1st Singapore Astronomy Olympiad
23 March 2013

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Name: ____

School: ____

Rules

- You may use pen or pencil to answer the paper.
- Do not write your working or answers on the question papers.
- Write your name and school acronym on all sheets of paper used. Number the pages used.
- Clearly label all your working and answers.
- Show your working clearly. Use of background knowledge is allowed, in which case state that you are doing so clearly, and how you use said knowledge to derive your answer. However, background knowledge alone may not gain you full marks if no working is shown.
- At the request of the invigilator, please produce a photo ID for verification of identity.
- Only the use of scientific calculators is allowed. Graphing calculators are not allowed.
- Use only the paper provided to write your answers in.
- Questions are independent of each other, e.g. question 1 & question 2 are independent. Parts within each question may or may not be linked.
- You have 4 hours to complete the paper. If you are late, no time extension is granted.
- State all relevant assumptions made, in your working.
- Penalty: -0.5 mark for each final answer given to the wrong significant figure, maximum penalty for the entire paper is of 1 mark.
- The number of marks for each part question is given in [] at the end of the question.
- You may leave any time upon submission of their solutions and question paper.
- Cheating and allowing others to cheat are grounds for immediate disqualification.
- Notes are not allowed.
- Question papers must be returned together with their scripts.
- At this point, please raise your hand to clarify any doubts now, if any.

Constants:
1 AU = 1.5×10^{11} m
1 megaton = 4.2×10^{15} J
1 Mpc = 3.1×10^{22} m
Mass of Sun = 2.0×10^{30} kg
Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
H_0 = 70 km s^{-1} Mpc^{-1}
G = 6.67×10^{-11} m^3 kg^{-1} s^{-2}
1" = 2.54 cm

Question 1

a) Rigel has RA 5h 15m and declination -8°. Sirius has RA 6h 45m and declination -16°. If observed from San Francisco, USA, which of the stars would cross the local meridian earlier, and how much earlier? If the information provided is insufficient to fully answer the question, state what piece of information is necessary in order to evaluate an answer. [2] (*Note: if you answer only which star crosses earlier without working, no marks will be awarded.*)

b) New Zealand is to the south of latitude 16°S. When Sirius crosses the meridian in New Zealand, what will be its altitude and azimuth? Give your answer in terms of δ and L, where δ is the declination of Sirius and L is the latitude of New Zealand. [3]

c) The Moon's orbit is tilted at 5° with respect to the ecliptic. What is the maximum altitude at which it is ever possible to find the Moon when observing at San Francisco, 37°N? [3]

Question 2

One possible trajectory for sending a spacecraft to Mars is an elliptical orbit with Earth at its perihelion and Mars at its aphelion. The craft would be launched out of low-Earth orbit by a quick burst of rockets into solar orbit, and would then "coast" until rockets fire again to slow it down into Mars orbit. Neglecting the acceleration and deceleration phases, how long would it take to get from low-Earth orbit to Mars along this "minimal-energy" trajectory? State all necessary assumptions you make. Sketch the trajectory with the orbits of Earth and Mars. [7]

Question 3

Zeroing a telescope is very important as it "synchronizes" your finderscope and optical tube assembly (OTA) to make your life easier. Suppose you have an 8" Cassegrain with focal length 2032 mm, an eyepiece of focal length 32 mm, and a 7X50 finderscope attached onto the OTA with a holder such that the edge of the OTA and the center of the finderscope are separated by 3.0". The apparent field of view of the eyepiece is 50° while the true field of view of the finderscope is 5°. Sketch the situation, and find the minimum distance of an object from the telescope such that after zeroing, a celestial object appearing in the center of the finderscope is still within the field of view of the OTA. [6]

Question 4

The Cassini division between Saturn's A and B rings is visible from Singapore with a moderate telescope under good conditions. The inner edge of the Cassini division – the Huygens gap – lies at a radius (from the center of Saturn) of approximately 117,580 km. The Maxwell gap between the B and C rings lies at a radius of approximately 89,400 km. What does Mimas, which orbits at a radius of about 186,000 km, have to do with gaps in the rings? Quantitatively explain your answer. [6]

