2nd Singapore Astronomy Olympiad

Organized by: Astronomy.SG
Sponsored by: DSO National Laboratories

22 March 2014

Name: ______________________________

School: ______________________________

Level: ______________________________

Rules

- Do not write your working or answers on the question papers.
- Write your name and school acronym on all sheets of paper used, including graph paper, and clearly label all your working and answers.
- Show your working clearly. Use of background knowledge is allowed, in which case you must clearly state that you are doing so. However, background knowledge alone may not gain you full marks if no working is shown.
- Only the use of scientific calculators is allowed. Graphing calculators are not allowed.
- Questions are independent of each other, e.g. question 1 & question 2 are independent. Parts within each question may or may not be linked.
- State all relevant assumptions made, in your working.
- Penalty: -0.5 mark for each final answer given to the wrong significant figure, maximum penalty for the entire paper is 3 marks.
- The number of marks for each part question is given in [] at the end of the question.
- Cheating and allowing others to cheat are grounds for immediate disqualification.
- Notes are not allowed.
- Question papers must be returned together with their scripts.

Physical constants:
Wein's displacement constant b = $2.9 \times 10^{-3} m \cdot K$
Solar luminosity $L_S = 3.83 \times 10^{26}$ W
Mass of Sun M_S = 1.989 $\times 10^{30}$ kg
Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$
Mass of He-4 = 4.002602 Da
Mass of He-3 = 3.0160293 Da
Mass of proton = 1.00728 Da
Mass of electron = 0.00055 Da
1 Da = 1.6605×10^{-27} kg
1 eV (electron-volt) = 1.602×10^{-19} J

1. Fig. 1 shows the light curve of a star with an orbiting planet. The Doppler shift of absorption lines in the atmosphere of the planet are taken at the start and at the end of the transit. The difference in radial velocity with respect to Earth is found to be 30 km/s. Assuming that the planet orbits the star in a circular orbit that is edge on as seen from Earth, estimate:

a) the radius of the orbit of the planet [6]

b) the mass of the star [2]

c) the radius of the star [3]

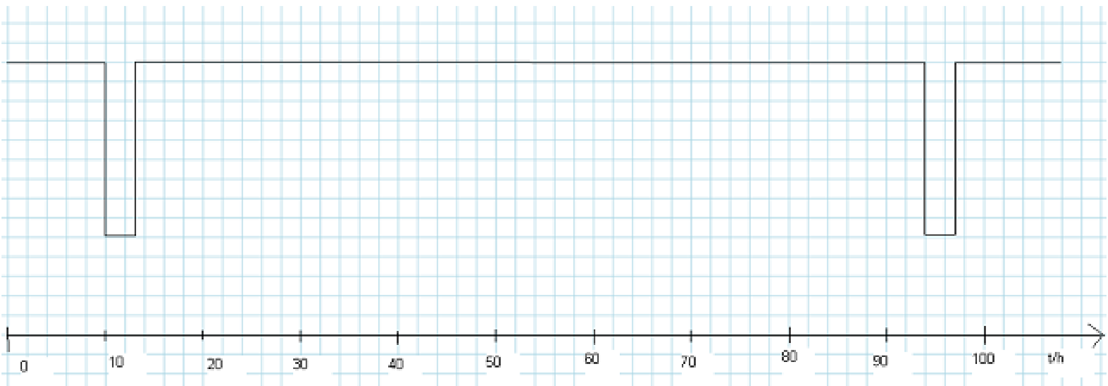

Figure 1: Light curve of a star.

2. Branches I and II of the proton-proton (pp) chain are shown in the images below, as a hint to how the pp chain operates. For a star of 0.9 M_S, assume 85% of the total energy is released via Branch I, while the other 15% comes from Branch II. In the pp I branch, prior to the fusion of two ^{3}He, the total energy released is 13.36 MeV. The total energy released in the entire pp II reaction is 25.2 MeV.

