pi r^2 \epsilon(r)$$

Outside any central generating regions, $L(r)$ becomes constant and approaches the surface luminosity of the star. This total outward power flow, $L(r)$, is related to the flux density outwards, $J(r)$, by

$$L(r) = 4\pi r^2 J(r)$$

where the flux density of radiant heat, $J(r)$, is

$$J(r) = -\frac{4ac}{3} \frac{T^3}{\rho\kappa} \frac{\Delta T}{\Delta r}$$

Hence, find the condition required for radiative temperature gradient to be equal to the critical gradient for convection. [2]

d) For the core of HD85512, it is estimated that $P = 1.7 \times 10^{16}$ Pa, $T = 13.7 \times 10^6$ K and $\kappa = 0.138 \text{m}^2 \text{kg}^{-1}$, find the minimum power generation per unit mass in order for convection to dominate. Given that the actual power generated is $1.35 \times 10^{-3} \text{W kg}^{-1}$, evaluate your answer. [1]

e) Without using the Stefan Boltzmann Law, estimate the number of monochromatic photons radiated by HD85512 per second. [4]

f) If one were to use the Stefan Boltzmann Law to compute the luminosity of HD85512, what will be the number of monochromatic photons radiated per second? [1]

3.3 Part 3: Star Death [10]

Since HD85512 is a main sequence star, like our Sun, it will evolve to a white dwarf.

a) Given that the total classical electronic energy of a white dwarf has the form

$$E_e = K \frac{N_e^{5/3}}{R^2 m_e}$$

where $K = 1.23 \times 10^{-68} \text{ kg}^3 \text{m}^4 \text{s}^{-2}$, R is the radius of the white dwarf, N_e is the total number of electrons in the white dwarf and m_e is the mass of electron.

Give a numerical estimate to the equilibrium radius of HD85512, when it eventually reaches the 'White Dwarf Stage'. [5]

Hint for part a): *You may be required to differentiate polynomial functions:*

$$\frac{d}{dx} x^n = nx^{n-1}$$

where n is a real number and x is a variable.

b) Assuming that the electron distribution is homogeneous, estimate the speed of the electrons, giving your answer in terms of the speed of the light, c . [3]

Hint for part b): *The electrons are spaced so closely to each other that we may use the de-Broglie hypothesis where the de-Broglie wavelength is given to be twice the inter-electron separation. The de-Broglie equation is given to be*

$$\lambda_{dB} = \frac{h}{p}$$

where λ_{dB} is the de-Broglie wavelength of the particle, h is the Planck's constant and p is the particle's momentum.

c) A relativistic correction has to be done (hint to previous part to check if your answer is relativistic). In this limit, the electronic energy is now

$$E_{e,rel} = \frac{\pi^2 \hbar c}{4^{4/3}} \left(\frac{3}{\pi} \right)^{5/3} \frac{N_e^{4/3}}{R}$$

Perform a similar analysis as before, estimate the critical mass of HD85512, M_c , in terms of M_\odot . [2]

Note that if the star is above M_c , the star will contract. Check whether M_c is dependent on the parameters of HD85512. Moreover, your answer might differ from the well-known Chandrasekhar limit because of the many slipshod approximations used in this question. The proper derivation is beyond the scope of the Olympiad.

We thus conclude our question here. Looks like Tommy has to work harder in order to understand the exoplanetary system of HD85512.

