5th Singapore Astronomy Olympiad Suggested Answers

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Provided by 5th and 6th SAO Committees

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Q1 LIGO: A window into Binary Black Holes (18m)

<table>
<tr><td></td><td>Question-setter Remarks:
Strain (which is dimensionless) is a measure of the stretching of space, defined as the change in length divided by the initial length of space. Eg. if 1 meter of space became 1.5 meter, the strain would be 0.5.</td><td></td></tr>
<tr><td>a)</td><td>Candidates are allowed to use any measure of wavelength (crest-crest, trough-trough, zero-point crossing, etc.) to determine the answer, but it has to be around the region of maximum amplitude.

Given that, the accepted range of answers for the period T is
$0.0040\ s \leq T_{gw} \leq 0.0075\ s$.

Then, the corresponding range for frequency f_{gw} is
$250.00\ Hz \geq f_{gw} = \frac{1}{T_{gw}} \geq 133.3\ Hz$

Henceforth, we use the value $f_{gw} = 150.0\ Hz$.

Observe that as this binary system is composed of two black holes of equal mass, the system returns to an identical configuration after half an orbit. Hence, the actual orbital frequency ω is half that of the gravitational wave frequency. Then,

$\omega = 2\pi f_{orbit} = \pi f_{gw}$
$= \pi(150.0\ Hz)$
$= \mathbf{471\ rad\ s^{-1}}$

Hence, the allowed range for ω is
$418.9\ rad\ s^{-1} \leq \omega \leq 785.4\ rad\ s^{-1}$.

Award full credit to any answer within this range.</td><td>B1

M1

A1</td></tr>
<tr><td>b)</td><td>From (a), $f_{max} = f_{gw} = 150.0\ Hz$.

Rearranging the formula given, we get
$f_{max}^{-\frac{8}{3}} = \left(\frac{8\pi}{5}\right)^{\frac{8}{3}} \left(\frac{GM}{c^3}\right)^{\frac{5}{3}} (t_{merger} - t_{max})$
$\left(\frac{GM}{c^3}\right)^5 = \left(\frac{5}{8\pi f_{max}}\right)^8 (t_{merger} - t_{max})^{-3}$
$M = \left(\frac{5}{8\pi f_{max}}\right)^{\frac{8}{5}} (t_{merger} - t_{max})^{-\frac{3}{5}} \left(\frac{c^3}{G}\right)$
$= \left(\frac{5}{8\pi(150.0\ s^{-1})}\right)^{\frac{8}{5}} (0.4620\ s - 0.4230\ s)^{-\frac{3}{5}} \left(\frac{(3.00 \times 10^8\ m\ s^{-1})^3}{6.67 \times 10^{-11}\ kg^{-1}\ m^3\ s^{-1}}\right)$
$= \mathbf{7.06 \times 10^{31}\ kg = 35.5\ solar\ masses}$</td><td>A2</td></tr>
</table>

c)	The gravitational force between the black holes provide the centripetal force for their mutual orbital motion (in the barycentric frame), thus $$M\omega^2 r = \frac{GM^2}{(2r)^2}$$ $$\Rightarrow r = \left(\frac{GM}{4\omega^2}\right)^{\frac{1}{3}}$$ $$= \left(\frac{(6.67 \times 10^{-11}\ kg^{-1}\ m^3\ s^{-1})(7.06 \times 10^{31}\ kg)}{4(471\ s^{-1})^2}\right)^{\frac{1}{3}}$$ $$= \mathbf{1.74 \times 10^5\ m = 174\ km}$$	**A2**
d)	White dwarves have diameters $2R$ on the order of that of Earth (12756 km), much larger than the orbital radius r that was just calculated; hence it is **not possible** for this to be a binary white dwarf system. Although a neutron star is on the order of tens of km across, the mass limit for a neutron star is at most 3 solar masses, much less than the mass that was just calculated (35.5 *solar masses*), suggesting the system is likely to be a **binary black hole**. **Award 1m for refuting each hypothesis that the system is composed of binary white dwarves, or binary neutron stars. Both observations must be distinct; refuting both based on mass will only net 1 mark.**	**A1** **A1**
e)	The Schwarzschild radius of a black hole is the radius at which the escape velocity is equal to the speed of light (calculated non-relativistically), i.e. $$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$ $$\Rightarrow r = \frac{2GM}{v^2}$$ $Sub\ v = c\ and\ r = r_S,$ $$r_S = \frac{2GM}{c^2}$$ $$= \frac{2(6.67 \times 10^{-11}\ kg^{-1}\ m^3\ s^{-2})(7.06 \times 10^{31}\ kg)}{(3 \times 10^8\ m\ s^{-1})^2}$$ $$= \mathbf{1.04 \times 10^5\ m = 104\ km}$$	**A1**
f)	The case of two black holes of equal mass M orbiting a common barycentre can be reduced to that of a single body of reduced mass μ orbiting a fixed mass M at a distance $r = r_S$, where $$\mu = \frac{M_1 M_2}{M_1 + M_2}$$ $$= \frac{M^2}{2M}$$ $$= \frac{M}{2}$$	**M1**

