# $3^{rd}$ Singapore Astronomy Olympiad
# 21 March 2015

*Organized by*

Astronomy.SG

*Sponsored by*

DSO NATIONAL LABORATORIES

I hereby consent to provide my details to the organizing and sponsoring organizations, *Astronomy.SG* and *DSO National Laboratories*:

Full Name:________________________________

School:________________________________

NRIC/FIN:________________________________

Email address:________________________________

Signature:________________________________

For official use only

| Question | Score |
|---|---|
| 1 | /9 |
| 2 | /10 |
| 3 | /15 |
| 4 | /16 |
| 5 | /20 |
| 6 | /20 |

| Question | Score |
|---|---|
| 1 | /9 |
| 2 | /10 |
| 3 | /15 |
| 4 | /16 |
| 5 | /20 |
| 6 | /20 |

Rules

- Students may use pen or pencil to answer the paper.
- Any working or answers on the question papers will not be considered.
- Only the use of scientific calculators is allowed. Graphing/programmable calculators are not allowed.
- Students are to bring their own stationery.
- Students have 3 hours to complete the paper. If a student is late, no time extension is granted.
- Students should **clearly state their assumptions** in their working, if any.
- Students may leave any time upon submission of their solutions and question paper.
- Cheating or allowing others to cheat are grounds for immediate disqualification.
- No notes are allowed.
- Question papers must be returned together with their scripts. The cover page is to be attached to the answer script.

Constants / Values

Gravitational constant, G: 6.67 X $10^{-11}$ $m^3kg^{-1}s^{-2}$
Hubble constant, $H_0$: 67.8 $kms^{-1}Mpc^{-1}$
Stefan-Boltzmann constant, σ: 5.67 X $10^{-8}$ $Wm^{-2}K^{-4}$
Wein's displacement constant, *b*: 2.90 X $10^{-3}$ mK
1 Astronomical Unit = 1.5 X $10^{11}$ m
Radius of Earth: 6,370 km
Solar mass: 2.0 X $10^{30}$ kg
Solar luminosity: 3.85 X $10^{26}$ W
Solar absolute bolometric magnitude: 4.75

Formulae

Magnetic flux, $\varphi = BR^2$, where $B$ is the magnetic field strength and $R$ the radius
Binding Energy ~ $GM^2R^{-1}$, where $G$ is the gravitational constant and $M$ the mass
Angular momentum, $J \sim \Omega R^2$, where $\Omega$ is the angular rate of rotation in radian/sec

**<u>Question 1</u>** [9 marks]

A protostar that has just begun its Hayashi track has a surface temperature of 3000 K and a diameter of $1.0 \times 10^{11}$ m. If an observer on Earth could have seen the protostar then with the naked eye, it would have a magnitude of 5.0. Since then, the protostar contracted and became a main sequence star with diameter $1.4 \times 10^{9}$ m and surface temperature 5780 K. Assume the distance between the Earth and the star has not changed significantly since the time that it was a protostar. Calculate, in any order:

a) the apparent magnitude of the main sequence star

b) the minimum size of the telescope required to see the main sequence star

c) the approximate distance between the star and the earth

d) the peak wavelength of the light emitted by the main sequence star

**<u>Question 2</u>** [10 marks]

The following data describe an imaginary comet X.

a) During perihelion of X, Earth is directly between X and the Sun. A parallax measurement of the comet from two observatories directly opposite each other on earth (i.e. at antipodes) shows a parallax of 35". Calculate the perihelion distance of X in AU.

b) The aphelion distance of X is 28.5 AU. Calculate its orbital period in years.

c) It is known that the comet moves faster near the sun and slower when further away from the sun. Calculate, in any order:

   i) the orbital velocity at perihelion

   ii) the rate at which an area is swept by a line joining the sun and the comet at perihelion

   iii) the rate at which an area is swept by a line joining the sun and the comet at aphelion

**Question 3** [15 marks]

The apparent positions of three major objects in the solar system over a period of 1 calendar year is given in Table 1 (over the next 2 pages).

a) Identify object A.

b) Identify object B.

