5^{th} Singapore Astronomy Olympiad

18 March 2017

Organized by:

In partnership with:

I hereby consent to provide my details to the organizer, Astronomy.SG.

Full Name: ____

School: ____

NRIC/FIN: ____

Email address: ____

Signature: ____

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Question	**Score**
1	/18
2	/17
3	/13
4	/19
Practical	/23
Total	/90

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<u>Rules:</u>

- Students may use pen (preferably with blue or black ink) or pencil to answer the paper.
- Students are to write their answers on the blank paper provided, as well as on the practical portion of this question paper. Any working or answers on the rest of the question paper will not be considered.
- Only the use of scientific calculators is allowed.
- Graphing/programmable calculators are **not** allowed.
- Students must use their own stationery.
- Students have 3.5 hours to complete the paper. If a student is late, no time extension is granted.
- Students may leave any time upon submission of their solutions and question paper.
- Cheating or allowing others to cheat are grounds for immediate disqualification.
- No notes or help-sheets are allowed.
- A constants sheet is provided separately for the students' use.
- Students should **clearly state their assumptions** in their working, if any.
- Students should do their best to attempt all questions and maximize their total marks.

- <u>Upon completion of the paper, do the following:</u>
  1. Separate the cover page AND the practical portion from the question paper.
  2. Staple together: The cover page, the practical, and your answers for the rest of the questions. This will be your answer script.
  3. Return Question papers to the 'Question papers' pile.
  4. Return your answer script to the 'Answers scripts' pile.
  5. You may either return the constants sheet together with the Question paper or choose to keep it for future reference.

Q1. LIGO: A window into Binary Black Holes

The first direct gravitational-wave detection was made by the Advanced Laser Interferometer Gravitational Wave Observatory (LIGO) on September 14, 2015. The signal, dubbed GW150914, was strong enough to be apparent in the filtered detector strain data. In this question, we can analyse (using Newtonian Physics, thankfully!) certain features of the system from which the waves originated from. Note that you do not need to know the intricacies of LIGO's detectors, nor do you need to know how interferometry or General Relativity works for this question. Below is a graph of LIGO data from the detector in Hanford, Washington.