	As the energy radiated is equal to the loss in orbital energy, then, $E_{gw} = -\Delta E$ $= E_i - E_f$ $= 0 - \left(-\frac{GM\mu}{2r_s}\right)$ $= -\frac{GM^2}{4r_s}$ $= -\frac{(6.67\times10^{-11}\ m^3\ kg^{-1}\ s^{-2})(7.06\times10^{31}\ kg)^2}{4(1.04\times10^5\ m)}$ $= \mathbf{7.99\times10^{47}\ J}$ **Answers utilising a sum of energies approach only receive partial credit as the orbital angular velocity ω of the two black holes calculated in part (a) at the time of maximum amplitude (1) differs from the orbital angular velocity ω' at merger (where the amplitude falls off) and (2) has a large uncertainty due to it being an estimate.**	**M1** **A1**
g)	The luminosity of the system is trivially equal to $L_{gw} = \frac{E_{gw}}{T}$ The flux received is then $b_{gw} = \frac{L_{gw}}{4\pi d^2}$ Comparing the ratio of the fluxes received from the Sun (the solar constant) to that from the binary black hole system, we have the apparent magnitude of the system, $m_{gw} - m_{Sun} = -2.5\lg\left(\frac{b_{gw}}{b_{Sun}}\right)$ $m_{gw} = m_{Sun} - 2.5\lg\left(\frac{L_{gw}}{4\pi d^2 T}\times\frac{4\pi a_E^2}{L_{Sun}}\right)$ $= m_{Sun} - 2.5\lg\left(\frac{L_{gw}a_E^2}{L_{Sun}d^2T}\right)$ $= -26.74 - 2.5\lg\left(\frac{(7.992\times10^{47}\ J)(1.496\times10^{11}\ m)^2\left(\frac{1}{0.20\ s}\right)}{(3.826\times10^{26}\ W)(4\times10^8\times3.086\times10^{16}\ m)^2}\right)$ $= \mathbf{-12.2}$ **Award the second 1m as well if the solar constant $b_{Sun} = L_{Sun}/4\pi a_E^2$ is calculated explicitly.**	**M1** **M1** **M1** **A1**

Q2 Sunrise (17m)

<table>
<tr>
<td>a)</td>
<td>As Singapore's local (civil) time is ahead of its geographical longitude (local solar time), we need to apply a correction Δt to the time of local solar noon

$$\Delta t = LT - \frac{\lambda}{360}(24^h)$$
$$= 8^{\mathrm{h}} - \frac{103.8}{360}(24^h)$$
$$= 1.08^h$$

This implies on average, the Sun is highest above the horizon (culminates) 1.08 hours after local noon, at which the civil time is $\mathbf{1:04:48\ pm}$.</td>
<td>M1

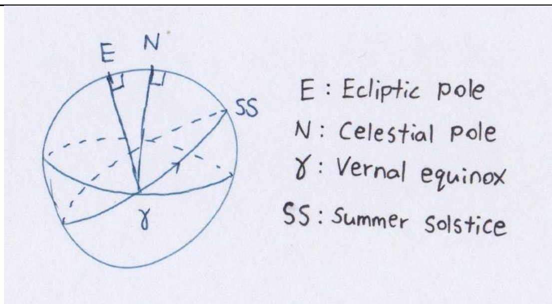
A1</td>
</tr>
<tr>
<td>b)</td>
<td>The time of sunrise occurs when the upper limb of the Sun appears over the horizon in the morning. Due to the angular size of the Sun and atmospheric refraction, this occurs at an angle $\Delta\theta$ below the geometric horizon,

$$\Delta\theta = \theta_{Sun} + \theta_{atm}$$
$$= 16' + 34'$$
$$= 50'$$

As Singapore is approximately at the equator, this corresponds to a correction of the time of sunrise by

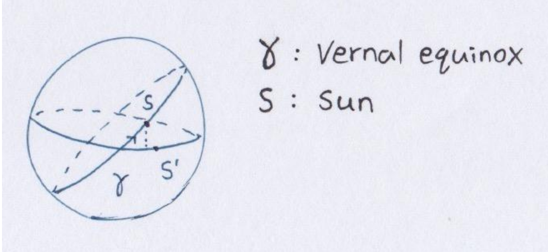
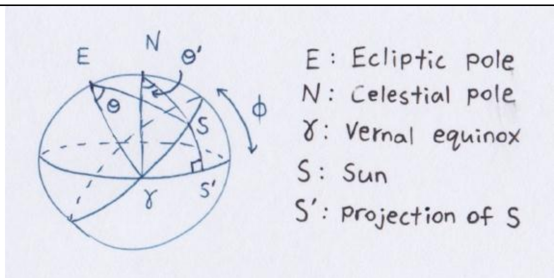
$$\Delta t = \frac{50'}{360 \times 60'}(24^h)$$
$$= 0.0556^{\mathrm{h}} = 3.33\ mins$$

Thus, the Sun rises about 6 hours 3.33 mins before culmination (at local solar noon), at **7:01:28 am**.</td>
<td>A1

A1</td>
</tr>
<tr>
<td>c)</td>
<td>

At the Summer Solstice, the Sun would have traversed exactly 90° from along the Vernal Equinox; this corresponds to a change in right ascension of exactly 90°, when projected onto the Celestial Equator (CE). Thus, sunrise occurs **neither earlier nor later than usual**.</td>
<td>M1

A1</td>
</tr>
</table>

d)	 The Sun travels along the ecliptic, which is inclined with respect to the Celestial Equator. Thus, it displaces eastwards at a rate slower than if it were on the CE (the projection of the angular velocity of the Sun parallel to the CE is smaller in magnitude due to the component of angular velocity in declination). The Sun would therefore be positioned slightly to the West to the next day, and as such, would rise **earlier**.	**A1** **A1**
e)	 With reference to the diagram, let $\theta = 2\pi x, 0 \leq x < \frac{1}{4}$ Consider the spherical triangle $\gamma SS'$; from the cotangent four-part rule we have $\cos\theta' \cos\phi = \cot\theta \sin\theta' - \cot\frac{\pi}{2}\sin\phi$ $\frac{\cos\phi}{\cot\theta} = \tan\theta'$ $\Rightarrow \theta' = \arctan(\tan\theta\cos\phi)$ **The above relation can also be derived from first principles (spherical cosine rule, polar triangles) and will score full credit.** The measured hour angle is given by $\alpha = \alpha_0 + \theta - \theta'$ Expressing in terms of x, hence shown $\alpha = \alpha_0 + 2\pi x - \arctan(\tan 2\pi x \cos\phi)$	**M1** **M1** **M1** **A1**
f)	<u>Solution 1: Numerical solution</u> The Sun rises earliest when α is a maximum, hence evaluating $\cos\phi$, $\cos\phi = \cos 23°30' = 0.917060$ Through numerical iteration, we find that the maxima occurs when $x = 0.1284, \alpha = 0.043278$	**M1** **M1**