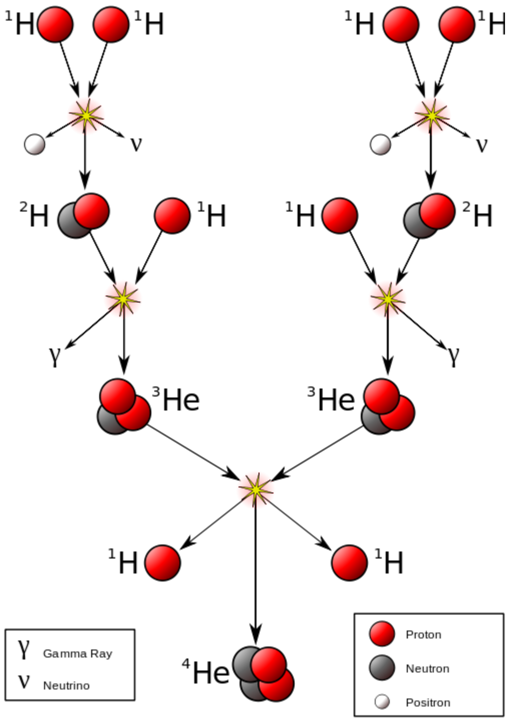

Figure 2: Branch I of the proton-proton chain. Source: Wikipedia.

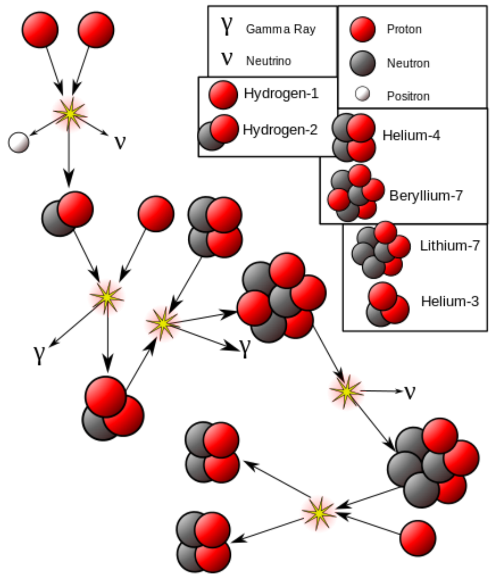

Figure 3: Branch II of the proton-proton chain. Source: Wikipedia.

a) Calculate the average release of energy per proton fusion reaction (i.e. for every 2 protons that fuse together) in a star undergoing nuclear fusion as described above. [5]

b) Calculate as a fraction of its mass, how much this star radiates, given that 9% of its hydrogen is being "burned". [3]

c) Estimate the lifetime of this star. [2]

3. From Earth, a galaxy is observed to have a redshift of z = 1, while a quasar is observed to have a redshift of z = 2. Given that both objects were observed along the same line of sight from Earth, what is the redshift of the quasar as observed from that galaxy? [2]

4. Fig. 4 shows the observed optical to submillimeter spectral energy distribution of HD107146. Assume that this is a system with a star at the centre and a ring of dust around it.

a) What is the effective surface temperature of the star? [2]

b) Suppose the star has luminosity of 1 L_{Sun} and the emissivities of the star and the dust are all $\epsilon = 1$. Let R be the radius of the ring of dust. Estimate R. Assume spherical dust particles. [4]

c) Sketch the Hertzsprung-Russell diagram, and plot this star on it. [2]

d) Derive the equation for the isoradius line (i.e. line of constant radius), and sketch 2 isoradius lines on your HR diagram. [3]