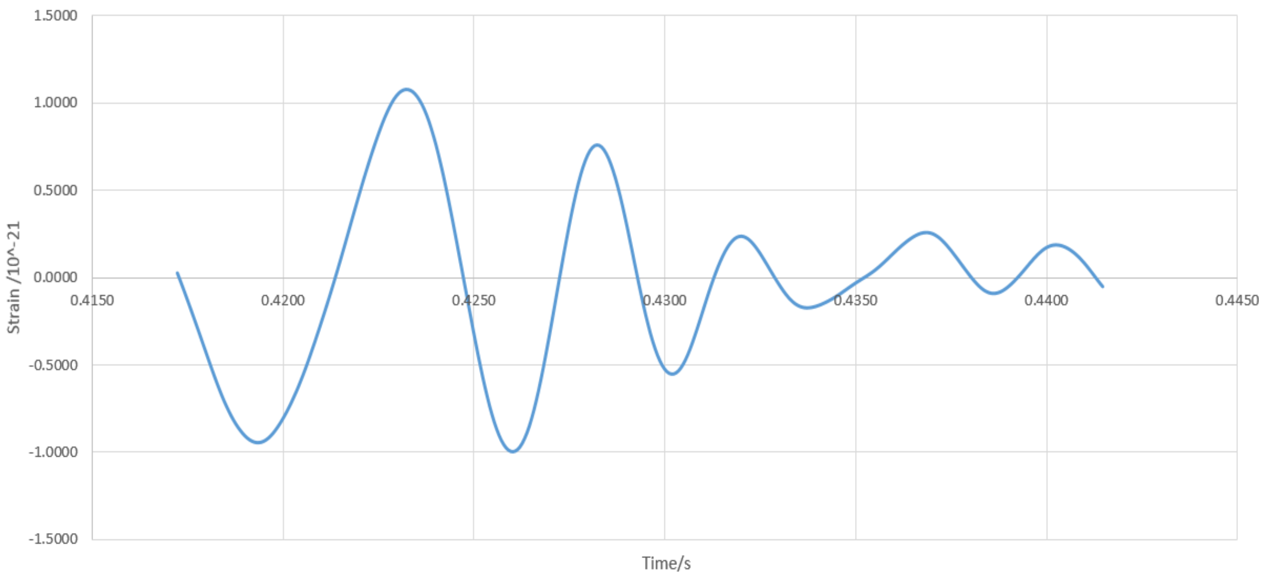

Figure 1: Zoomed in LIGO Data from Hanford, Washington.

Figure 1 above describes the exact final moments before two black holes merge into each other. Strain is a measure of the stretching of space caused by the black hole system. The first part of the graph, where gravitational waves reach peak amplitude, describes the end of the 'inspiral' stage, when the black holes' orbits around each other shrink to a minimum. This is followed by what's known as the 'ringdown' stage, where the amplitude decays off, and the two black holes gradually merge together as one.

<u>Part 1:</u>

You may imagine the two black holes to be identical Schwarzschild (ie. Non rotating, non-charged) black holes, revolving around each other in a 'common' circular orbit, and that the radius of the orbit is slowly shrinking.

a) Using the graph, estimate the frequency of gravitational waves at <u>maximum amplitude</u>. [1]

b) Hence, determine the **angular frequency** of the orbit at the same point in time. [2]

The rate of loss of orbital energy through gravitational waves in the nonrelativistic approximation can be calculated using results derived by Einstein. We need only use the eventual result, which links the frequency of gravitational waves at maximum amplitude (f_{max}) to the time of maximum amplitude(t_{max}), the mass of each black hole (M), and other constants (G, c, π):

$$f_{max}^{-\frac{8}{3}} = \left(\frac{8\pi}{5}\right)^{\frac{8}{3}} \left(\frac{GM}{c^3}\right)^{\frac{5}{3}} (t_{merger} - t_{max})$$

Note that t_{merger} is the theoretical time when the black holes merge completely. You may take it to be at 0.4620 seconds in this case, which is beyond the graph's range.

c) Hence, determine the mass of each black hole in solar masses. [2]

d) Determine the radius R of the orbit at the point of maximum wave amplitude. [2]

e) Comment on and justify/refute whether this system could actually be a binary system composed of white dwarves, or neutron stars, as opposed to black holes. [2]

Part 2:

f) From Part 1, What is the event horizon, r_s of each black hole, the radius beyond which light cannot escape? [2]

g) We can approximate the energy radiated out as gravitational waves by considering the loss in orbital energy (defined as the sum of kinetic energy and gravitational potential energy of the black holes) from when the black holes are infinitely far apart, till when each of the black holes' centre has crossed the other black hole's event horizon (ie. Merger). By this definition, calculate the total energy radiated out as gravitational waves. [3]

h) Let us assume (and in fact this is fair) that all of the energy in (f) was radiated over 0.20 seconds. We can also ascertain that the luminosity distance to the binary black hole system is 400 Mpc. Suppose all the energy in the gravitational waves was converted to light and radiated at a constant rate over the above time period. What would be the apparent magnitude of the binary black hole system? Use the Sun's apparent magnitude and the Sun's brightness as seen from Earth as a reference. [4]

Q2. Sunrise

We all know that the Sun rises in the East, travels in an arc across the sky, reaches right above our heads at noon and sets in the West. As such, it would be reasonable to think that the Sun rises at 6am every day, exactly six hours before noon. However, the time of sunrise in Singapore is actually closer to 7am, and to make matters more complicated, it fluctuates throughout the year. Let us consider the effects that contribute to the actual observed time of sunrise:

- Singapore is located at a longitude of 103.8°E but follows time zone UTC+0800. The Sun is not directly overhead at noon.
- The Sun cannot be treated as a point source. It has an angular diameter of 32 arcminutes. Sunrise is defined as the time at which the top edge of the Sun is in line with the horizon.
- For celestial objects near the horizon, the curvature of the atmosphere bends light downwards by an angle of about 34 arcminutes. As such, the Sun appears to rise before it is geometrically in line with the horizon.
- The Sun does not travel exactly along the celestial equator. Instead, it travels along the plane of the ecliptic, at an angle of 23.5° from the celestial equator.
- The Earth does not orbit the Sun in a perfect circle. The eccentricity of its orbit is 0.0167. Therefore, the Sun does not move across the sky at a constant speed.