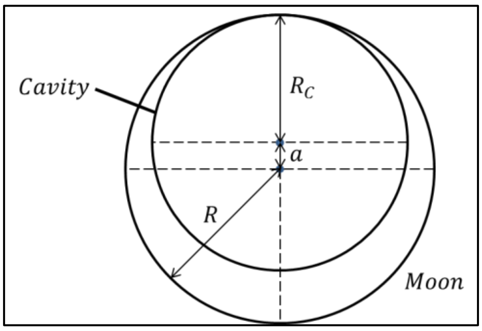
	Solution 2: Analytical solution using calculus From part (e), we have for $0 \leq x < 1/4$, $\alpha = \alpha_0 + 2\pi x - \arctan(\tan 2\pi x \cos\phi)$ First differentiate α with respect to x, $\frac{d\alpha}{dx} = 2\pi - \frac{2\pi\cos\phi\sec^2(2\pi x)}{1 + (\tan(2\pi x)\cos\phi)^2}$ $= 2\pi - \frac{2\pi\cos\phi\sec^2(2\pi x)}{1 + \tan^2(2\pi x)\cos^2\phi}$ To find the maxima, set $d\alpha/dx = 0$, then $\frac{2\pi\cos\phi\sec^2(2\pi x)}{1 + \tan^2(2\pi x)\cos^2\phi} = 2\pi$ $2\pi(1 + \tan^2(2\pi x)\cos^2\phi) = 2\pi\cos\phi\sec^2(2\pi x)$ $\cos^2(2\pi x) + \sin^2(2\pi x)\cos^2\phi = \cos\phi$	M1
	Introduce $\sin^2(2\pi x)$ on the LHS, then using difference of squares, $\sin^2(2\pi x) + \cos^2(2\pi x) - \sin^2(2\pi x) + \sin^2(2\pi x)\cos^2\phi = \cos\phi$ $\sin^2(2\pi x) + \cos^2(2\pi x) - (1 - \cos^2\phi)\sin^2(2\pi x) = \cos\phi$ $1 - \cos\phi = (1 - \cos^2\phi)\sin^2(2\pi x)$ $\sin^2(2\pi x) = \frac{1 - \cos\phi}{(1 + \cos\phi)(1 - \cos\phi)}$ $\sin(2\pi x) = \sqrt{\frac{1}{(1 + \cos\phi)}}$ $\Rightarrow x = \frac{\arcsin\sqrt{\frac{1}{(1 + \cos\phi)}}}{2\pi}$ $= \frac{\arcsin\sqrt{\frac{1}{(1 + \cos 23°30')}}}{2\pi}$ $= 0.1284$	M1
	Both solutions This corresponds to a time correction Δt of $\Delta t = \frac{0.043278}{2\pi}(24^h) = 9.92\ mins$ By symmetry, another maxima should occur at $x = 0.128 + 0.500 = 0.628$	
	Therefore the Sun rises earliest at <u>$\boldsymbol{x = 0.128}$ **and** $\boldsymbol{x = 0.628}$</u>, 9.92 mins before the time calculated in (b), at **6:51:33 am**.	M1 A1

Q3 Assassination classroom (13m)

<table>
<tr><td>a)</td><td>At a radius r from the centre of the Moon, the gravitational field is equal to the mass contained within r (shell theorem). We can write this in vector form,
$$g = \frac{GM}{r^3}\vec{r}$$
$$\because M = \frac{4\pi r^3 \rho}{3}, \therefore g = \frac{4G\pi\rho\vec{r}}{3}$$
Let us introduce a spherical cavity inside the solid sphere, with centre displaced by $\vec{a}$ from the centre of the sphere.

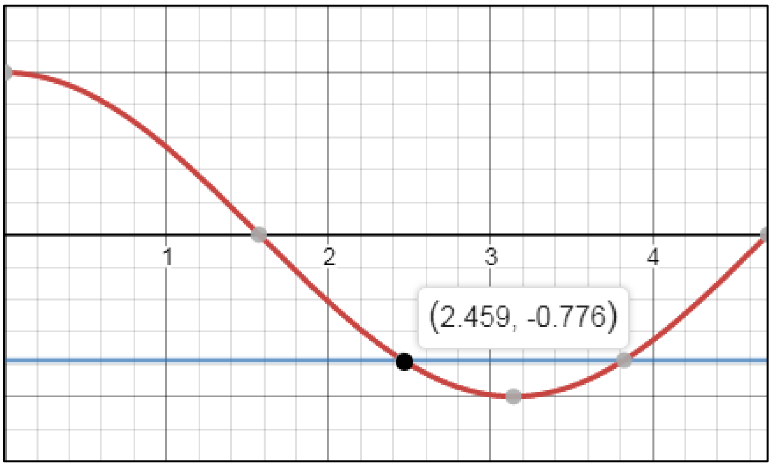
Then consider a point X inside the spherical cavity, with displacement vector $\vec{x}$ from the centre of the cavity. We can write the gravitational field at X as the g-field contributed by the solid sphere subtracted by the g-field contributed by the spherical cavity (similar to a sphere of negative mass),
$$g = \frac{4G\pi\rho(\vec{a}+\vec{x})}{3} - \frac{4G\pi\rho\vec{x}}{3}$$
$$= \frac{4G\pi\vec{a}}{3}$$
which is independent of $\vec{x}$. Hence proven.