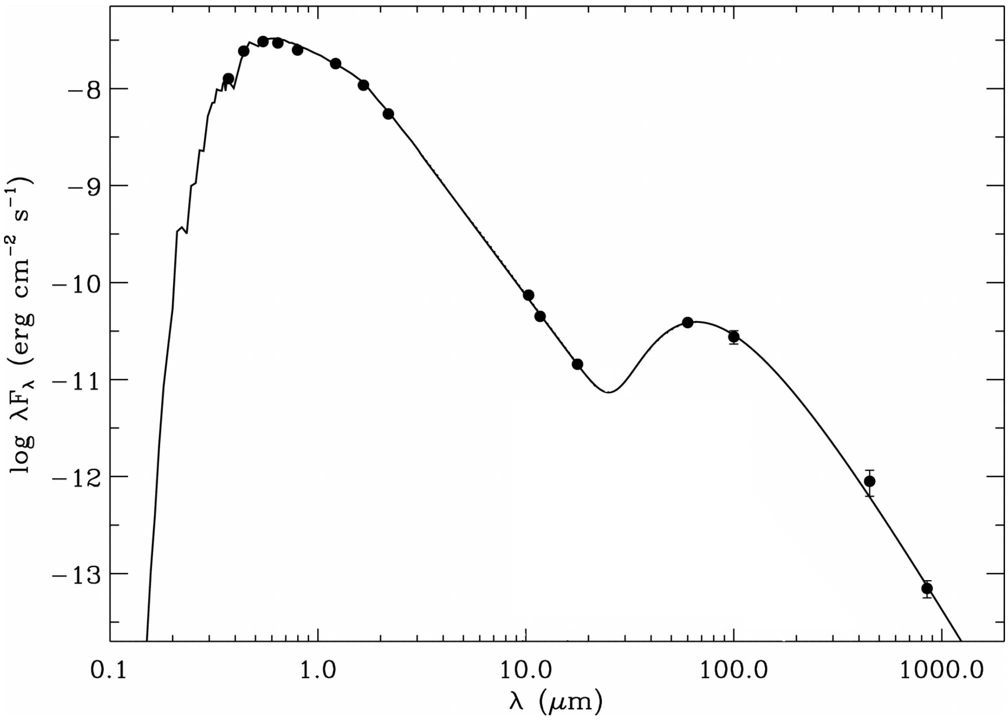

Figure 4: Observed spectral energy distribution of HD107146. Modified from Williams, J. P., et al., 2004, ApJ, 604:414-419.

5. Consider the two types of Cepheids: Type I and Type II Cepheids. Suppose that the difference between Hubble's measurement of the Hubble constant as 500km/s/Mpc and the current value of approximately 70km/s/Mpc could be entirely attributed to him having observed the "wrong" type of Cepheid. Find the difference in absolute magnitudes between these two types of Cepheids. [9]

6. A galaxy with an apparent magnitude in the V band ($\lambda_c = 555.0$ nm, width $\delta\lambda =$ 800 angstroms) of $m_V = 22.5$ mag is observed with a 200-inch telescope. How many photons per second are collected from this galaxy over the 200-inch diameter mirror? For reference, Vega is a 0 magnitude star, with a flux of 1000 photons/s/cm^2/Angstrom at Earth's surface. [5]

7.
a) Starting from Newton's Law of Gravitation and the equation of motion, ***show that there is a conserved quantity*** $\ell = \frac{\mathrm{d}\theta}{\mathrm{d}t} r^2$. By evaluating the infinitesimal area swept by the line joining the two bodies in the infinitesimal time $\mathrm{d}t$, ***show that Kepler's Third Law holds the form***:

$$GMT^2 = 4\pi a^3,$$

where G, M, T, and a are the Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$, mass of the central body, period of orbit, and semi–major axis of the orbit respectively. Hint: Area of the ellipse is πab, $b = a\sqrt{1-e^2}$, where b, and e are the semi–minor axis, and eccentricity respectively. You may also wish to consider using other conservation laws. [11]

b) Using the following data table of the Jupiter–Galilean Moon distance, ***deduce the mass of Jupiter***. Remember to ***include some error analysis*** in your deduction. From your analysis, ***identify the Galilean moons***. Hint: Part (a). [16]

