1st Singapore Astronomy Olympiad

Solutions

Penalty: -0.5 mark for each final answer to the wrong sf, up to penalty of 1 mark.

1a. Rigel [1] crosses earlier by 1.5 hours [1]. Any star crosses the meridian when sidereal time is equal to the star's RA. Thus Sirius will cross the meridian at ST 6:45 and Rigel at ST 5:15. Neglecting the small difference (about 10 seconds) between a sidereal hour and a solar hour, we see that Rigel will cross the meridian about 1.5 hours before Sirius. Note that where we observe is irrelevant, so long as we can see these stars.

1b. Sirius is north of the zenith (at a higher declination) of New Zealand, so its altitude A is [2]

$$A = 90^\circ - |\delta - L| \tag{1}$$

North of the Zenith, Sirius is at azimuth 0°. [1]

1c. The Moon achieves its maximal altitude for the day (or night) when it crosses our meridian. At that point its zenith angle is the difference between our latitude and the Moon's declination. So the maximal altitude will occur at meridian crossing when the Moon's declination is as close to 37°. Since the Moon's orbit is tilted 5° to the ecliptic, it can thus be found, at various times, on the ecliptic, or as much as 5° North or South of the ecliptic. The ecliptic, in turn, is tilted 23.5° to the celestial equator, so lies between declination 23.5° and -23.5°. The Moon's declination is thus closest to 37° when it is 5° North of the ecliptic at the ecliptic's northernmost point, at declination 28.5°. [1]

When it crosses the meridian that night it will lie $37 - 28.5 = 8.5^\circ$ south of the Zenith. [1]

Its altitude will thus be $90 - 8.5 = 81.5^\circ \approx 82^\circ$. [1]

2. Assume Earth and Mars have circular orbits [0.5] of radii 1.00 AU and 1.52 AU [0.5] respectively.

Sketch of the trajectory [2]:

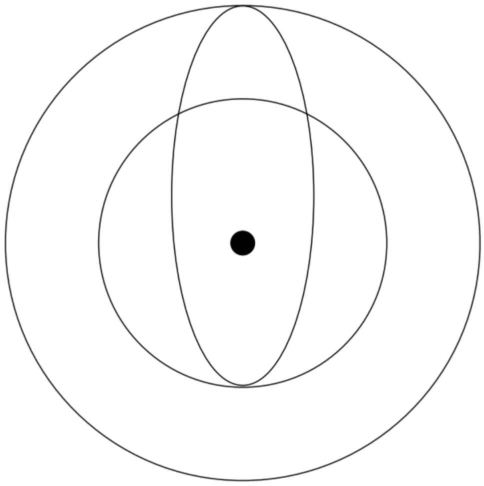

Figure 1: Orbits of Earth, Mars and spacecraft, not drawn to scale. Image from Duke University, "Introduction to Astronomy".

From Fig. 1, the major axis of our ellipse that goes through the aphelion, perihelion, and both foci, has length 2.52 AU. The semimajor axis is thus half of this. [1]

With engines off, the craft is in orbit around the Sun, and we can use Keplers third law for solar orbits [1]:

$$\left(\frac{P}{1yr}\right)^2 = \left(\frac{a}{1AU}\right)^3 \tag{2}$$

we find that the period of this orbit is $(2.52/2)^{3/2} = 1.41435$years. [1]

This is the time it would take the craft, left to its own devices, to return to the point whence it was launched (though Earth would no longer be there). By symmetry, the one-way trip from this initial point to aphelion would take half this time, or about 0.71 years [1]. Sf will depend on what assumption the student used for the radii of Earth and Mars.

3. (Sketch such that) The edge of the field of view (FoV) of the OTA must be parallel to the line of sight of the finderscope [1].

With the specifications given, the apparent FoV, θ_A, of optical tube can be converted to radians [1]:

$$\theta_A = 50^\circ = 0.873\text{rad}. \tag{3}$$

The magnification M of the OTA is [1]

$$M = \frac{2032}{32} \tag{4}$$
$$= 63.5\times. \tag{5}$$

The true FOV, θ_T, is then [1]

$$\theta_T = \frac{\theta_A}{M} \tag{6}$$
$$= \frac{0.873}{63.5} \tag{7}$$
$$= 1.37\times 10^{-2}\text{rad}. \tag{8}$$

Since we're considering only half the true FOV, at a separation distance of 7" = 17.78 cm [1] (7" because it's 1" finderscope-OTA separation + 4" half-OTA diameter), the minimum distance D will be [1]

$$D = \frac{0.1778}{\tan\left(\frac{\theta_T}{2}\right)} \tag{9}$$
$$= 26\text{m (check sf, 1 or 2 sf is ok)}. \tag{10}$$

Note that the important FoV is not the finderscope's, but the optical tube. This is because the celestial object will still be along the line of sight of the finderscope and it doesn't matter how big that FoV is. The numbers 7×50 and 5 degrees for finderscope specifications are not needed for this question.