4 Data Analysis: Seeing Double [40]

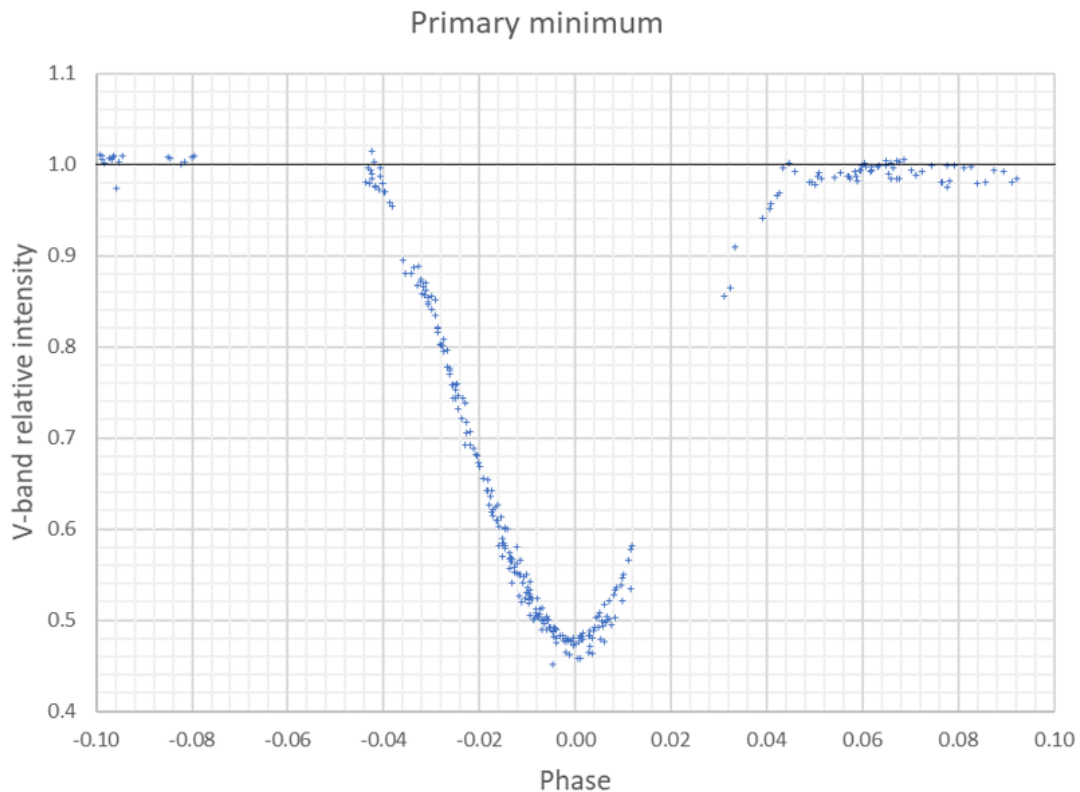
Z Herculis is an 7th magnitude eclipsing binary. You set up photoelectric detectors to detect changes in its apparent brightness, in both the B-band and V-band. Over the course of your observation, you manage to record both a set of deep primary minima, and a set of shallower secondary minima. You retrieve the timings and magnitudes of the V-band minima in the table below:

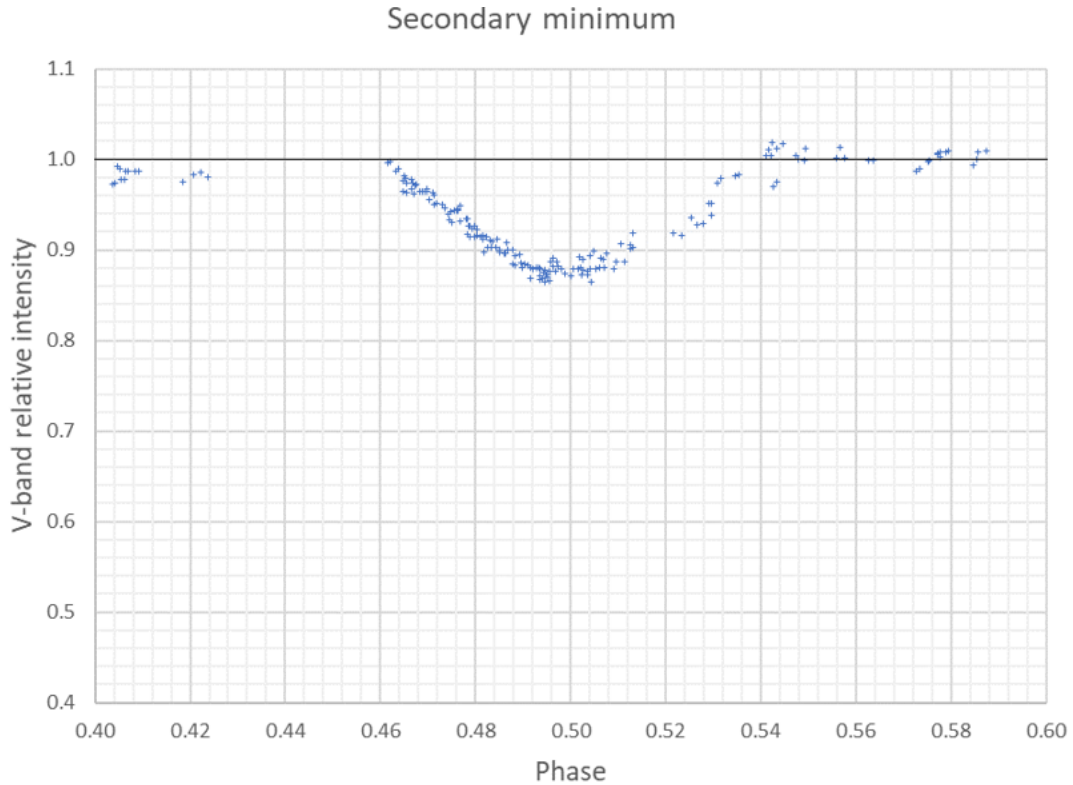
D/days	$\Delta m(V)$	P
44066.4870	0.876	0
44068.4697	0.221	
44070.4973	0.890	
44072.4988	0.231	
44078.4522	0.937	
44082.4446	0.864	
44088.4629	0.200	
44092.4240	0.231	
44106.4225	0.921	