<u>Alternative Marking Scheme</u>
1m – Participant demonstrates understanding cavity can be thought of as “negative mass”, but doesn’t continue
Max 1m – Participant uses integrals correctly but does not do so to completion due to complicated formula
2m – Other proper proofs showing the g-field is constant within the cavity</td><td>M1

A1</td></tr>
<tr><td>b)</td><td>Let R be the radius of the Moon, M the mass of the Moon. Then we have
$$\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$$


The radius of the cavity is
$$R_c = (0.7)^{\frac{1}{3}}R$$
$$= 1.542 \times 10^6\ m$$
Award this 1m for the correct value of R_C.</td><td>M1</td></tr>
</table>

	Then the distance between the centre of the Moon and the centre of the cavity is $$a = R - R_C = R - (0.7)^{\frac{1}{3}}R = R(1-(0.7)^{\frac{1}{3}}) = (1.737 \times 10^6\ m)(1-(0.7)^{\frac{1}{3}}) = 1.947 \times 10^5\ m$$ Then the magnitude of the gravitational field within the cavity is $$g_C = \frac{4G\pi\rho a}{3} = \frac{4G\pi}{3}\left(\frac{3M}{4\pi R^3}\right)R(1-(0.7)^{\frac{1}{3}}) = \frac{GM}{R^2}(1-(0.7)^{\frac{1}{3}}) = \frac{(6.67\times 10^{-11}\ kg^{-1}\ m^3\ s^{-2})(7.347\times 10^{22}\ kg)}{(1.737\times 10^6\ m)}(1-(0.7)^{\frac{1}{3}}) = 0.1821\ m\ s^{-2}$$	**M1**
	Using the kinematic equations, with $s = 2R_C, a = g_C, u = 0$, $$s = ut + \frac{1}{2}at^2 \Rightarrow 2R_C = \frac{1}{2}g_C t^2$$	**M1**
	$$t^2 = \frac{4R_C}{g_C}$$ $$\therefore t = \sqrt{\frac{4R_C}{g_C}} = \sqrt{\frac{4(0.7)^{\frac{1}{3}}R^3}{GM(1-(0.7)^{\frac{1}{3}})}} = \sqrt{\frac{4(0.7)^{\frac{1}{3}}(1.73\times 10^6\ m)^3}{(6.67\times 10^{-11}\ kg^{-1}\ m^3\ s^{-2})(7.347\times 10^{22}\ kg)(1-(0.7)^{\frac{1}{3}})}} = \mathbf{5820\ s}$$	**A1**
	<u>**Alternative Marking Scheme**</u> **4m – Participant integrates correct expressions to achieve correct final value** **2m – Participant integrates correct expressions but achieves incorrect final value**	
c)	Observe that the egg will fall straight through the Moon's centre – using the formula for gravitational acceleration, when the egg is a distance r from the Moon's centre, we have	

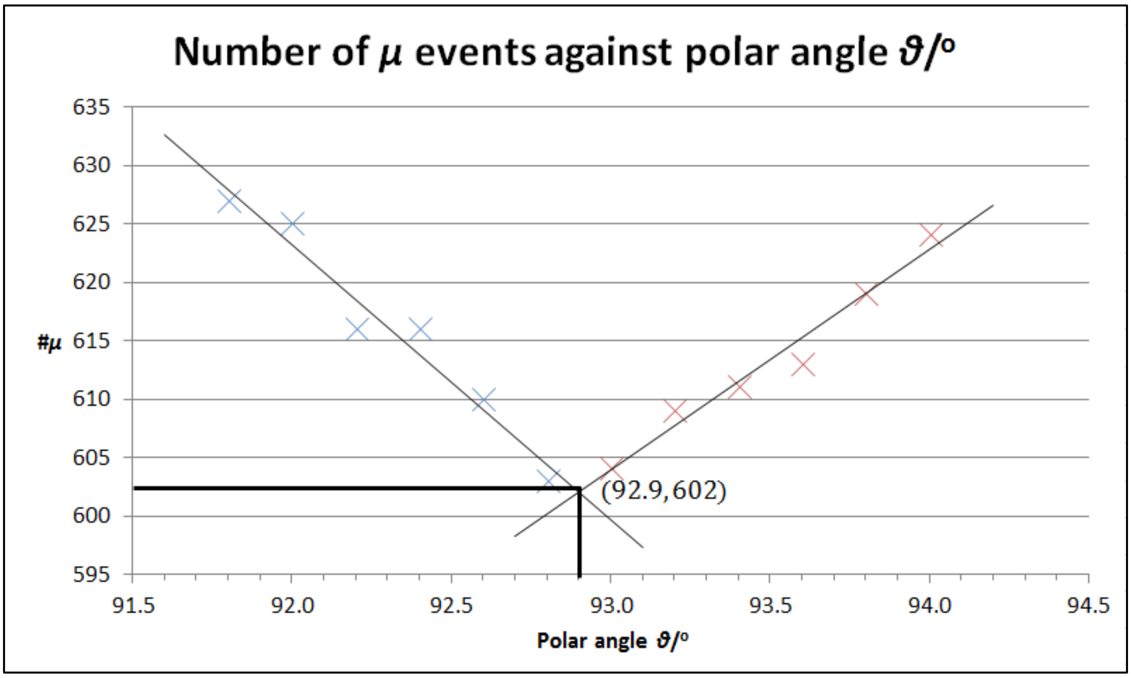
	$$g(r) = \frac{GM_{int}}{r^2} = \frac{G(\frac{4}{3}\pi r^3 \rho)}{r^2} = \frac{4\pi G\rho}{3} r$$	**M1**
	Observe again that the gravitational acceleration depends linearly on the radius r, implying the egg's motion is described by simple harmonic motion	
	For the egg's motion in SHM, $$\omega^2 = \frac{4\pi G\rho}{3} = \frac{4\pi G}{3}\left(\frac{3M}{4\pi R^3}\right) = \frac{GM}{R^3}$$	**M1**
	This 1m is also awarded for calculation of ω or period T.	
	Now, the fraction of a cycle the egg completes before hitting the ground, we draw the graph of the displacement of the egg, and a horizontal line corresponding to the displacement of the egg when it hits the ground, $$y = \cos x = \cos(2\pi\varphi)$$ $$y = \frac{R - 2R\left(0.7^{\frac{1}{3}}\right)}{R} = 1 - 2\left(0.7^{\frac{1}{3}}\right)$$	
		
	Since this is SHM, the phase (angle) is then, $$\varphi = \frac{\arccos\left(1 - 2\left(0.7^{\frac{1}{3}}\right)\right)}{2\pi} = 0.3913$$	**M1**
	This 1m is only awarded if the phase is calculated correctly.	