In the questions that follow, we will consider the first four effects, but neglect the last. Take Singapore to be located exactly on the Equator.

a) On average, when is the Sun highest above the horizon? [2]

b) On average, what time does sunrise occur? [2]

c) At the summer solstice, does sunrise occur earlier or later than usual? Explain with a suitable diagram. [2]

d) At the vernal equinox, will sunrise occur earlier or later the next day? Explain with a suitable diagram. [2]

e) The hour angle (celestial longitude relative to the observer) of the Sun as measured at the same time every day is given by the following expression.

$$\alpha = \alpha_0 + 2\pi x - \tan^{-1}(\tan 2\pi x \cos\phi), \qquad 0 \le x < \frac{1}{4}$$

α_0 is the hour angle of the Sun as measured on the day of the vernal equinox.
x is the fraction of the year that has passed from the preceding vernal equinox.
ϕ is the inclination of the ecliptic.
All angles are expressed in radians. Derive the above expression. [4]

f) At which point in the year does the Sun rise earliest, and at what time does it occur? You may give your answer in terms of the parameter x. [5]

Q3. Assassination Classroom

The following comic is from the manga Assassination Classroom (暗殺教室):

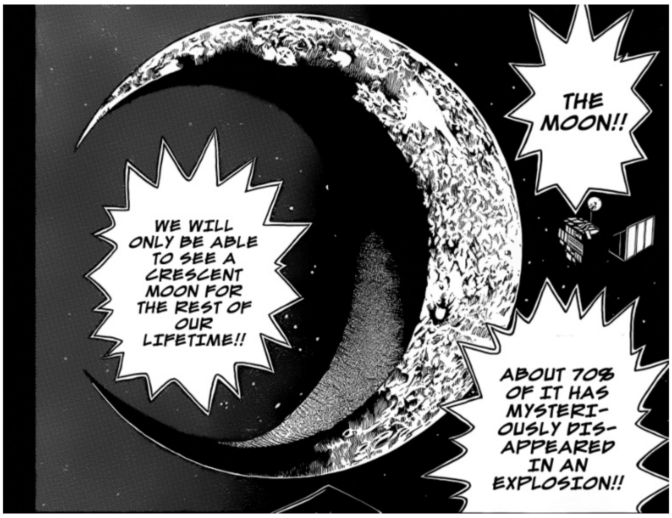

Figure 2: A comic frame from Assassination Classroom

Imagine the resulting moon as a sphere with a spherical cavity inside of it that is tangent to the surface at X (Figure 2, left). The cavity used to contain exactly 70% of the mass of the moon. You may assume the moon to be of uniform density, and not rotating.

a) Show that the gravitational field within the cavity is constant. [2]

b) Hence or otherwise, consider an egg released from X. It falls through the cavity and smashes on the surface at O. How long would this journey take? [4]

c) Let us rewind back to the original moon (without the cavity). Suppose there is a tunnel (of tiny width) passing from X to the same point of impact O (Figure 3, right). Consider an egg released from X into the tunnel which falls and smashes at the bottom at O. How long would this journey take? [5]

d) What is the ratio of the speed of the egg in part b and c? [2]

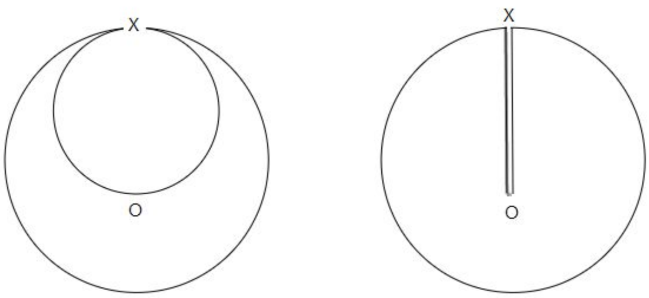

Figure 3: Cross section of the moon after (left), and before (right) the explosion.