Julian Date	Moon #1	Moon #2	Moon #3	Moon #4	Julian Date	Moon #1	Moon #2	Moon #3	Moon #4
2456658.50	-12.00	-4.73	-1.55	-2.19	2456668.50	5.70	-1.63	-2.99	2.86
2456658.75	-11.40	-4.48	-3.11	-2.91	2456668.75	no data	no data	no data	no data
2456659.00	-10.75	-3.39	-4.52	-1.46	2456669.00	3.36	-4.47	0.22	-1.29
2456659.25	-9.90	-1.65	-5.75	1.08	2456669.25	2.15	-4.72	1.85	-2.86
2456659.50	-9.10	0.38	-6.65	2.84	2456669.50	0.90	-4.10	3.40	-2.31
2456659.75	-8.05	2.36	-7.22	2.50	2456669.75	no data	-2.67	4.75	-0.03
2456660.00	no data	no data	no data	no data	2456670.00	-1.56	-0.79	5.90	2.26
2456660.25	-5.90	4.58	-7.40	-2.07	2456670.25	no data	no data	no data	no data
2456660.50	-4.75	4.45	-6.95	-2.91	2456670.50	-4.00	3.09	7.32	1.40
2456660.75	-3.55	3.40	-6.13	-1.63	2456670.75	-5.15	4.28	7.50	-1.13
2456661.00	-2.34	1.70	-5.03	0.90	2456671.00	-6.28	4.65	7.28	-2.81
2456661.25	-1.09	no data	-3.69	2.76	2456671.25	-7.35	4.05	6.78	-2.44
2456661.50	0.17	-2.33	-2.16	2.59	2456671.50	-8.45	2.71	5.90	-0.22
2456661.75	1.42	-3.86	-0.55	0.54	2456671.75	-9.30	0.81	4.75	2.15
2456662.00	2.65	-4.67	1.11	-1.92	2456672.00	-10.20	-1.27	3.38	2.94
2456662.25	3.88	-4.58	2.71	-2.94	2456672.25	-10.95	-3.10	1.85	1.58
2456662.50	no data	no data	no data	no data	2456672.50	-11.60	-4.33	no data	-0.95
2456662.75	6.15	-2.01	5.45	0.71	2456672.75	-12.15	-4.73	-1.41	-2.76
2456663.00	7.25	0.00	6.43	2.69	2456673.00	-12.60	-4.28	-2.97	-2.53
2456663.25	8.15	2.01	7.10	2.67	2456673.25	-12.95	-3.02	-4.42	-0.41
2456663.50	9.15	3.61	7.45	0.70	2456673.50	-13.20	-1.16	-5.65	2.01
2456663.75	9.95	4.52	7.45	-1.77	2456673.75	no data	no data	no data	no data
2456664.00	10.75	4.55	7.05	-2.92	2456674.00	-13.30	2.80	-7.20	1.74
2456664.25	11.40	3.66	6.32	-1.92	2456674.25	-13.20	4.12	-7.47	-0.75
2456664.50	11.90	2.05	5.32	0.55	2456674.50	-12.95	4.63	-7.40	-2.69
2456664.75	12.40	no data	4.02	2.58	2456674.75	-12.65	4.25	-6.97	-2.64
2456665.00	12.70	-2.00	2.56	2.75	2456675.00	-12.20	3.03	-6.22	-0.61
2456665.25	12.95	-3.63	0.98	0.90	2456675.25	-11.65	1.17	-5.15	1.88
2456665.50	13.05	-4.57	-0.68	-1.60	2456675.50	-10.95	-0.88	-3.81	2.96
2456665.75	13.00	-4.68	-2.28	-2.92	2456675.75	no data	no data	no data	no data
2456666.00	12.85	-3.86	-3.78	-2.06	2456676.00	-9.35	-4.17	-0.69	no data
2456666.25	12.60	-2.36	-5.10	0.35	2456676.25	-8.45	-4.75	0.98	-2.60
2456666.50	12.20	-0.38	-6.18	2.51	2456676.50	-7.45	-4.45	2.58	-2.72
2456666.75	11.70	1.65	-6.95	2.81	2456676.75	-6.30	-3.30	4.05	-0.79
2456667.00	11.10	3.36	-7.40	1.08	2456677.00	-5.18	-1.52	5.35	1.73
2456667.25	10.40	4.43	-7.50	-1.45	2456677.25	no data	no data	no data	no data
2456667.50	9.60	4.60	-7.22	-2.90	2456677.50	-2.75	2.47	7.05	2.03
2456667.75	8.75	3.87	-6.60	-2.20	2456677.75	no data	no data	no data	no data
2456668.00	7.75	2.39	-5.63	0.16	2456678.00	-0.28	4.63	7.43	-2.50
2456668.25	6.80	no data	-4.45	2.41	2456678.25	0.98	4.40	7.07	-2.78

Figure 5: Distances of the Galilean moons away from the center of Jupiter as seen from Earth in units of Jupiter Diameters ($D_J = 1.43 \times 10^8$ m) recorded over 3 weeks. Negative (positive) distances indicate that moons are on the East (West) of Jupiter.