	Thus the time taken is simply $t = \varphi\left(\frac{2\pi}{\omega}\right)$ $= 2\pi\varphi\sqrt{\frac{R^3}{GM}}$ $= 0.3919(2\pi)\sqrt{\frac{(1.737 \times 10^6\ m)^3}{(6.67 \times 10^{-11}\ kg^{-1}\ m^3\ s^{-2})(7.347 \times 10^{22}\ kg)}}$ $= \mathbf{2550s}$ **<u>Alternative Marking Scheme</u>** **4m – Participant integrates correct expressions to achieve correct final value** **2m – Participant integrates correct expressions but achieves incorrect final value** **The 1m in part (b) for R_C – Participant finds the value $2R_C = 3084578\ m$ in either part (b) or part (c)**	**A1**
d)	Observe that when the egg fell from one end of the cavity to the other end of the cavity, its gravitational potential with respect to the cavity did not change: as if the cavity didn't affect the energy of the egg! The gravitational potential of the egg that fell through the cavity is thus the same as that of the egg in the narrow tunnel. **Hence they both had the same speed when they landed, i.e. the ratio is 1**. **<u>Alternative Marking Scheme</u>** **1m for each value – Participant calculates the final velocity for each of the two cases, hence showing they are the same.** **2m – Participant provides another correct (physics-wise) explanation and proves the statement.**	**A2**

Q4 Muon tomography (19m)

Part 1: Graph of angle to muon incidents

Plot the points on the given graph paper (see attached spreadsheet for the plot).



Observe that there is a minima somewhere between the points $(92.8°, 603)$ and $(93.0°, 604)$. Hence, divide the data set into two and plot two linear best-fit lines for each subset of data.

Reading off from the intersection of the two lines, the intersection point corresponds to $(92.90, 602)$.

Marking Points:

(1) Points are plotted correctly with decent use of space (minimally 70% of area of graph paper) **M1**

(2) Graph is properly titled with units **M1**

(3) Axes are correctly labelled with units **M1**

(4) Two best-fit trend lines are drawn. **All other curves (quadratic, exponential) score no credit.** **M1**

(5) Intersection point of both lines clearly marked out, with $\theta = 92.90° \pm 0.02°$ and $\#\mu = 602 \pm 0.5$. **No credit at all is awarded if intersection is not within <u>both</u> the allowed tolerances.** **A2**

Part 2: Analysis to find distance to apex of pyramid

At the polar angle $\theta = 92.90°$, the muon flux is

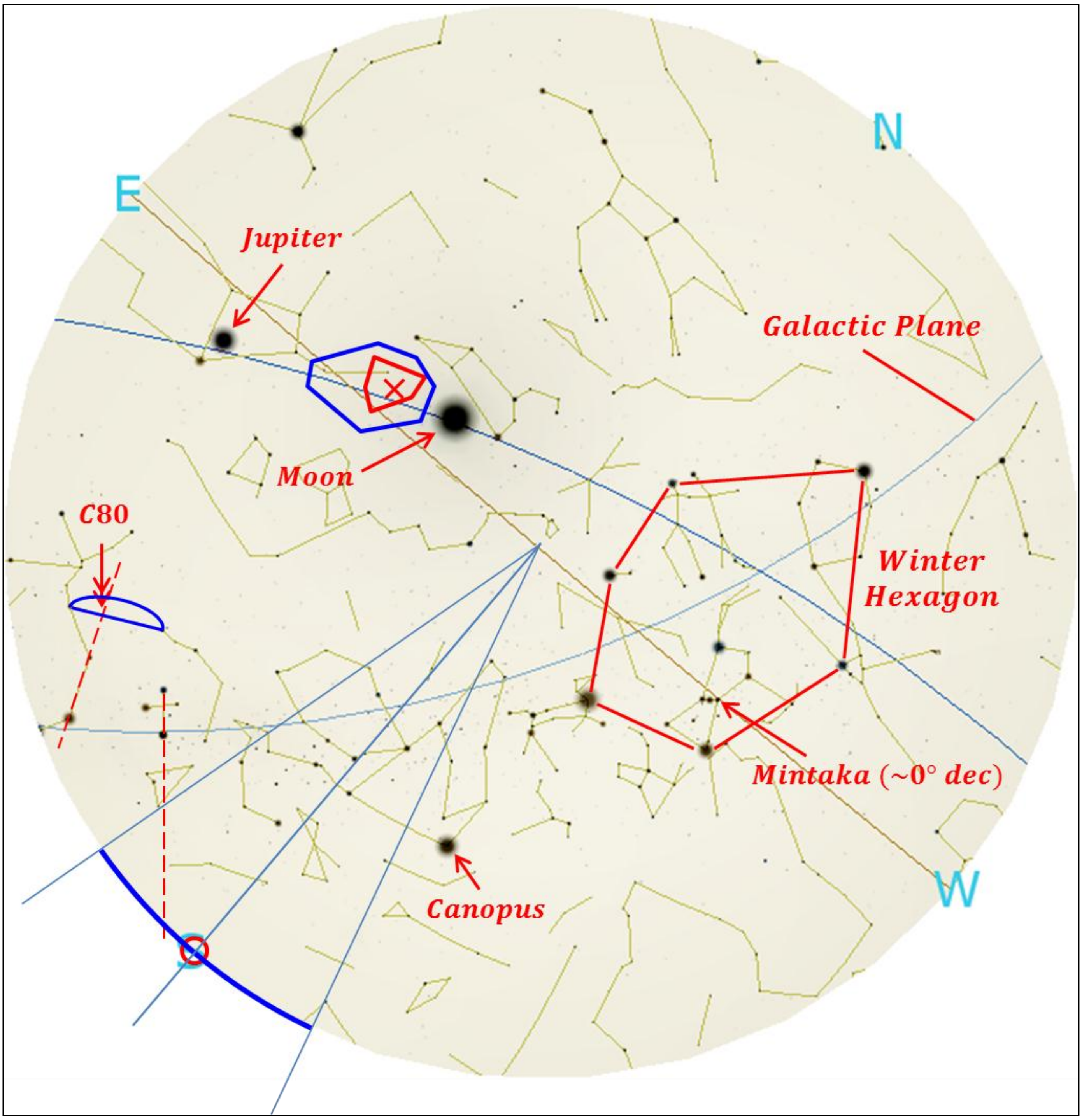
$$j = \frac{\#\mu}{T}$$
$$= \frac{602.0}{6 \times 60 \times 60}$$
$$= 0.02787\ events\ s^{-1}$$ **M1**