c) What is the approximate distance (to the nearest A.U.) of object C (from Earth) on 01 July?

d) Which of the following options is closest to the ratio:

$$\frac{\text{angular diameter of object B on 22 Mar}}{\text{angular diameter of object B on 22 Oct}}$$

i) $1+\sqrt{2}$
ii) $\sqrt{2}-1$
iii) $\sqrt{2}$
iv) 1
v) 2

Table 1

| | Object A | | Object B | | Object C | |
|---|---|---|---|---|---|---|
| Date | Right Ascension | Declination | Right Ascension | Declination | Right Ascension | Declination |
| Jan-01 | $18^h44^m45^s$ | -23°02'19" | $19^h52^m49^s$ | -18°15'18" | $00^h32^m14^s$ | +02°43'56" |
| Jan-08 | $19^h15^m31^s$ | -22°18'10" | $19^h36^m47^s$ | -17°07'24" | $00^h32^m37^s$ | +02°46'43" |
| Jan-15 | $19^h45^m52^s$ | -21°12'35" | $19^h18^m25^s$ | -16°18'27" | $00^h33^m09^s$ | +02°50'27" |
| Jan-22 | $20^h15^m41^s$ | -19°47'06" | $19^h03^m27^s$ | -15°52'19" | $00^h33^m50^s$ | +02°55'05" |
| Feb-01 | $20^h57^m12^s$ | -17°14'11" | $18^h55^m10^s$ | -15°52'11" | $00^h35^m02^s$ | +03°03'10" |
| Feb-08 | $21^h25^m28^s$ | -15°08'40" | $18^h59^m20^s$ | -16°08'25" | $00^h36^m02^s$ | +03°09'44" |
| Feb-15 | $21^h53^m04^s$ | -12°50'39" | $19^h10^m30^s$ | -16°27'07" | $00^h37^m07^s$ | +03°16'59" |
| Feb-22 | $22^h20^m04^s$ | -10°22'27" | $19^h27^m12^s$ | -16°39'15" | $00^h38^m19^s$ | +03°24'48" |
| Mar-01 | $22^h46^m35^s$ | -07°46'22" | $19^h48^m06^s$ | -16°37'49" | $00^h39^m36^s$ | +03°33'08" |
| Mar-08 | $23^h12^m40^s$ | -05°04'46" | $20^h12^m02^s$ | -16°17'59" | $00^h40^m57^s$ | +03°41'51" |
| Mar-15 | $23^h38^m26^s$ | -02°19'57" | $20^h38^m08^s$ | -15°36'48" | $00^h42^m21^s$ | +03°50'53" |
| Mar-22 | $00^h03^m59^s$ | +00°25'58" | $21^h05^m42^s$ | -14°32'50" | $00^h43^m47^s$ | +04°00'08" |
| Apr-01 | $00^h40^m24^s$ | +04°20'55" | $21^h46^m38^s$ | -12°22'22" | $00^h45^m53^s$ | +04°13'31" |
| Apr-08 | $01^h05^m58^s$ | +07°01'01" | $22^h15^m50^s$ | -10°25'47" | $00^h47^m21^s$ | +04°22'52" |
| Apr-15 | $01^h31^m43^s$ | +09°35'14" | $22^h45^m14^s$ | -08°11'00" | $00^h48^m49^s$ | +04°32'07" |
| Apr-22 | $01^h57^m45^s$ | +12°01'40" | $23^h14^m43^s$ | -05°40'50" | $00^h50^m16^s$ | +04°41'10" |
| May-01 | $02^h31^m46^s$ | +14°55'30" | $23^h52^m45^s$ | -02°10'26" | $00^h52^m03^s$ | +04°52'22" |
| May-08 | $02^h58^m42^s$ | +16°57'06" | $00^h22^m27^s$ | +00°42'12" | $00^h53^m23^s$ | +05°00'38" |
| May-15 | $03^h26^m06^s$ | +18°44'49" | $00^h52^m22^s$ | +03°38'49" | $00^h54^m39^s$ | +05°08'25" |
| May-22 | $03^h53^m57^s$ | +20°17'02" | $01^h22^m39^s$ | +06°35'46" | $00^h55^m49^s$ | +05°15'39" |
| Jun-01 | $04^h34^m30^s$ | +21°59'01" | $02^h06^m49^s$ | +10°41'31" | $00^h57^m20^s$ | +05°24'55" |
| Jun-08 | $05^h03^m17^s$ | +22°47'51" | $02^h38^m33^s$ | +13°23'10" | $00^h58^m16^s$ | +05°30'33" |
| Jun-15 | $05^h32^m17^s$ | +23°17'05" | $03^h11^m05^s$ | +15°51'48" | $00^h59^m05^s$ | +05°35'25" |
| Jun-22 | $06^h01^m24^s$ | +23°26'13" | $03^h44^m32^s$ | +18°03'28" | $00^h59^m46^s$ | +05°39'29" |