We could go into a deeper discussion where the distance is actually measured from that special point inside the telescope that is at the focal points of both the objective and the eyepiece. But considering that the telescope itself has some physical length, specifically as a Cassegrain the 2 metres worth of focal length is folded inside the OTA, the actual distance is a little bit off from the value given. That said, since 2 << 26, we'll approximate the distance to about 26m and leave it at that. (If a student discusses this, I'm thinking of giving a Special Mention - RE.)

4. Gravitational interactions of the ring particles with Mimas created the gaps. [1]

Taking the ratios of the orbital radii [1]:

$$\left(\frac{R_{\text{Mimas}}}{R_{\text{Huygens}}}\right)^3 = \left(\frac{186,000}{117,580}\right)^3 \tag{11}$$
$$= 3.96 \tag{12}$$

and [1]

$$\left(\frac{R_{\text{Mimas}}}{R_{\text{Maxwell}}}\right)^3 = \left(\frac{186,000}{89,400}\right)^3 \quad (13)$$

$$= 9.01 \quad (14)$$

Since all are orbiting the same planet, we have a Kepler relation [1]

$$P^2 = KR^3 \quad (15)$$

where K is the constant of proportionality, and the ratios are close enough to perfect squares to suggest strongly that the Huygens gap is due to a 2:1 resonance with Mimas [1], and the Maxwell gap to a 3:1 resonance [1].

5. a) Heavy element lines imply that δ Cephei is a Pop I star, so the Pop I calibration curve should be used [1].

Ignoring interstellar extinction (this is the assumption) [1],

the luminosity is about $1000L_{sol}$ [1].

Brightness, luminosity and distance are related by [1]

$$b = \frac{L}{4\pi D^2} \quad (16)$$

and scaling to solar units [2]:

$$D = \sqrt{\frac{L/L_{sol}}{b/b_{sol}}} \quad (17)$$

$$= \sqrt{\frac{10^3}{5.4 \times 10^{-13}}} \quad (18)$$

$$= 4.3 \times 10^7 AU \quad (19)$$

$$\approx 200\text{pc. (check sf, 1 or 2 sf is ok)} \quad (20)$$

b) Students may use a value of luminosity of $60 - 100L_{sol}$ [1]. In the following calculation, I'll just use a value of $80L_{sol}$.

Extinction decreases the apparent brightness of a star. Only 69% of the light from δ Sanfrancisco makes it through the interstellar dust and gas between the star and Earth. This light is what produces the apparent brightness we observe. For a variable star, of course, brightness and luminosity change. Taking a median brightness of $3.45 \times 10^{-15} b_{sol}$ as a representative value, this is now related to distance and luminosity by a similar equation as in part (a) [1]:

$$D = \sqrt{\frac{0.69 \times 80}{3.45 \times 10^{-15}}} \tag{21}$$

$$\approx 600pc.\text{(check sf, 1 or 2 sf is ok)} \tag{22}$$

6. Image adapted from `http://www.nakedeyeplanets.com/`.

For parts (a) and (b), the "reasonable working" should show an understanding of the apparent retrograde motion, measurement of the apparent angular motion of the object across the sky e.g. in degrees per year, and realizing that it is an object in orbit in the solar system, moving in an approximate circular arc. If the participants, whether by experience or otherwise, realize from looking at the object and its path that it is actually the planet Saturn, and also happen to remember the facts/figures about Saturn's orbit, then they should be rewarded for that.

a) Full credit can be awarded for 8 to 9 A.U. without working. Partial credit can be given for 10 A.U. and/or reasonable working shown, arriving at a reasonable conclusion (final answer cannot be too far off) [3].

b) Full credit can be awarded for 10 to 11 A.U. without working. Again, partial credit can be given if reasonable working shown and/or answer is 2 A.U. more than answer from (a) [2].

c) Increasing (full credit, no working required) [2]. No partial credit.

d) Anywhere from 1 to 4 degrees would be an acceptable answer (must show some working required for full credit) [2]. Partial credit can be given for reasonably close estimates, that shows correct working from measurements of the "thickness" (this is difficult to measure by ruler, has large error bar) of the retrograde motion loop, and making use of their answer for distance to the object.

7. a) 0.02 or 0.03 radians. [0.5]

Uncertainty: ± 0.01 radians. [0.5]

b) First exclude the outliers, i.e. values of radial velocities that are much larger/smaller than the rest. These are galaxies that happen to be in the field of view, but are much further away/nearer, so they are not actually in the cluster of interest. [1]

Then find the mean radial velocity, which is 7027 km/s. [1]

Using Hubble's constant $H_0 = 70$km/s/Mpc, the distance of the cluster is $7027/70 = 100$ Mpc. [1] (1 or 2 sf is ok.)

c) R = $0.5 \times$ angular diameter found in part (a) $\times$ distance of cluster found from part (b). [1] (1 sf.)