Using the given equation for the incidence rate of high energy muons in excess of energy E_0, then this minimum energy is $j = (\frac{37}{E_0^{1.7}})$ $\Rightarrow E_0 = \left(\frac{37}{j}\right)^{\left(\frac{1}{1.7}\right)}$ $= \left(\frac{37}{0.02787}\right)^{\left(\frac{1}{1.7}\right)}$ $= 68.72\ GeV$	**M1**
Since the detector can only detect muons above an energy of $E_{det} = 1\ GeV$, then this implies the muons have lost an energy ΔE whilst travelling through the pyramid's interior of $\Delta E = E_0 - E_{det}$ $= 68.72 - 1.00$ $= 67.72\ GeV$	**M1**
The equation given for the energy loss of a muon as it travels through a distance Δx through the pyramid's interior of uniform density ρ is $\Delta E = (-2\ \frac{MeV\ cm^2}{g})\rho\Delta x$ Converting the units to those used in the question we have $\Delta E = \left(-2\ \frac{GeV}{1000} * \frac{m^2}{100^2} * \frac{10^3}{kg}\right)\rho\Delta x$ $= \left(-\frac{2}{10^4}\right)\rho\Delta x$	**M1**
Substituting the values $\Delta E = -67.72\ GeV$, $\rho = 2750\ kg\ m^{-3}$, we find $\Delta x = -\frac{10^4 \times \Delta E}{2\rho}$ $= -\frac{10^4(-67.72\ GeV)}{2(2750\ kg\ m^{-3})}$ $= 123.2\ m$	**A1**
<u>Part 3: Geometry</u> Hence the offset from the origin (centre of pyramid) in the x direction is just $X = \Delta x \cos\theta$ $= (123.2) \cos 92.90°$ $= \mathbf{6.23\ m}$	**M1**
The perpendicular distance to the apex of the pyramid is $h = \Delta x \sin\theta$ $= (123.2) \sin 92.90°$ $= 123.0$ m	**M1**
Thus, we are a fraction of the distance to the apex of the pyramid of height H	

$f = \frac{h}{H}$ $= \frac{123.0\ m}{140.0\ m}$ $= 0.8784$ This implies our offset along the y direction (along the diagonal of length L of the square pyramid of base length w) is $\Delta y = (1 - f)L$ $= (1 - f)\left(\frac{w}{2}\right)\sqrt{2}$ $= (1 - 0.8784)\left(\frac{230}{\sqrt{2}}\right)$ $= \mathbf{19.77\ m}$	**A1**
Part 4: Accuracy of Final Answer and Conclusion **Therefore, the detector was placed at $(\mathbf{6.23, 19.77})$.**	**A4**
Award 2m for answers of X within $\pm \mathbf{0.04}$ of the above value, and only 1m for answers of X within $\pm \mathbf{0.08}$ of the above value. **Award 2m for answers of Y within $\pm \mathbf{0.10}$ of the above value, and only 1m for answers of Y within $\pm \mathbf{0.20}$ of the above value.**	
However, observe that the situation is symmetric in the y-direction, **hence the detector may also be placed at $(\mathbf{6.23, -19.77})$**, as this would produce the same dataset.	**A1**

QP Practical astronomy (written) (23m)

Task A (11 marks)



a)	See above annotated star chart for the correct answer. **Award 1m for connecting <u>all</u> vertices correctly (Capella in Auriga, Aldebaran in Taurus, Rigel in Orion, Sirius in Canis Major, Procyon in Canis Minor, Pollux in Gemini). Castor and Betelgeuse are incorrect vertices.**	**A1**

b)	Observe that the long (vertical in the star chart) arm of the constellation Crux points approximately at the South Celestial Pole, which due to Singapore's equatorial location, crudely approximates the south cardinal point. See above annotated star chart for the answer. **Award 1m for marking out cardinal South correctly, anywhere along the blue arc along the horizon (corresponding to 15 degrees on either side of true cardinal South)**.	**A1**
c)	See above annotated star chart for the answer. **This 1m is awarded for a smooth arc that passes through any part of <u>all</u> of these constellations: Centaurus, Crux, Vela, Puppis, Monoceros, Auriga, which lie along the galactic plane.** **Reject answers that are straight lines (the galactic plane is inclined with respect to the celestial equator).** **Reject all answers where the end points of the arc are separated by less than 160° or more than 200° along the circumference of the star chart (since the galactic equator is a great circle on the celestial sphere)**.	**A1**
d)	See above annotated star chart for the answer. **Award 1m for marking out Canopus, the brightest star south of Canopus in the star chart.**	**A1**
e)	Recall that the brightest globular cluster visible in the night sky is C80, the Omega Centauri cluster in Centaurus. See above annotated star chart for the answer. **Award this 1m for any answers within the blue semi-ellipse.**	**A1**
f)	Recall that Orion lies along the celestial equator – and that one notable feature of the Belt of Orion are the declination of the belt stars, that is, their declination is close to 0°. As an estimate, we take *Mintaka's* declination to be 0°, such that it would culminate at the zenith (centre of the starchart) when the local sidereal time is equal to its right ascension. From part (b), we know that *Mintaka* is to the west of the local meridian, i.e. it has already culminated and thus its hour angle is $HA > 0$. Thus, we measure Mintaka's distance from the centre of the starchart (zenith), to estimate its hour angle, $d = 3.7\ cm$	