a) The table above is incomplete. P represents the number of periods elapsed since the first primary minimum at $D = 44066.4870$ days. Copy the table onto your answer sheet, and complete the column with the appropriate values for each data point. [4]

b) Using the values from your table, plot a suitable graph to determine the period T of the variation in Z Herculis' visual magnitude. Give your answer to 3 significant figures. [8]

With this estimate of the period, you are able to reconstruct the light curve of Z Herculis. The graphs shown depict the data around the primary and secondary minima, plotted as the relative intensity in the V-band versus the phase (proportion of orbital period).



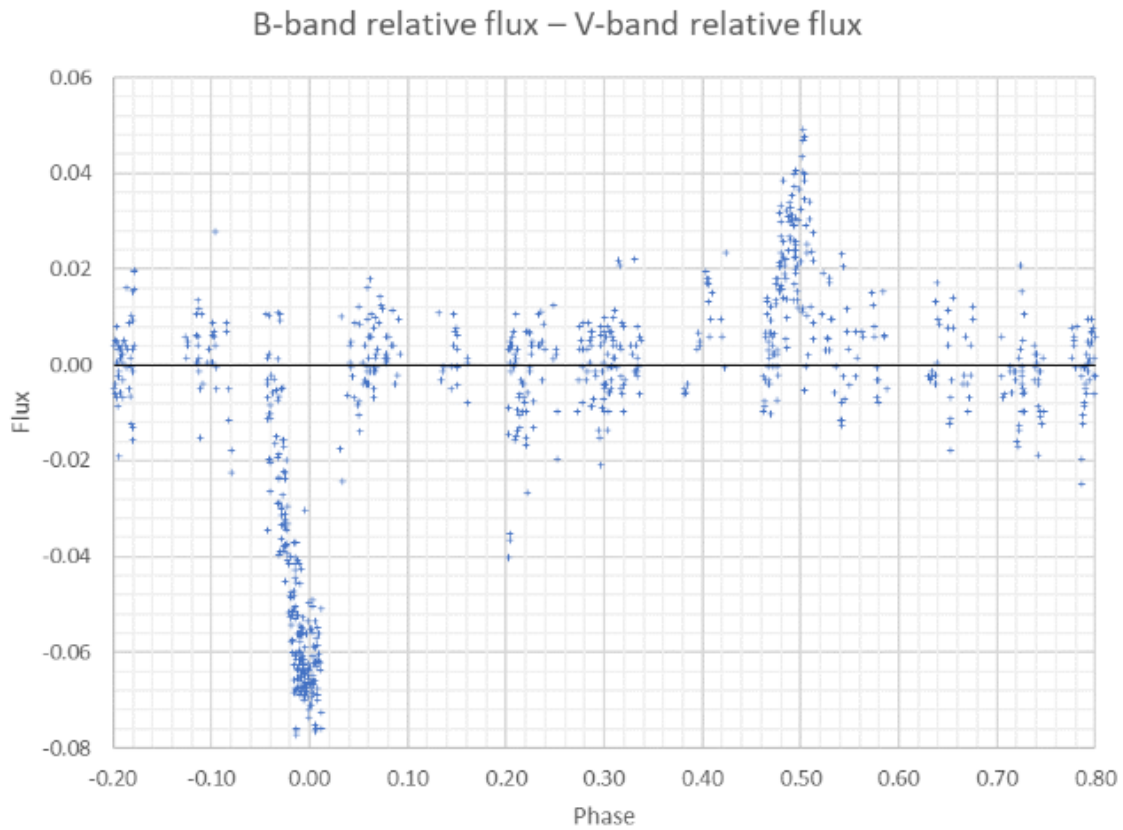


c) Estimate the following quantities from the light curves provided:

- τ_1 , duration of primary minimum as a fraction of the period [1]
- τ_2 , duration of secondary minimum as a fraction of the period [1]
- I_1 intensity of light at primary minimum [1]
- I_2 , intensity of light at secondary minimum [1]

d) Consider an eclipsing binary with two stars, one bigger and one smaller, where $k \leq 1$ is the ratio of their radii. When the smaller star is occulted by the larger star, a fraction $\alpha_0 \leq 1$ of the smaller star is obscured. Let Δi_{oc} be the fractional decrease in observed intensity when the larger star occults the smaller star, and Δi_{tr} be the fractional decrease in observed intensity when the smaller star transits the larger star. Assuming that the stars have uniform brightness across their discs, show that [2]

$$\alpha_0 = \Delta i_{oc} + \frac{\Delta i_{tr}}{k^2}$$



e) From your observational data, you also manage to draw a graph of the difference between the B-band relative intensity and V-band relative intensity of Z Herculis versus its phase. By analysing this graph, answer the following questions regarding the primary minimum:

- Is this an occultation or a transit? **[1]**
- Is the star being eclipsed the hotter or cooler star? **[1]**

f) Assume that the orbits of the stars are circular and being viewed edge-on from Earth (inclination = 90°), resulting in total eclipses ($\alpha_0 = 1$), and also that the stars have uniform brightness across their discs. From this point onwards, star A refers to the component of Z Herculis that is eclipsed during the primary minimum. Use the information obtained so far to derive the following quantities: **[5]**

- ρ_A , ratio of the radius of star A to the distance between the two stars
- ρ_B , ratio of the radius of star B to the distance between the two stars

g) Observe the shapes of the light curves at the primary and secondary minima. Do they support the assumption that total eclipses are being observed at the minima? Provide a brief explanation for your answer. **[2]**

You manage to find spectrometric data for Z Herculis. The radial velocities of its two components appear to vary sinusoidally with the same period as the combined visual magnitude. The table below lists the most positive and most negative values of radial velocity measured for each component star.

	$v_+/\text{km s}^{-1}$	$v_-/\text{km s}^{-1}$	$K/\text{km s}^{-1}$
Star A	40.5	-130.5	
Star B	60.0	-150.0	

We continue to use the same assumptions, that the orbits of the stars are circular and being viewed edge-on from Earth, and also that the stars have uniform brightness across their discs.

h) Find the amplitude of variation K in each star's radial velocity. **[2]**

i) Using the data, determine the following parameters: **[11]**

- d , the distance between the two stars
- a_A , the semi-major axis of A's orbit about the system barycentre
- a_B , the semi-major axis of B's orbit about the system barycentre
- m_A , the mass of Star A
- m_B , the mass of Star B
- R_A , the radius of Star A
- R_B , the radius of Star B

5 P(A) [10]

5.1 MCQ 1 [2]

On 15 Dec 2019, Darren will be on holiday in Sapporo, Japan (43.0621°N , 141.3544°E). Which of the following Deep Sky Objects (DSO) will he be unable to see? Assume perfect viewing conditions.

- (a) NGC 25 ($57^{\circ}01'15''$ S, RA 00h 09m 59.3s)
- (b) NGC 35 ($12^{\circ}01'15''$ S, RA 00h 11m 10.5s)
- (c) M93 ($23^{\circ}51'24''$ S, RA 07h 44m 30.0s)
- (d) M40 ($59^{\circ}23'50''$ N, RA 12h 23m 4.0s)

5.2 MCQ 2 [2]

Zhi Yuan wishes to host a Messier Marathon and brings out his 6" Equatorial Mount Newtonian Telescope. Below is a list of steps, in a disorganized fashion:

- (I) Attach optical tube assembly
- (II) Deploy tripod onto flat, stable ground
- (III) Mount accessories to telescope
- (IV) Attach counterweights to balance telescope
- (V) Place equatorial mount

Which of the following is the correct sequence of steps to accurately and safely set up his telescope?