Table 1 (continued)

| | Object A | | Object B | | Object C | |
|---|---|---|---|---|---|---|
| Date | Right Ascension | Declination | Right Ascension | Declination | Right Ascension | Declination |
| Jul-01 | $06^h38^m47^s$ | +23°08'16" | $04^h28^m53^s$ | +20°21'37" | $01^h00^m27^s$ | +05°43'27" |
| Jul-08 | $07^h07^m38^s$ | +22°31'38" | $05^h04^m19^s$ | +21°40'22" | $01^h00^m49^s$ | +05°45'32" |
| Jul-15 | $07^h36^m09^s$ | +21°35'57" | $05^h40^m26^s$ | +22°30'44" | $01^h01^m03^s$ | +05°46'42" |
| Jul-22 | $08^h04^m17^s$ | +20°22'14" | $06^h17^m01^s$ | +22°50'29" | $01^h01^m08^s$ | +05°46'58" |
| Aug-01 | $08^h43^m40^s$ | +18°08'16" | $07^h09^m31^s$ | +22°23'19" | $01^h00^m59^s$ | +05°45'45" |
| Aug-08 | $09^h10^m38^s$ | +16°16'39" | $07^h46^m01^s$ | +21°25'40" | $01^h00^m42^s$ | +05°43'50" |
| Aug-15 | $09^h37^m06^s$ | +14°12'13" | $08^h22^m03^s$ | +19°57'36" | $01^h00^m17^s$ | +05°41'04" |
| Aug-22 | $10^h03^m09^s$ | +11°56'42" | $08^h57^m26^s$ | +18°01'24" | $00^h59^m45^s$ | +05°37'30" |
| Sep-01 | $10^h39^m45^s$ | +08°27'28" | $09^h46^m39^s$ | +14°32'44" | $00^h58^m46^s$ | +05°31'10" |
| Sep-08 | $11^h05^m03^s$ | +05°52'41" | $10^h20^m12^s$ | +11°41'51" | $00^h57^m57^s$ | +05°25'59" |
| Sep-15 | $11^h30^m11^s$ | +03°13'07" | $10^h53^m06^s$ | +08°34'57" | $00^h57^m03^s$ | +05°20'17" |
| Sep-22 | $11^h55^m17^s$ | +00°30'36" | $11^h25^m29^s$ | +05°16'05" | $00^h56^m05^s$ | +05°14'11" |
| Oct-01 | $12^h27^m42^s$ | -02°59'34" | $12^h06^m43^s$ | +00°49'28" | $00^h54^m46^s$ | +05°05'57" |
| Oct-08 | $12^h53^m09^s$ | -05°41'25" | $12^h38^m43^s$ | -02°41'06" | $00^h53^m43^s$ | +04°59'25" |
| Oct-15 | $13^h18^m56^s$ | -08°19'41" | $13^h10^m55^s$ | -06°09'53" | $00^h52^m40^s$ | +04°52'54" |
| Oct-22 | $13^h45^m11^s$ | -10°52'15" | $13^h43^m34^s$ | -09°32'35" | $00^h51^m39^s$ | +04°46'33" |
| Nov-01 | $14^h23^m38^s$ | -14°15'55" | $14^h31^m23^s$ | -14°02'41" | $00^h50^m16^s$ | +04°38'02" |
| Nov-08 | $14^h51^m18^s$ | -16°25'14" | $15^h05^m54^s$ | -16°51'47" | $00^h49^m23^s$ | +04°32'38" |
| Nov-15 | $15^h19^m40^s$ | -18°21'02" | $15^h41^m24^s$ | -19°19'15" | $00^h48^m35^s$ | +04°27'51" |
| Nov-22 | $15^h48^m44^s$ | -20°01'05" | $16^h17^m53^s$ | -21°20'40" | $00^h47^m54^s$ | +04°23'45" |
| Dec-01 | $16^h27^m02^s$ | -21°43'09" | $17^h06^m06^s$ | -23°12'09" | $00^h47^m13^s$ | +04°19'41" |
| Dec-08 | $16^h57^m26^s$ | -22°39'32" | $17^h44^m18^s$ | -24°00'11" | $00^h46^m50^s$ | +04°17'32" |
| Dec-15 | $17^h28^m14^s$ | -23°14'12" | $18^h22^m49^s$ | -24°12'12" | $00^h46^m35^s$ | +04°16'20" |
| Dec-22 | $17^h59^m16^s$ | -23°26'13" | $19^h01^m14^s$ | -23°47'36" | $00^h46^m30^s$ | +04°16'06" |
| Jan-01 | $18^h43^m37^s$ | -23°03'32" | $19^h55^m17^s$ | -22°10'51" | $00^h46^m40^s$ | +04°17'29" |