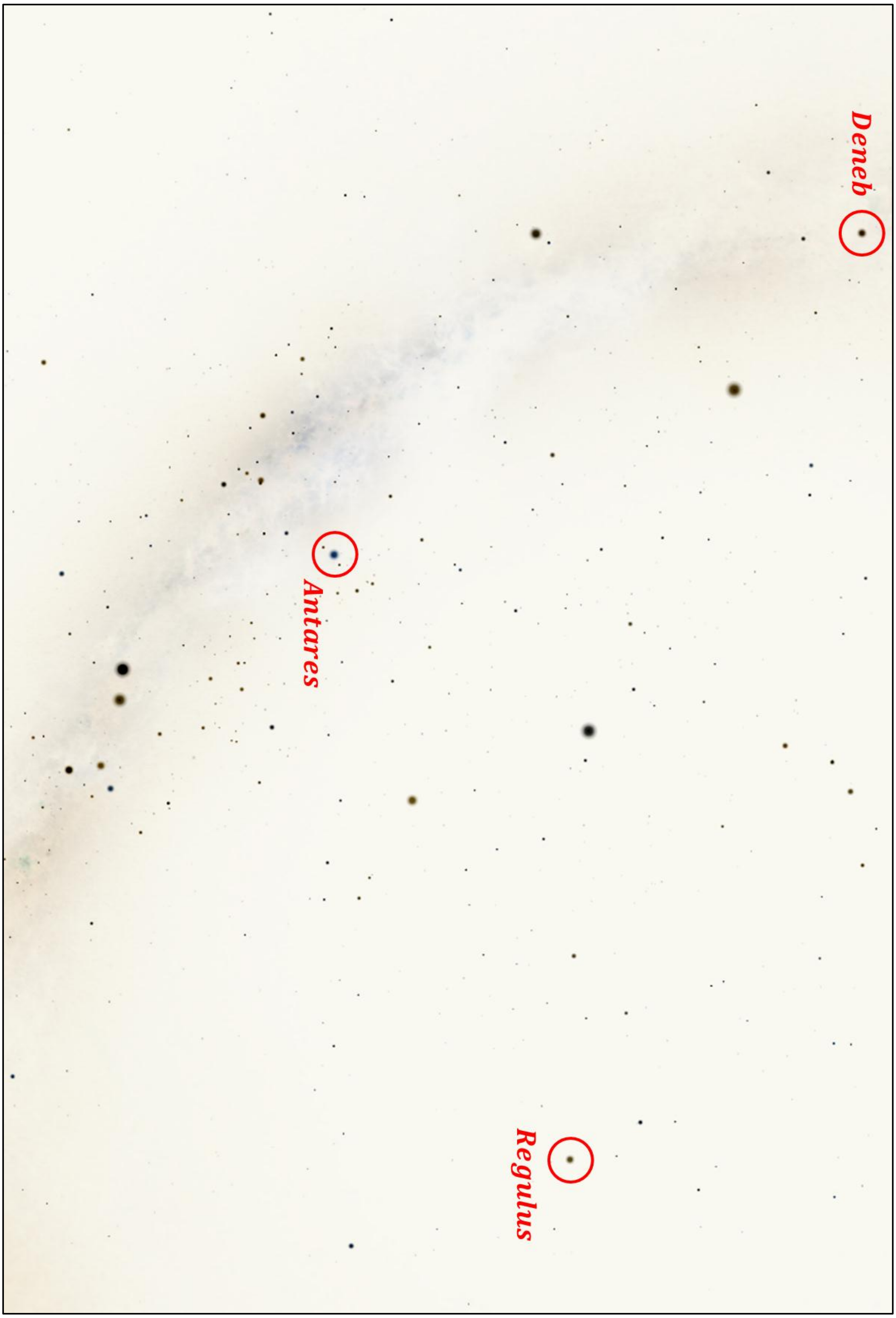
	Then the fraction outward from the centre of the starchart of radius r is $$f = \frac{d}{r} = \frac{3.7\ cm}{8.7\ cm} = 0.4252$$ Then, to a first approximation, ignoring distortion from the stereographic projection, the hour angle HA of *Mintaka* from the zenith is approximately $$HA = (90°)(0.4252) = 38.28° = 2.55^h$$ (In reality, the actual HA of Mintaka in the star chart is 3^h10^m.) Then the local sidereal time at the time of the starchart is $$LST = HA + \alpha = 2.55^h + 5^h33^m = 8^h06^m$$ As the star chart is dated 11 Mar 18, close to the vernal equinox, we can take the LST_{0000} at local solar midnight as approximately $12^h - 10 \times 4^m = 11^h20^m$. Then, to find the local sidereal time at local civil midnight, we need to introduce a correction as Singapore's timezone (UTC+8) is not in alignment with its longitude, $$LST'_{0000} = LST_{0000} - \Delta LST = LST - \left(LT - \frac{\lambda}{360}(24^h)\right) = 11^h20^m - \left(8^h - \frac{103.8°}{360°}(24^h)\right) = 10^h12^m$$ Therefore, the time before midnight ΔT is simply the difference in the local sidereal times at the time of the starchart and at local civil midnight $$\Delta T = LST'_{0000} - LST = 10^h12^m - 8^h06^m = 2^h06^m$$ Therefore, the local time of the star chart is 0000h – 0206 = 2154h ~ **2200h** to the nearest 15 minutes. **The actual local time of the star chart is 2230h, i.e. 10.30pm.** Based on experience, a much simpler estimate around 2200h – 2300h can be recalled given the altitude of Jupiter above the horizon in the prevailing weeks. **Award 2m for answers within 1.5h (inclusive) of the actual local time.** **Award only 1m for answers within 3h (inclusive) of the actual local time.**	**A2**