Q4. Muon Tomography

In 1965, Luiz Alvarez proposed to use Muon tomography to scan the Great pyramids of Giza for hidden chambers. He published the results in the Feb. 6, 1970 paper titled "Search for Hidden Chambers in the Pyramids". His methodology was a novel approach to using cosmic rays as a method of detecting the thickness of materials. From here, muons are written as μ.

μ (heavier cousins of electrons) are charged particles that are generated by cosmic rays when they strike the atmosphere. As it travels through matter, a retarding force acts on the μ. While low energy μ rapidly lose energy as they travel, energetic μ (E > 1 GeV) lose a very constant amount of energy as they pass through materials. The amount of energy loss depends on the material's density. The approximate energy loss of the μ as it travels a given distance Δx through a material of density ρ is given by:

$$\Delta E = \left(-2\ \frac{MeVcm^2}{g}\right)(\Delta x)(\rho)$$

Let us consider a detector that counts μ from a certain direction above the energy 1 GeV. The incidence rate of high energy μ with $E > E_0$ (where E is the μ's energy as is emitted) is dependent on energy, but approximately constant over time. It is given by $\frac{37}{s}\left(\frac{GeV}{E_0}\right)^{1.7}$. E.g. for μ with energies above 10 GeV, the detector measures on average 0.738 μ /s.

Set up a Cartesian coordinate system, with the origin at the center of the base of the pyramid and the axes pointing to the 4 corners (Figure 4). The detector was placed somewhere on the base of the pyramid with its rotational axis aligned parallel to the y axis. The detector is set to detect μ from some polar angle θ, measured from the x axis (Figure 5). You can assume the path length of the μ as it travels through the pyramid is given by the straight line distance to the pyramid's surface and that the cavity the detector array resides is small in comparison to the entirety of the pyramid itself. The number of μ events captured within a time period of 6 hours is below:

$\theta(°)$	91.8	92.0	92.2	92.4	92.6	92.8	93.0	93.2	93.4	93.6	93.8	94.0
$\#\mu$	627	625	616	616	610	603	604	609	611	613	619	624

Table 1: Number of μ events against polar angle

Your task is to plot a suitable graph [6] and determine where the detector was placed in the base of the pyramid [13]. The pyramid is square with base length 230m and height 140m. Assume it is made entirely of limestone of density 2750 kg/m^3 with no hidden chambers or cavities. Report your answer as an (x,y) coordinate, in meters to 2 d.p. Do also note that error analysis is not required.