<u>Question 4</u> [16 marks]

Bosscha Observatory is the oldest observatory in Indonesia. It is located in West Java (coordinates: 6.82° S, 107.6°E) at an altitude of 1.3 km above sea level.

a) What is the minimum declination of a star such that it is circumpolar for this observer?

b) On June 28 2014, an observer at Bosscha Observatory noted the Moon was in the third quarter phase. For how long was the Moon above the horizon? Give your answer as a range, precise to the minute.

c) The same observer observed a star to culminate at an altitude of 54° 30". The star's proper motion components were observed to be $\mu_\alpha$ = -0.0374"/year, and $\mu_\delta$ = 1.21"/year. Calculate its total proper motion.

d) Given further that the star exhibited a blueshift of 7.6 km/s and the parallax was 0.376", calculate the star's velocity relative to the Sun.

e) Then, find the star's total proper motion at the time of its closest approach.

<u>Question 5</u> [20 marks]

Spectrographic measurements of an exoplanet "Dee Two" shows a radial Doppler shift around its parent star "Dee". The Doppler shift follows a sinusoidal oscillation with oscillation period of 80 days and amplitude ranging from -30 km/s to 30 km/s. Assuming that the mass of star is much greater than the mass of the exoplanet,

a) Calculate the lower limit for the orbital radius of the planet

b) Calculate the lower limit for the mass of the star

Optical measurements of the extrasolar system also reveal that it is a transiting system and we observe a sketch of its light curve as shown below in Fig. 1:

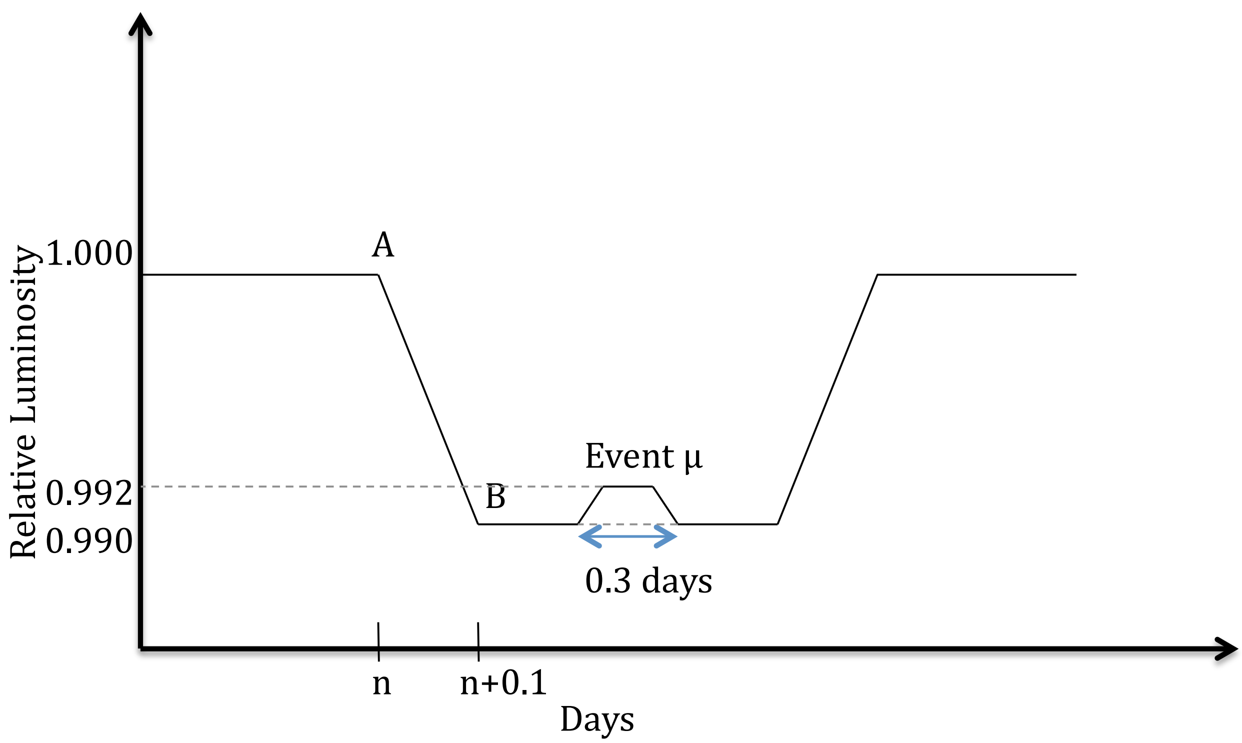


Fig. 1: Sketch of the Dee system light curve

c) Calculate the radius of Dee Two.

d) Calculate the radius of Dee.

e) Given further information that the extrasolar system is 4.5 light years away from Earth, calculate the maximum inclination of the exoplanet's orbital plane to us and then calculate the maximum limit of the star's mass. Hint: Recalculate the exoplanet's orbital radius from (a) in order to factor in the transit phenomenon. You may wish to make use of the following trigonometric identity:

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

There are 2 possible explanations given for Event $\mu$ in Fig. 1.

Hypothesis one: A triple transit event where an exo-moon transits an exoplanet transiting a star.

Hypothesis two: The exoplanet transits past a starspot on the star.

f) For Hypothesis one, calculate the radius of the exo-moon.

g) For Hypothesis two, calculate the width of the starspot,

h) Considering that luminosity and spectrograph of an exo-moon is difficult to resolve with current technology, suggest how astronomers can distinguish between the phenomenon of an exo-moon from that of a starspot transit.

**Question 6** [20 marks]

Pock is a star with the following initial characteristics:

Mass: 1.5 solar masses
Radius: $10^6$ km
Period: 20 days
Magnetic field strength: $10^{-2}$ T
Distance to Earth: 1.0 kpc

a) Pock undergoes a collapse to form a neutron star. During this collapse, the volume of Pock decreases by 15 orders of magnitude. After the collapse,

i) What is Pock's radius?

ii) What is Pock's rotational period?

iii) What is Pock's magnetic field strength?

iv) What is the minimum period of Pock?

b) Pock collides with its identical twin, Puck, to form a more massive pulsar named "Pick". Neutron stars follow the mass-radius relationship $RM^{1/3}=k$, where $k$ is a constant. What is the final binding energy of Pick, relative to the total initial binding energy of Pock and Puck?

c) Pick emits radiation from its poles, with the beams shaped as a double cone with an opening angle (half-angle of the cone) of 3°. The angle between Pick's rotation and emission axis is denoted as $\boldsymbol{\vartheta}$. What is the probability of detecting Pick's pulses?

d) As it turns out, Pick's pulses were detected on Earth, and the absolute bolometric magnitude of Pick was measured to be -3.5. What is the luminosity of Pick?

**The End**