Question 5

a) δ Cephei varies in brightness with a period of 5.4 days. Its spectrum contains plenty of heavy element lines. On average, it appears 5.4×10^{-13} times as bright as the Sun. Using Fig. 1, find the distance to this star, stating which calibration curve you used and any necessary assumptions made. [6]

b) δ Sanfrancisco is a (fictitious) RR Lyrae variable. Its apparent brightness varies from 2.0×10^{-15} to 4.9×10^{-15} times that of the Sun with a period of 0.69 days. Interstellar extinction dims the star by 31%. Using Fig. 1, estimate the distance to δ Sanfrancisco. [2]

Question 6

Fig. 2 is a star map showing the apparent path of an object across the sky from September 2006 to December 2013.

a) What is the approximate distance of the object from the Earth on 1 Apr 2011? [3]

b) What is the approximate distance of the object from the Earth on 15 Sep 2009? [2]

c) From 2006 to 2013, is the distance between the object and the Sun increasing, decreasing, or is there no observable change? [2]

d) The object is moving in a plane that makes an angle θ with the ecliptic. Find θ in degrees. [2]

Question 7

In this problem, you are to estimate the total mass M_{tot} of a galaxy cluster using the virial theorem. The virial theorem relates the average kinetic energy $\langle T \rangle$ and potential energy $\langle U \rangle$ of a self-gravitating system:

$$\langle U \rangle = -2\langle T \rangle$$

Take the average kinetic energy of the cluster to be

$$\langle T \rangle = \frac{1}{2} M_{tot} \langle v^2 \rangle$$

where $\langle v^2 \rangle$ is the mean square velocity of cluster material. You can estimate $\langle v^2 \rangle$ using the variance of the velocities of the cluster member galaxies about the mean cluster velocity. The potential energy of the cluster can be taken to be

$$\langle U \rangle = -\alpha \frac{G {M_{tot}}^2}{R}$$

where R is half the diameter of the cluster and α is a numeric factor of order unity which depends on how mass is distributed within the cluster; for this problem, use $\alpha = 0.5$.

a) Estimate the angular diameter of the cluster shown in Fig. 3, in radians. Include a suggested uncertainty for your estimate. [1]

b) Table 1 gives the measured radial velocities for some of the galaxies in Fig. 3. Find the distance of the cluster away from us, in Mpc. Note that not all the galaxies in Table 1 belong to the cluster. [3]

c) What is the half-diameter R of the cluster, in Mpc? [1]

d) Again using Table 1, find the variance of the radial velocities of the cluster member galaxies about the mean cluster radial velocity, in km^2/s^2. [2]

e) Assuming the velocity distribution is isotropic (i.e. same in all directions), estimate $\langle v^2 \rangle$. [1]

f) Using the virial theorem, write down an expression for M_{tot} in terms of quantities estimated from the data. [1]

g) Hence evaluate M_{tot} , giving your answer in solar masses. What is the primary contributor of error/uncertainty to your value for M_{tot} ? [2]

h) From Fig. 3, estimate the number of galaxies in the cluster. If each galaxy has a total mass in stars of about 10^{11} solar masses, what is the total mass of all stars in the galaxy cluster? Comment on your answer in relation to M_{tot}. [2]

Question 8

Fig. 4 shows radial velocities for a binary star system. In this question, suppose that this is an eclipsing binary.

a) The surface temperatures of the primary (more massive and larger) star and secondary (less massive and smaller) star, respectively, in this binary system are T_A and T_B, and their radii are R_A and R_B (with $R_A > R_B$), respectively. Estimate the brightness of the system when the secondary eclipses the primary, and similarly for when the primary eclipses the secondary, relative to the brightness of the system between eclipses, in terms of T_A, T_B, R_A and R_B (or other variables of your choice that are dependent on T_A, T_B, R_A and R_B if you find that helps to simplify the expression). [6]

b) Write down all the value(s) of ϕ (within the values in Fig. 4) at which eclipse(s) occur. For each value of ϕ, state also if the measured light curve for the system would dip "slightly" or "significantly". [2]

c) Given the values of T_A = 9700K , T_B = 5800K , R_A = 3.04R_{sol} , R_B = 0.90 R_{sol} . Find the length t_H of the horizontal segment of the dip in total luminosity during an eclipse, and the length t_S of the segment of the light curve during which the luminosity decreases to its minimum value during an eclipse. [6]

