5th Japan Astronomy Olympiad National Finals

Theoretical Problems

February 22, 2025 13:15–15:45

Instructions

1. Do not open this problem booklet until instructed to begin.
2. This booklet contains 15 pages in total. If any pages are missing, out of order, or have unclear printing, raise your hand and notify the proctor.
3. Use only a black pencil or black mechanical pencil for all answers.
4. On the answer sheet, write not only the final answer but also the calculation process and explanations that lead to it, as necessary.
5. Write your examinee number in the designated field on the answer sheet. Do not write your examinee number anywhere else.
6. Write your answers in the designated answer area for each problem. If it is unclear which sub-question a portion of your answer corresponds to, it may receive no credit. You do not need to answer sub-questions in order, as long as it is clear which sub-question each answer addresses.
7. Do not write any irrelevant characters, symbols, or marks in the answer area.
8. If physical constants needed for calculations are not explicitly stated in the problem, refer to the values in the appendix constant table.
9. For numerical answers where significant figures are not specified, use your own judgment to round to an appropriate number of significant figures. Answers with too many or too few significant figures may be penalized.
10. The margins of this booklet may be used for scratch work, but do not tear any pages.
11. Do not take the answer sheet home.
12. After the exam ends, take the problem booklet, constant table, and calculation paper home.
13. Questions about the problems will not be answered. If you believe there is an error in a problem, note it on the answer sheet; it will be taken into consideration during grading.

Problem 1 (130 points)

Answer the following independent questions (Questions 1–4).

Question 1 (30 points)

Briefly explain each of the following three terms in about two lines each.

(1) Solar wind

(2) Population II (stellar population II)

(3) Cosmic Microwave Background (CMB) radiation

Question 2 (30 points)

Read the following passage about double stars and binary stars, then answer questions (1)–(5).

Mizar in Ursa Major is a star of approximately 2nd magnitude, with a 4th-magnitude star, Alcor, located approximately 12 arcmin away. Because the angular separation between Mizar and Alcor is small, observing them with the naked eye is near the limit of human visual resolution. However, using a telescope or other means to increase angular resolution (visual acuity), Mizar and Alcor can be resolved as separate stars. A pair of stars that appear close together on the celestial sphere in this way is called a *double star*.

Double stars are divided into two types: *optical doubles*, where two stars that have no physical relationship happen to lie in nearly the same direction as seen from Earth, and *binary stars*, where two spatially close stars are gravitationally bound and orbit their common center of mass. The following methods are used to distinguish optical doubles from binary stars.

Method A: Measure the annual parallax. If the two stars are found to be at completely different distances, they cannot be a binary.

Method B: If the pair is a binary, the stars should orbit their common center of mass; if this orbital motion is detected as an apparent motion, it is strong evidence that they are a binary.

(1) In distance measurement by annual parallax (Method A), there is a maximum measurable distance corresponding to the minimum detectable parallax. If the minimum detectable parallax is 0.020 mas ($1\,\text{mas} = 10^{-3}\,\text{arcsec}$), find the maximum measurable distance in parsecs (pc) to 2 significant figures.

(2) Besides Methods A and B, briefly describe in about two lines a spectroscopic method for determining whether a double star is a binary.

(3) Assume that the naked-eye stars visible in the night sky are randomly distributed across the whole sky. We estimate the expected number of optical double stars over the whole sky. Let the total number of stars visible to the naked eye across the whole sky be 9×10^3, and define a pair of stars as an "optical double" if their angular separation is 12 arcmin or less.

(3-1) Let p be the probability that a star b falls within an angular distance $\theta = 12\,\text{arcmin}$ of a given star a. By considering the ratio of the area of a circle of radius 12 arcmin centered on star a to the area of the whole sky, find p to 2 significant figures. Since θ is sufficiently small, you may ignore the curvature of the sphere. The total area of the whole sky is 4π sr.

(3-2) Using the result of (3-1), find the expected number of optical double star pairs across the whole sky to 1 significant figure.