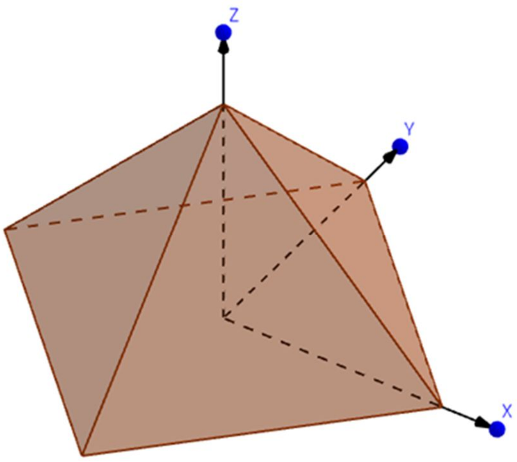

Figure 4: Cartesian Coordinates

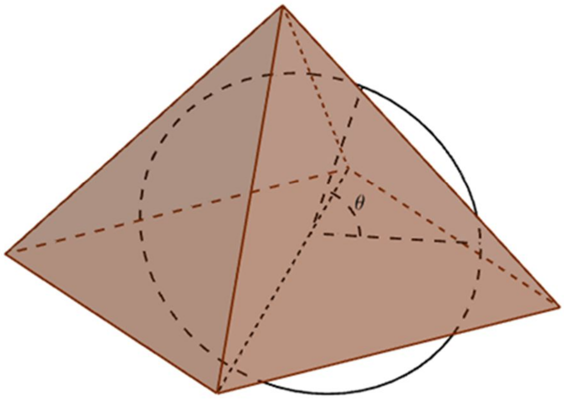

Figure 5: Cylindrical Coordinates

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Practical Portion (Written)

Instructions to Candidates
The practical segment comprises four separate tasks. Write or mark out your answers in the indicated area. At the end, separate this portion from the question paper and submit it together with your answers for the other questions.

You are recommended to spend no more than 30 minutes on this segment.

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Task	Score
A	/11
B	/3
C	/3
D	/6
Practical Total	/23

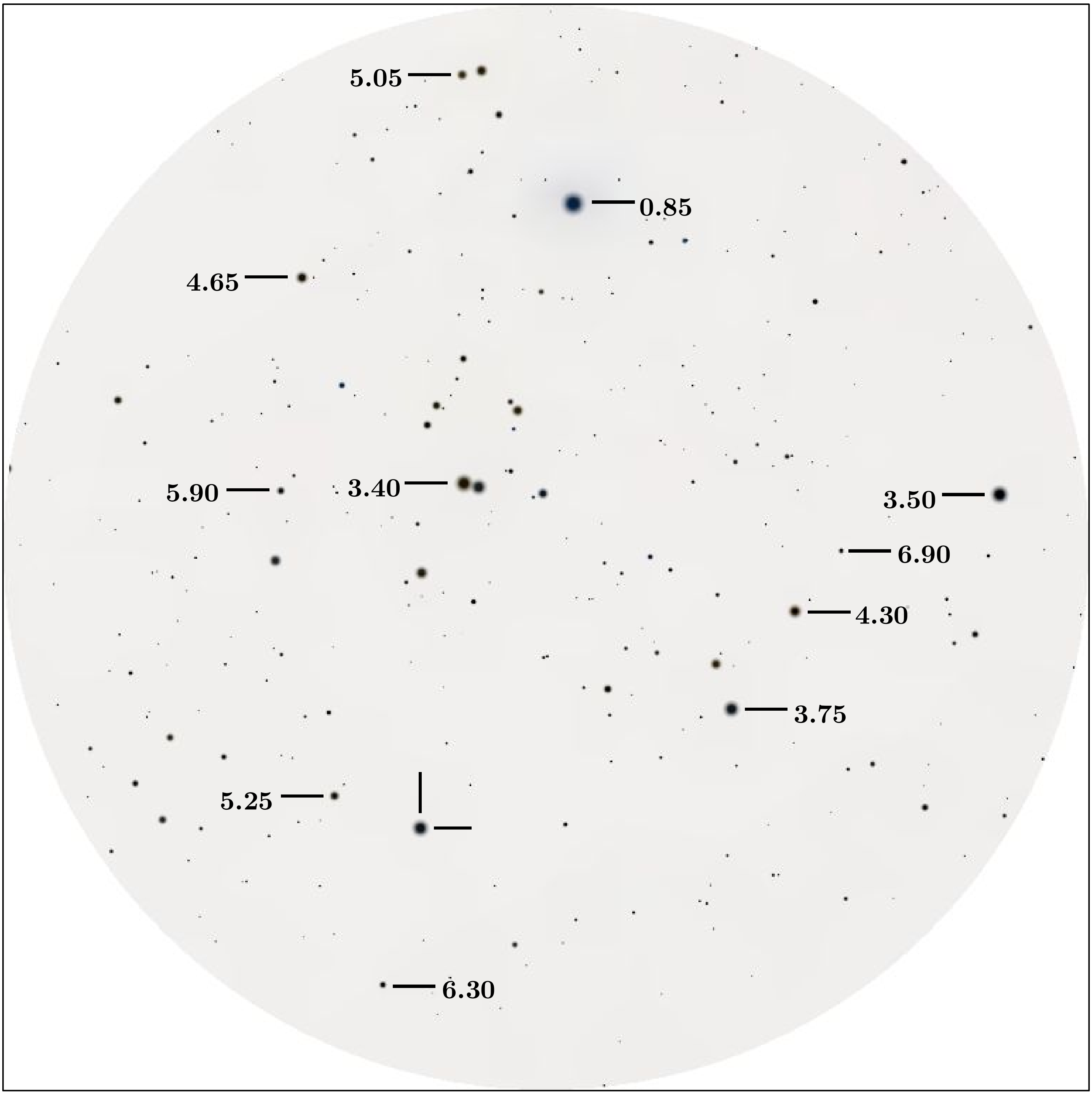
Task A (11 marks)