The End

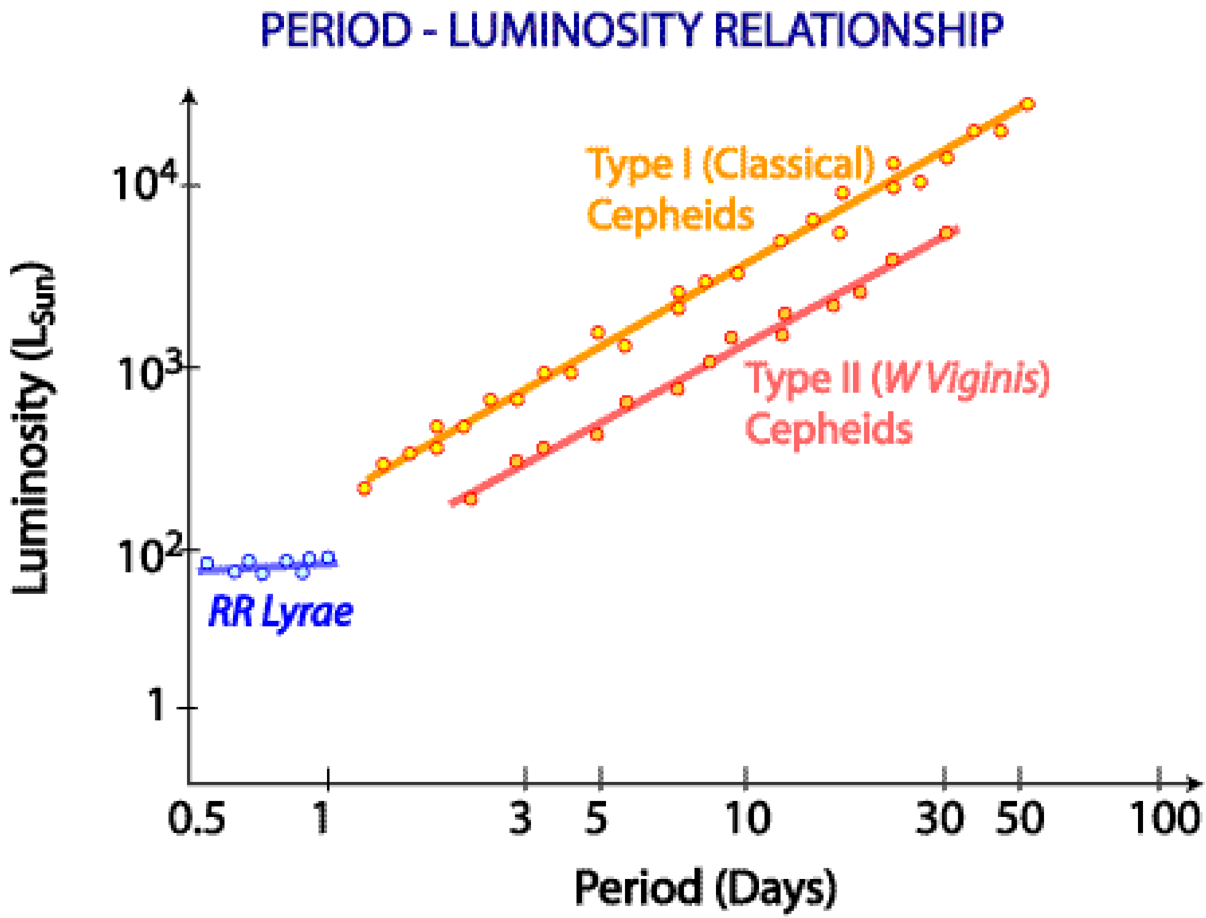

Fig. 1: Period-luminosity relationship of variable stars. Image from http://outreach.atnf.csiro.au/education/senior/astrophysics/images/binvar/plrelnceph.gif.