- (a) II, V, IV, I, III
- (b) II, V, I, IV, III
- (c) II, V, I, III, IV
- (d) II, IV, V, I, III

5.3 MCQ 3 [2]

For visual observation, which telescope configuration would give the brightest image? Assume equal quality of optics.

- (a) 300mm length, 80mm aperture Schmidt-Cassegrain
- (b) 1000mm length, 150mm aperture Refractor
- (c) 600mm length, 150mm aperture Newtonian Reflector
- (d) Binoculars with $20\times$ magnification and 50 mm aperture

5.4 MCQ 4 [2]

To preserve night-adapted vision, astronomers carry around a special type of torchlight to illuminate their surroundings. What colour is the light?

- (a) UV Blacklight
- (b) Red
- (c) Green
- (d) Blue

5.5 MCQ 5 [2]

An astrophotographer, Zi Ming, wishes to image the ISS as it transits the Sun, using his mirrorless camera and a telephoto lens. What settings should Zi Ming prioritise to get the clearest image of the ISS?

- (a) Shutter Speed
- (b) ISO (Sensitivity)
- (c) Aperture
- (d) Temperature

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6 P(B) [28]

Fill in your Participant Code:

After completing this section, detach this section and attach them together with the rest of your answer sheet.

6.1 Constellation Identification [6]

For the Image P and Q below:

- Circle one star and identify its name.
- Circle one Deep Sky Object and identify its name.
- Identify one constellation and link the stars of the constellation together.

Image P: [3]



Image Q: [3]



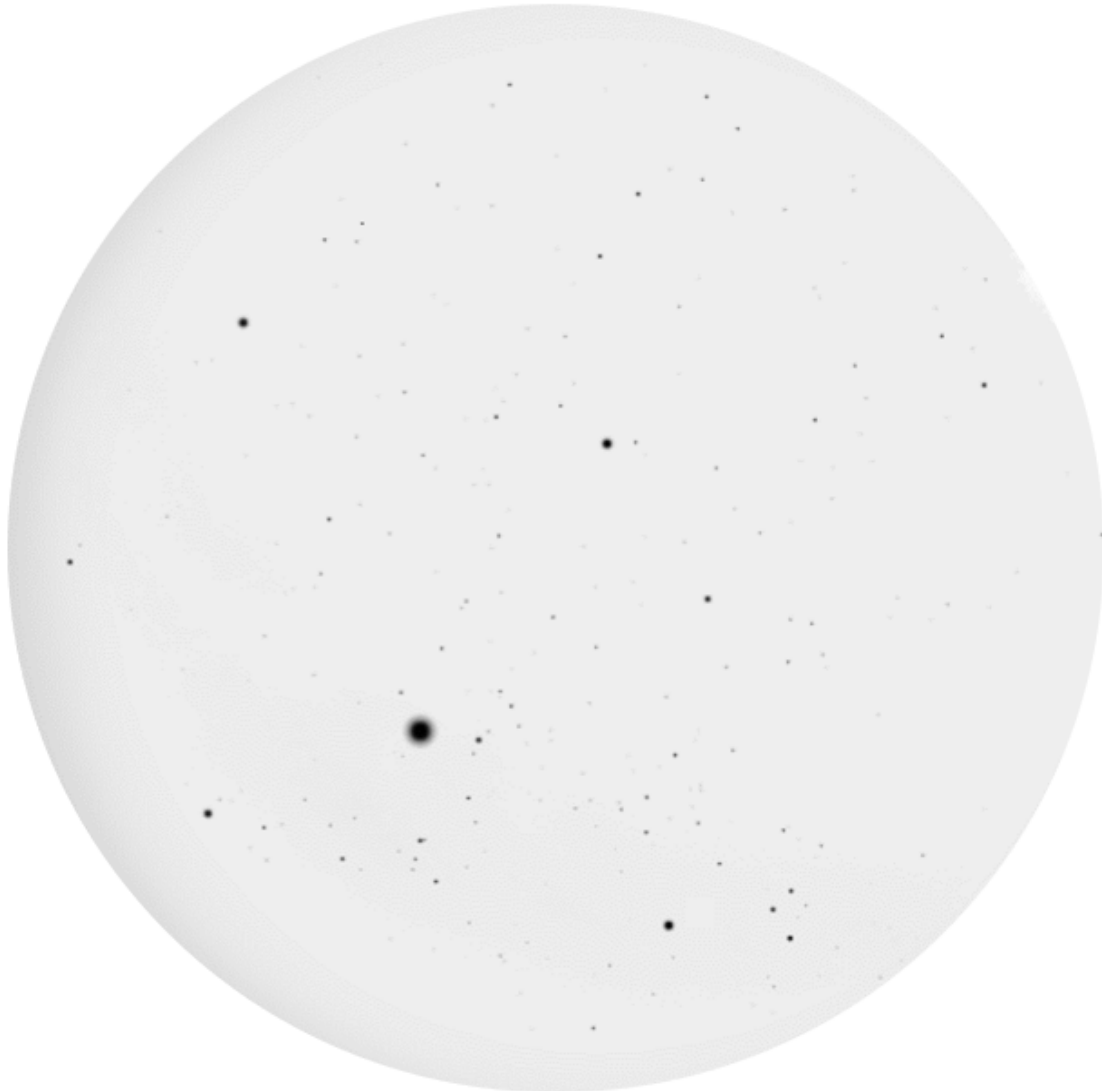
6.2 Winter Sky [9]