E.g. for angular diameter of 0.02 radians, R = 1 Mpc.

d) Again excluding the outliers, the data yields 592234 km^2/s^2; round it off appropriately to 5.9 or 6×10^5 km^2/s^2 (1 or 2 sf). There may also be differences (5%) due to students dividing by N or N-1 when calculating variance; let's not penalize for this. [2]

e) For 3 dimensions, $< v^2 > = 3 \times$ answer from part (d) $= 1.8 \times 10^6$ km^2/s^2. [1] (1 or 2 sf.)

f) [1]

$$M_{tot} = \frac{< v^2 > R}{\alpha G} \tag{23}$$

g) Substituting values into Eq. 23,

$$\begin{aligned} M_{tot} &= \frac{1.6 \times 10^6 \text{km}^2/\text{s}^2 \times 1 Mpc}{0.5 \times 6.67 \times 10^{-11}} && (24) \\ &= 8 \times 10^{24} M_{sol} && (25) \end{aligned}$$

or whatever value obtained based on the previous answers [1] (1 or 2 sf). The primary contributor of error/uncertainty would be from the estimate of the angular diameter of the cluster [1].

h) 10^2 or a few hundred galaxies, so total mass of all stars $10^{13} M_{sol}$ [1] (1 or 2 sf). Accept answers of this order.

Most of the mass is in the form of dark matter [1].

OR: An alternative answer that Zwicky postulated is that the cluster is not in equilibrium [1].

OR: Any reasonable answer deemed fit by the marker.

8. For an eclipsing binary system where the stars are in circular orbits, eclipses occur when the stars are lined up with our line of sight so their radial motion vanishes. This is not strictly true for non-circular orbits.

a) The luminosities of the two stars, L_A and L_B, can be found from their temperatures T_A and T_B and radii R_A and R_B [2]:

$$\frac{L_A}{L_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 \tag{26}$$

Lets evaluate the second case first (easier), that of the primary eclipsing the secondary. Between eclipses the total luminosity of the system is $L_A + L_B$. When the primary eclipses the secondary the smaller star is completely invisible and the total luminosity is reduced to L_A. This corresponds to a dimming by a factor of [1]:

$$\frac{L_A}{L_A + L_B} \tag{27}$$

When the secondary eclipses the primary the eclipse is not total but annular. We can see some of A since B is smaller. Of the luminous disk of A, a fraction $\left(\frac{R_B}{R_A}\right)^2$ is obscured. The total luminosity is now [2]:

$$L_B + L_A\left(1 - \left(\frac{R_B}{R_A}\right)^2\right) \tag{28}$$

corresponding to a reduced brightness [1]:

$$\frac{L_B + L_A\left(1 - \left(\frac{R_B}{R_A}\right)^2\right)}{L_A + L_B} \tag{29}$$

b) $\phi = 0$; “significantly”.

$\phi = 0.5$; “slightly”.

$\phi = 1$; “significantly”.

1 mark for getting all values of ϕ correct. 1 mark for labeling “slightly/significantly” correctly.

Partial credit: 1 mark for getting two values of ϕ and both their corresponding “slightly/significantly”.

Other answers: no credit.

c) When the stars are eclipsing each other they are moving at right angles to our line of sight, in opposite directions, so the relative speed is $v = v_A + v_B = 150$km/s (160 km/s is also acceptable) [2].

The horizontal segment of the light curve during an eclipse corresponds to the time during which the smaller star moves across the disk of the larger one. During this segment, the center of the small star travels a distance of $2(R_A - R_B)$ across the face of the larger star, which takes a time t_H given by [1]:

$$t_H = \frac{2(R_A - R_B)}{v} \tag{30}$$

From the time the other eclipse begins until it is complete, the small star travels a distance of $2R_B$ across the face of the larger star, which takes a time t_S [1]:

$$t_S = \frac{2R_B}{v} \tag{31}$$

Solar radius was not given, so the final answer may be in terms of the solar radii. Students, if they recall (but are not expected to), the solar radius is 7×10^5 km. That said, a "trick" is that students should recall the apparent angular size of the Sun is about $0.5°$, so by using the small-angle formula, the Sun's diameter can be calculated as

$$\frac{(0.5 \times 3600)''}{206265} = \frac{2R_{sol}}{1\text{AU}} \tag{32}$$

$$R_{sol} = 6.5 \times 10^5 \text{km} \tag{33}$$

Substituting all the relevant numbers, and using $v = 150$km/s and R_{sol} is in km, $t_H = 0.029R_{sol}$s/km $= 20,000s$ [1] and $t_S = 0.012R_{sol}$s/km $= 8,400s$ [1].

End

These solutions have been prepared by Soh Rong'en, Lim Yu Xian, Ng Kia Boon, Wang Sihao and An Nguyen.