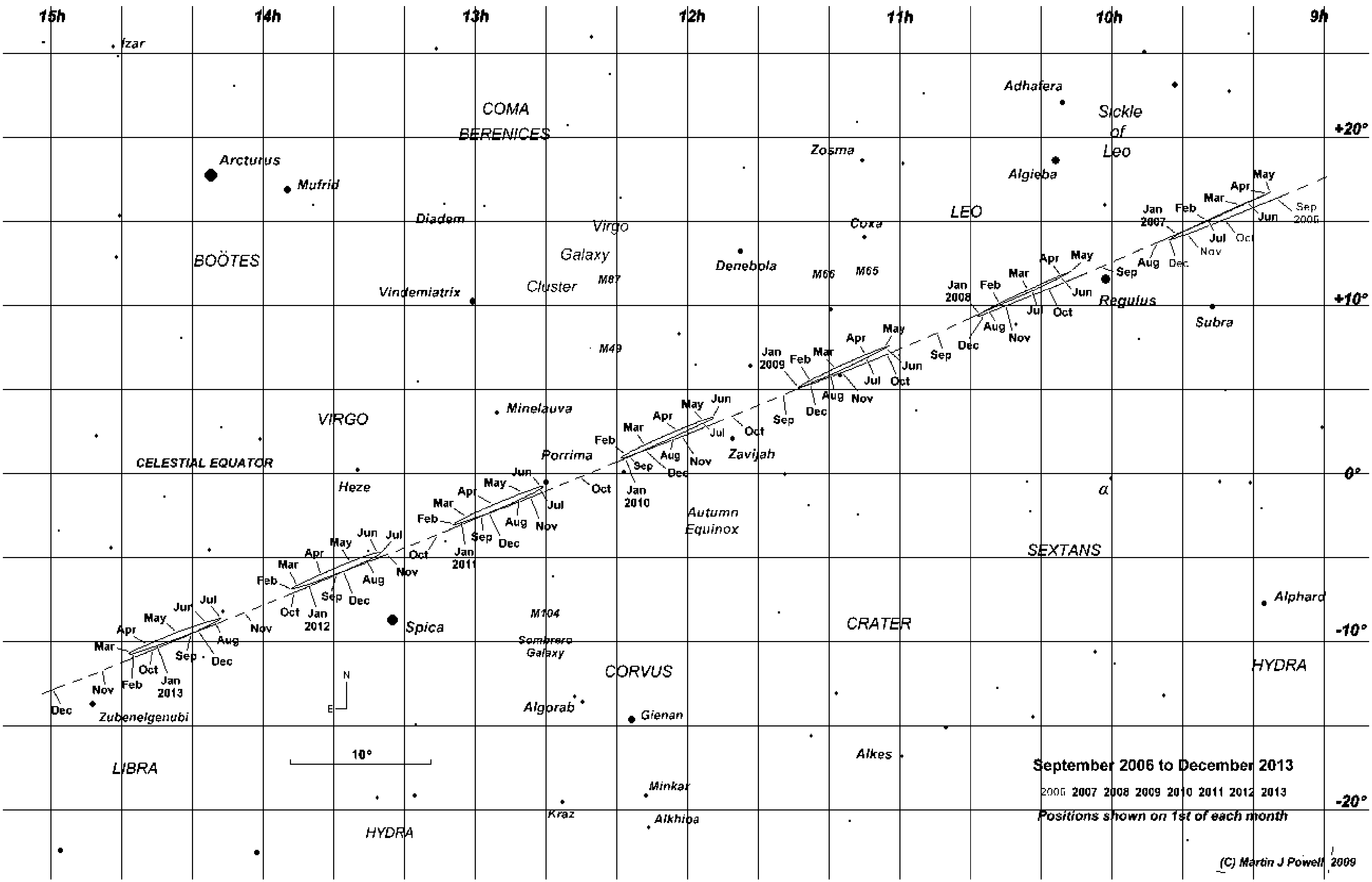

Fig. 2: Star map showing the apparent path of an object across the sky from September 2006 to December 2013.