- (a) Circle and identify the star Canopus in the image. [1]
- (b) Connect the stars that make up the Winter Hexagon. [1]
- (c) Locate the position of Beehive cluster in the photo by putting a "X". [1]
- (d) Link and identify the constellation that represents the Nemean Lion which was killed by Hercules. [2]
- (e) Along the horizon (circumference of the star chart), mark out the approximate location of cardinal West, with a "W". [2]
- (f) Estimate the latitude of the observer (To the nearest 10°). [2]



6.3 Summer Sky [13]

- (a) Label and identify the two missing stars on the star chart. [4]
- (b) Label and identify any planet(s) on the star chart. [4]
- (c) Draw and label the plane of the Milky Way with “G”. [1]
- (d) Link and Label the constellation “*Corvus*”. [2]
- (e) Trace out the local meridian with a solid arc, and label it “M”. [2]



7 P(C) [22]

7.1 Part 1 [7]

Shown below is the Orion Atlas 10 EQ-G GoTo Newtonian Telescope of 1200mm focal length



- On the image, indicate the finder scope with a circle and label it with “F”. [1]
- “*The Atlas 10 EQ-G reflector features has a focal ratio of $f/4.7$ for breathtaking view*”. Explain what it means by focal ratio of $f/4.7$. [1]
- Briefly describe the structure of a Newtonian Telescope. [2]
- Indicate the direction of motion in declination by drawing a circular arrow around the appropriate axes. [1]
- Explain why it is preferred to use an Astro-modified DSLR to image Eta Carina Nebula using this telescope as compared to unmodified camera. [2]

7.2 Part 2 [12]

Shown below is the Celestron SkyMaster Giant 15x70 Binoculars.



- (a) Explain what is meant by exit pupil and calculate the exit pupil size of this binoculars. [2]
- (b) Calculate the magnification for this binoculars. [1]
- (c) The binoculars is waterproof, and nitrogen purged for use in all weather conditions. Explain why the binocular is nitrogen purged. [2]
- (d) Suppose on the day of SAO (March 23th), you are tasked to organize a stargazing event in Singapore (8pm – 11pm) with telescopes, binoculars and other commonly found objects in astronomy clubs. The committee passed you a list of objects to observe.
 - Pleiades
 - Orion Nebula
 - M41
 - Jewel Box Cluster
 - Moon
 - Mars

Choose any 4, briefly explain where you can roughly find the object, object's appearance. Due credit will be given if you explain why objects cannot be seen or are too difficult to be seen in Singapore. [8]

7.3 Part 3 [3]

You are given a list of the following planetary nebulae for an upcoming observatory session near the Equator. (Suppose all can be seen and the weather is clear).

Object	RA	Dec	Apparent Mag
M27 (Dumbbell Nebula)	19h 59m	22°43'	7.5
M57 (Ring Nebula)	18h 53m	33 °01'	8.8
M97 (Owl Nebula)	11h 14m	55°01'	9.9
M42 (Orion Nebula)	5h 35m	-5°23'	4.0
M11 (Wild Duck Cluster)	18h 51m	6°16'	5.8

- (a) Suppose that during one night, you notice that the Owl Nebula is setting. Other than the Owl Nebula, what objects are above the horizon right now? **[2]**
- (b) On one night, you notice a particular object in your list is near the Zenith. You also notice that Orion Nebula is setting in the West. What is this particular object? **[1]**

End of Paper