g)	First, observe that all stars and objects in the star chart have their sizes scale by their brightnesses in the night sky. Then, we notice there are two objects larger in size in the star chart than *Sirius*, the brightest star in the night sky. Given that the Moon is hinted at being present by part (h), therefore there is likely to be one planet visible. Participants are expected to recall from experience, that during the weeks leading up to the 5th SAO, Jupiter rises shortly after twilight and is visible – hence the planet is likely to be **Jupiter**. See above annotated star chart for the location of Jupiter. **Award the first 1m for correct identification of Jupiter as the visible planet, and the next 1m for correct location of Jupiter (even if another planet was identified in its place).**	**A1** **A1**
h)	At the same local (civil) time on the next day, the night sky would have rotated westward by an amount approximately 4^m in RA. However, due to the Moon's orbital motion about the Earth (anti-clockwise), it moves eastward relative to the fixed stars and its RA increases by approximately $\Delta RA = \frac{360°}{T_{sid}} = \frac{360°}{27.3\ days} = 13.19\ °/day \approx 53^m/day$ Since the Moon's orbital plane is similar to the rest of the planets, it will **thus appear eastward along the ecliptic by about 50 mins**, or given the scale of the starchart, about 1 cm eastward along the ecliptic. See the annotated star chart for the location of the Moon on the next day. **Award 2m if the Moon if the answer is located within the red polygon (about 50^m eastward and $3 - 5°$ north of the ecliptic). Award only 1m if the answer is located within the blue polygon (eastward of its position in the starchart and close to the ecliptic).**	**A2**

Task B (3 marks)

a)	Observe that the field of view shows no distinct nebulae, galaxy or tight cluster of stars (globular cluster), hence the object is likely an open star cluster. From the V-shape arrangement of the stars (forming the head of the bull), the object is **(The) Hyades (star cluster)/C(aldwell) 41**.	**A1**
b)	By comparison, the star in the crosshairs is intermediate in brightess between the star of apparent magnitude 3.50 and the star of apparent magnitude 3.75. Hence, the apparent magnitude of the star is **3.65**. **All answers between 3.50 and 3.75 (inclusive) score full credit**.	**A1**
c)	Since this deep-sky object is the Hyades, an open star cluster relatively close to the Sun, its angular size in the sky should be significant. Given the V-shaped arrangement of stars in Taurus can be easily seen in the night sky, the true FoV of this *instrument* is $\underline{\mathbf{6.7^\circ}}$, and the instrument is likely to be either a **finder scope or pair of binoculars**. **Award this 1m only if the estimated FoV is between $\mathbf{5.0^\circ - 8.0^\circ}$ and the instrument is one of the two above**.	**A1**

Task C (3 marks)



First, observe that the constellation of *Leo* is incomplete, and missing its brightest star *Regulus*. Next, observe that the constellation of *Scorpius* is missing its brightest star *Antares*, whose brightness in the night sky typically rivals that of β *Centaurus (Hadar)*. Lastly, observe that two of the three stars in the Summer Triangle, *Vega* in *Lyra*, *Altair* in *Aquila* are visible – but not the third star *Deneb* in *Cygnus*, which will complete the asterism to form an approximate right-angled triangle. See above annotated star chart for the locations and names of the three missing stars. **Award 1m for each missing star correctly identified and marked out within 3mm from its actual position (within the respective red circle)**.	**A1** **A1** **A1**

Task D (6 marks)

a)	The comic strips are to be read downwards then rightwards. Observe that in each panel, the position of the innermost moon (which says "Hi!") remains unchanged, i.e. each panel progresses by 1 period of the innermost moon (Io). Hence, we see that the second moon's period is twice that of the first moon's, the third moon's period is four times that of the first moon's. **Award 1m if all four moons are identified correctly:** Io (innermost Galilean moon) – "Hi!" Europa (second Galilean moon, smallest radius) – "What's your name?" Ganymede (third Galilean moon, largest radius) – "MOOOOOON." Callisto (outermost Galilean moon) – "So annoying", "YESSSS!" The comic strip depicts the **1:2:4 Laplace resonance (of orbital periods) between Io, Europa and Ganymede. Callisto is not part of the orbital resonance**. **This resonance is preserved by mutual gravitational and tidal interactions between the moons.** As each comic pane progresses by one Io orbital period, these three moons return to their initial configurations every four comic panes. **Award 2m for explaining the orbital resonance between the inner three moons with their relative periods identified.** **No credit is awarded for the phenomena of "gravitational capture" or "escape" – only the last panel of the comic strip depicts this, and there are no details in the panel to physically explain this phenomenon.**	 **A1** **A1** **A1**
b)	**There will be an increased contrast of M31 overall relative to the background sky**, as the filter reduces the light pollution signal more than the Hα signal from stars in M31. This is because artificial light pollution tends to be primarily composed of a few discrete wavelengths (e.g. from LP sodium lamps), whereas stars emit like blackbodies, where Hα is part of the continuum. **The Hα image will also show the spiral arms of M31 and H II regions more prominently** as compared to the real colour image. Star formation regions are concentrated in the spiral arms due to mass density waves. The young, hot stars emit plenty of ultraviolet radiation, partially ionising the surrounding molecular cloud, forming the H II regions. Due to the opacity of the molecular cloud, **a significant amount of the continuum radiation from these massive stars are absorbed and subsequently re-radiated as Hα.** Hence, as star formation regions emit significantly in Hα, **their contrast is enhanced in Hα images as the continuum emission from other older stars in the disc and the galactic bulge (core) is filtered out**. **Accept all other well-reasoned answers, provided they are supported by discussion of relevant spectra and sound astrophysical processes.**	**A1** **A1** **A1**