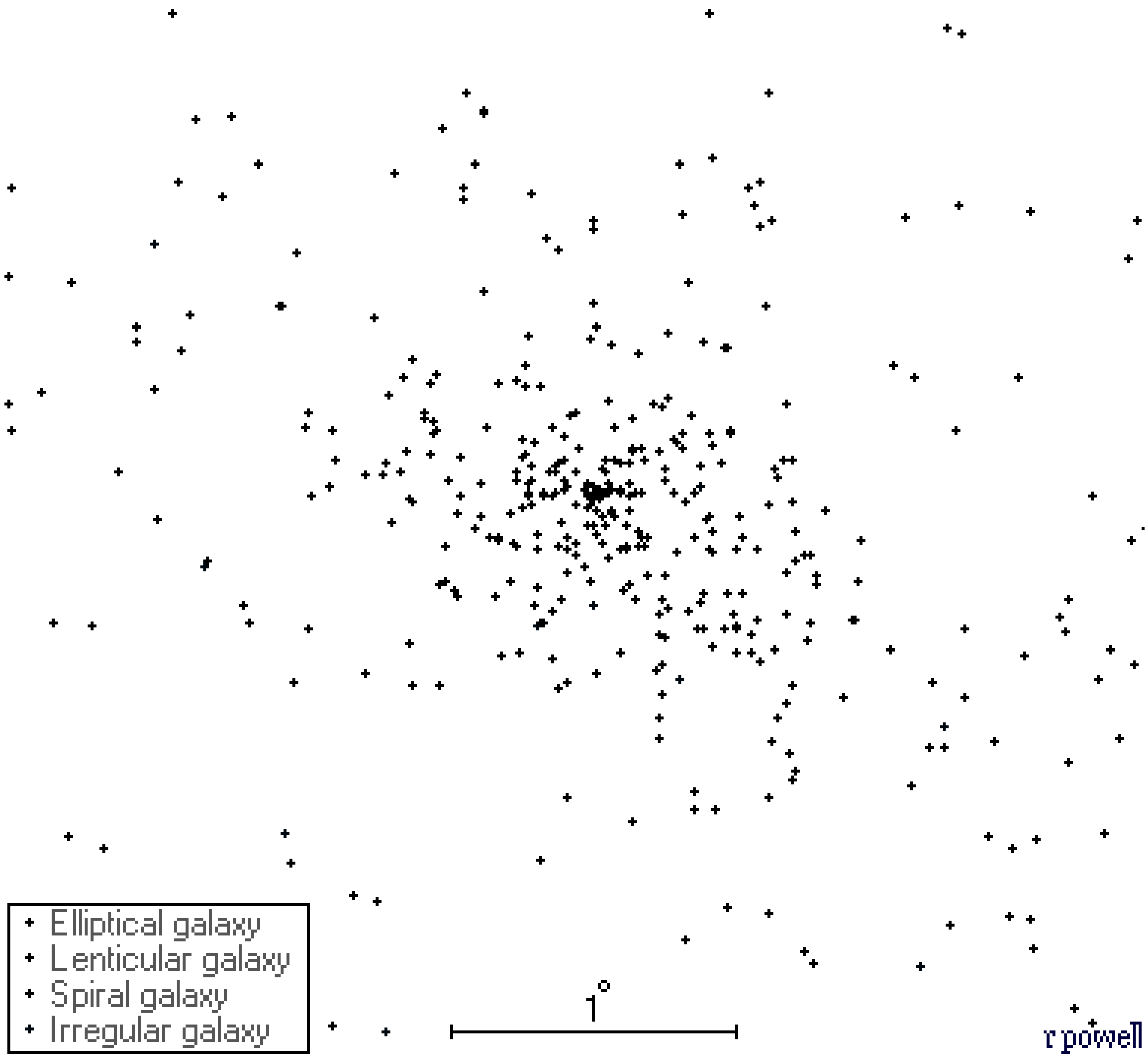

Fig. 3: Map of a 4×4 degree^2 area of the sky, with the field of view centered on a galaxy cluster. Image from http://www.atlasoftheuniverse.com/superc/com.html.

Table 1: Radial velocities for galaxies. Adapted from Biviano et al., Astron. Astrophys. Suppl. Ser. 111, 265 (1995).