The star chart below shows the night sky tonight at an unknown time, from Singapore (UTC +0800), at latitude 1° 17’ N and longitude 103° 51’ E. Complete the questions on the following page.

Question	M	Answer Here
(a) On the star chart, mark out the asterism, the **Winter Hexagon**, with solid lines.	[1]	
(b) Along the horizon (circumference of the star chart), mark out the location of **cardinal South**, with a hollow circle ∘.	[1]	
(c) Trace out the **galactic plane** with a solid curve.	[1]	
(d) Mark out the star, **Canopus**, **α Car**, with an arrow →. The tip of the arrow should point unambiguously at the star.	[1]	
(e) Mark out the position of the brightest globular cluster in the night sky, with a two-headed arrow ↠.	[1]	
(f) Estimate the **local time** of the star chart, to the nearest 15 minutes. You are given that the star **Mintaka**, **δ Ori**, has right ascension of $5^{\mathrm{h}}\,33^{\mathrm{m}}$.	[2]	
(g) Mark out and identify any planets that are visible in the star chart. Write '*none visible*' to the right if no planets are visible.	[2]	
(h) Mark out the position of the **Moon** on the <u>following night</u> at the same local time, with a cross ×.	[2]	

Task B (3 marks)

The image below shows the view through an optical instrument, centered on a deep-sky object (DSO).