Figure 1–1 shows the cumulative histogram (a graph showing the total count from the smallest bin up to each bin) of angular separations between pairs of naked-eye stars, constructed from the Washington Double Star Catalog (Mason, B.D., et al., February 8, 2026 edition, `https://www.astro.gsu.edu/wds/`).[1]

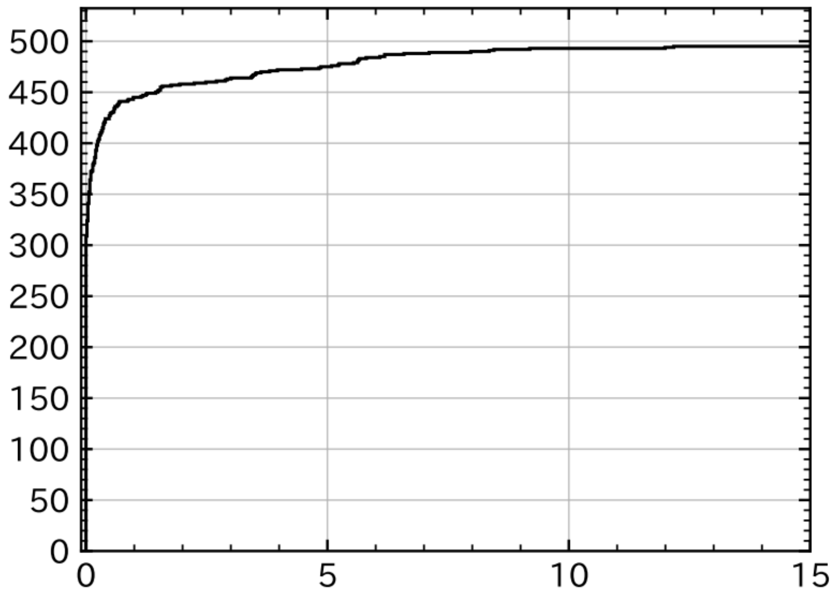

Figure 1–1: Cumulative histogram of angular separations between naked-eye stars. Horizontal axis: angular separation / arcmin (0 to 15); Vertical axis: cumulative count (0 to 500).

(4) From Figure 1–1, the number of double star pairs with apparent angular separation of 12 arcmin or less is how many times larger than the result of (3-2)? Give the answer to 1 significant figure.

One reason why the actual number of double stars does not agree with the result of (3-2) is that many binary stars exist in the universe, and many of the stars observed as double stars are in fact such binaries. Furthermore, the non-isotropic distribution of stars also affects the number of double stars.

(5) Suppose the Galactic plane occupies 20% of the total sky area, and 60% of all stars (by number) are uniformly distributed in the Galactic plane while the remaining 40% are uniformly distributed outside it. Using this two-region model, and considering only optical doubles formed within the same region (i.e., ignoring pairs consisting of one star in the Galactic plane and one outside), find the expected number of optical double star pairs across the whole sky to 1 significant figure.

[1] This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory.

Question 3 (40 points)

Planetary defense is an international effort to identify in advance the collision risk posed by near-Earth objects (NEOs) such as asteroids and comets, and to prevent damage. It is one of the important challenges in astronomy and space development as a preparation for low-frequency but extremely damaging natural disasters, such as the giant asteroid impact that caused the mass extinction at the end of the Cretaceous period, and the 2013 Chelyabinsk meteor impact.

Asteroids and comets that approach Earth's orbit are collectively called NEOs (Near-Earth Objects). Among these, objects that could cause a damaging Earth impact in the astronomically near future are called PHAs (Potentially Hazardous Asteroids). A PHA is defined as an object whose "minimum orbital intersection distance" (MOID) with Earth's orbit is 0.05 au or less, and whose absolute magnitude (the apparent magnitude when the object is placed 1 au from the Sun and observed from the Sun) is less than 22.0. Survey observations are conducted to discover and track PHAs as early as possible and determine their orbits precisely. Answer the following questions with these definitions in mind.

(1) The average albedo of asteroids is 0.13. What is the minimum diameter of a PHA (a perfect sphere with this albedo)? Assume that reflected light spreads isotropically over a solid angle of π sr.

A solar system object has six orbital elements describing its orbit and motion: semi-major axis a, eccentricity e, inclination i, longitude of ascending node Ω (the ecliptic longitude of the point where the orbit crosses from south to north of the ecliptic), argument of perihelion ω (the angle measured from the ascending node to perihelion), and true anomaly ν (the angle indicating how far the object has advanced from perihelion).

(2) Read the following passage and answer the questions.

From this question onward, assume that Earth's orbit is a circular orbit in the ecliptic plane with $a = 1$ au, $e = 0$, $i = 0°$.

We consider the minimum orbital intersection distance (MOID) between Earth's orbit and that of a given object. The MOID is found as the smaller of the two distances between the orbits along the line of intersection of their orbital planes. Since we assume Earth's orbit lies in the ecliptic plane, the line of intersection of the two orbital planes always passes through the ascending node, the descending node, and ____(A)____. Since these points lie in the ecliptic plane, the MOID does not depend on the inclination i. Also, since Ω indicates ____(B)____, the distance does not depend on Ω.