Galaxy ID (arbitrary)	Radial velocity (km/s)
1	6971
2	6628
3	5500
4	123
5	8116
6	6200
7	8135
8	6210
9	6626
10	114990
11	7732
12	6650
13	6543
14	94
15	6841
16	62822
17	7198
18	56
19	5681
20	8299
21	7046
22	7304
23	38272
24	7323
25	-202
26	8203
27	6488
28	7031
29	7604
30	7294

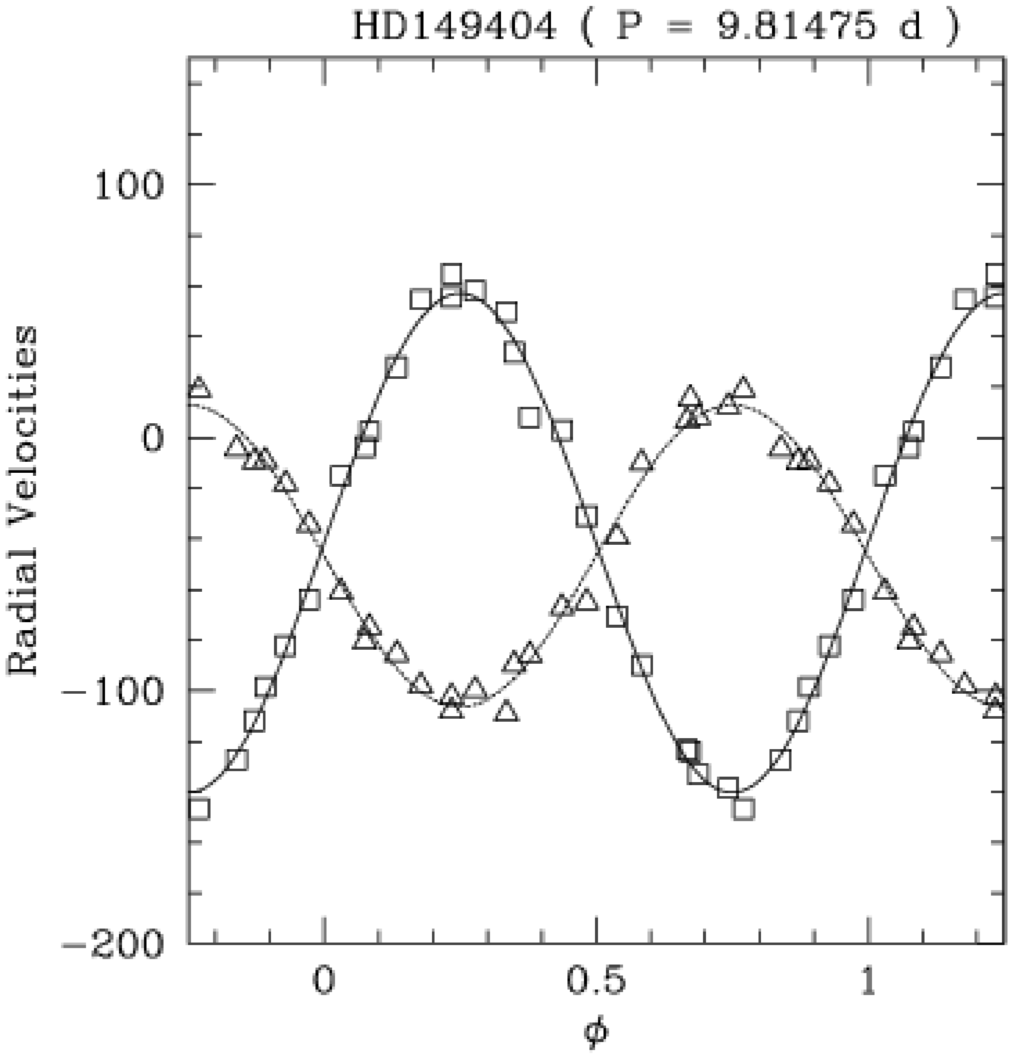

Fig. 4: Radial velocity measurements for an eclipsing binary star system, in km/s. The squares correspond to data from one of the stars while the triangles correspond to data from the other star. Image adapted from Rauw**,** et al.**,** A&A **368** (**1**) 212 (2001).