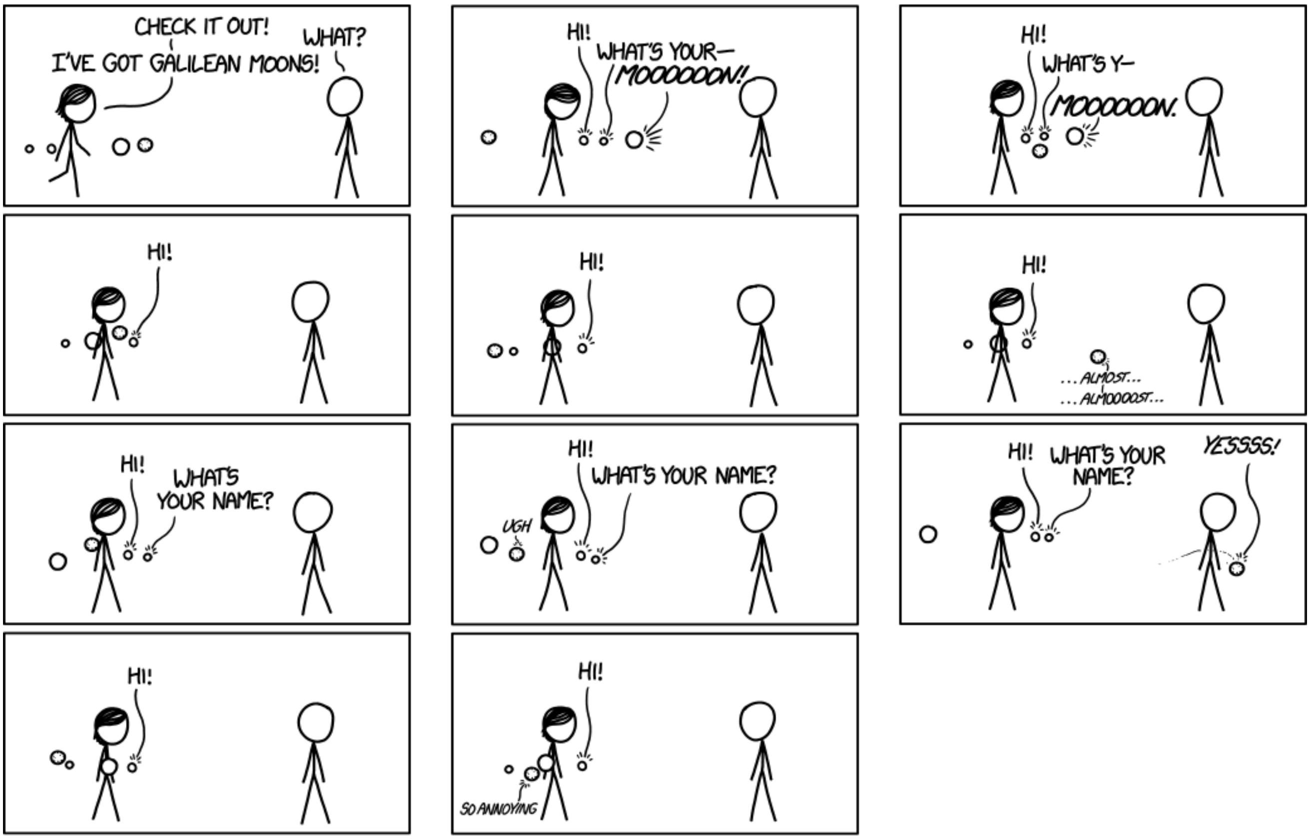
Question	M	Answer Here
(a) Name this DSO. Both the common name and the catalogued name of the DSO are accepted.	[1]	
(b) Estimate the apparent magnitude of the star indicated by the crosshairs, to the nearest first decimal place. The apparent magnitudes of some surrounding stars are provided.	[1]	
(c) Estimate the true field of view of this optical instrument, in arcseconds, to the nearest first decimal place. Hence, suggest the optical instrument used.	[1]	

Task C (3 marks)

The image below is a star chart of the summer sky, with three prominent stars removed. Name and mark out the approximate locations of each of these three stars.

Task D (6 marks)

The following are two qualitative questions regarding practical and observational astronomy. Respond to the questions within the boxes.



Caption: I'm SO glad I escaped. They almost had me caught in their weird... thing.

(a)	The above comic strip from *xkcd* depicts the four Galilean moons. Identify each moon (Callisto, Europa, Ganymede, Io) by what it says, and explain the phenomenon depicted by the comic strip.	[3]

Answer here:

(b)	Suppose we want to image M31, the Andromeda Galaxy, with a CCD. However, in Singapore, light pollution severely reduces the quality of the data captured. An astronomer suggests imaging M31 through a Hα filter instead of LRGB (luminance, red, green, blue) filters. With consideration to the spectrum of light pollution and M31, state and explain **two** ways which you would expect a colour composite image (combination of LRGB images) of M31 to differ from an image taken through a Hα filter, apart from colour.	**[3]**

Answer here:

References used for the 5th Singapore Astronomy Olympiad:

Q1

1. Data retrieved from: https://losc.ligo.org/events/GW150914/. Graph plotted in Microsoft Excel.

2: The LIGO Scientific collaboration, the Virgo Collaboration (2016, October 4). The basic physics of the binary black hole merger GW150914. *General Relativity and Quantum Cosmology*. Retrieved from: http://arxiv.org/abs/1608.01940

Practical Portion

1. Munroe, R. (2013, December 6). Galilean Moons [Cartoon]. Retrieved February 13, 2017, from https://xkcd.com/1300/