Here, the distance from the Sun at true anomaly ν on an elliptical orbit is given by:

$$r(\nu) = \frac{a(1-e^2)}{1+e\cos\nu}$$

At the ascending node and descending node, the true anomalies are (C) and (D), respectively. Substituting these into the above formula gives the object's distance from the Sun at these points. The differences from Earth's orbital distance (1 au) at each position are:

$$\left|\frac{a(1-e^2)}{1+e\cos\omega} - 1\right|, \quad \left|\frac{a(1-e^2)}{1-e\cos\omega} - 1\right|$$

and the smaller of these two is the MOID.

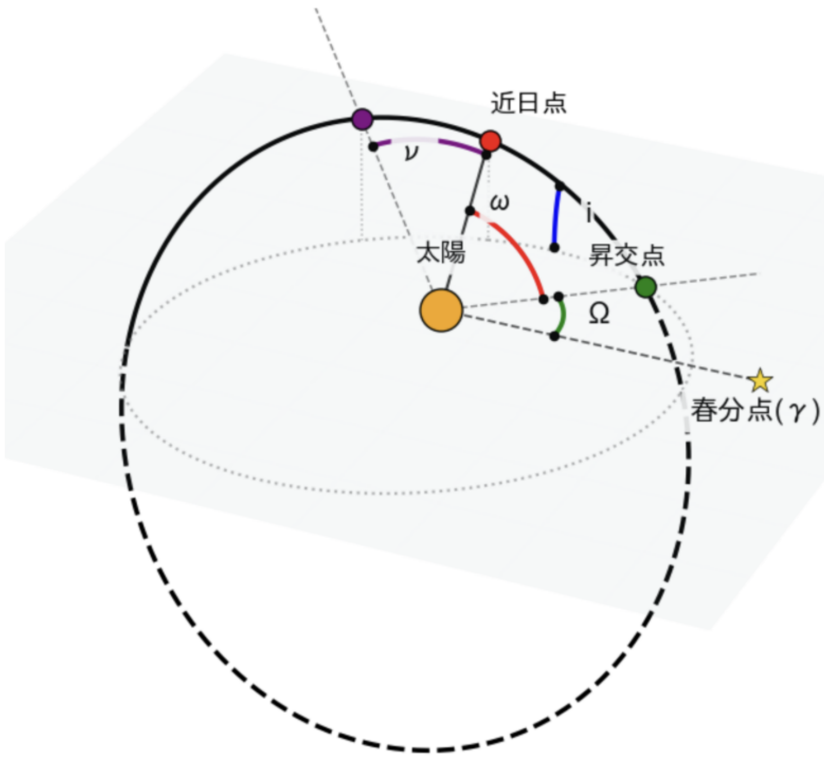

Figure 1–2: Diagram showing orbital elements: inclination i, longitude of ascending node Ω, argument of perihelion ω, and true anomaly ν.

(2-1) Choose the most appropriate phrase for blank **(A)** from the following options:

① The vernal equinox
② The perihelion of this object
③ The Sun
④ The north celestial pole

(2-2) Choose the most appropriate phrase for blank **(B)** from the following options:

① The direction of perihelion
② The distance from the Sun at perihelion
③ The semi-minor axis of the orbit
④ The orientation of the orbit

(2-3) Express the angles for blanks **(C)** and **(D)** using any necessary quantities from a, e, i, Ω, ω.

(2-4) Are the following two objects (a) and (b) PHAs? State your reasoning.

Object	a [au]	e	i [deg]	Ω [deg]	ω [deg]	Diameter [m]
a	1.324	0.280	1.621	69.081	162.803	330
b	1.2705	0.2852	2.328	194.133	233.951	4.1

(3) Consider the process of actually discovering PHAs through observation.

(3-1) Calculate the MOID with Earth's orbit for the following three orbits c, d, e.

Orbit	a [au]	e	ω [deg]
c	0.922	0.1910	126.40
d	0.920	0.1913	126.40
e	0.922	0.1913	126.40

(3-2) Determining an object's orbit requires long-term observations. Explain the reason using words from the following word bank:
Word bank: position, distance, velocity, degrees of freedom, observational accuracy

(3-3) When an object with a collision risk to Earth is found, choose the most appropriate graph (from ①–④ below) showing how the collision probability changes with observation period, and state your reasoning.

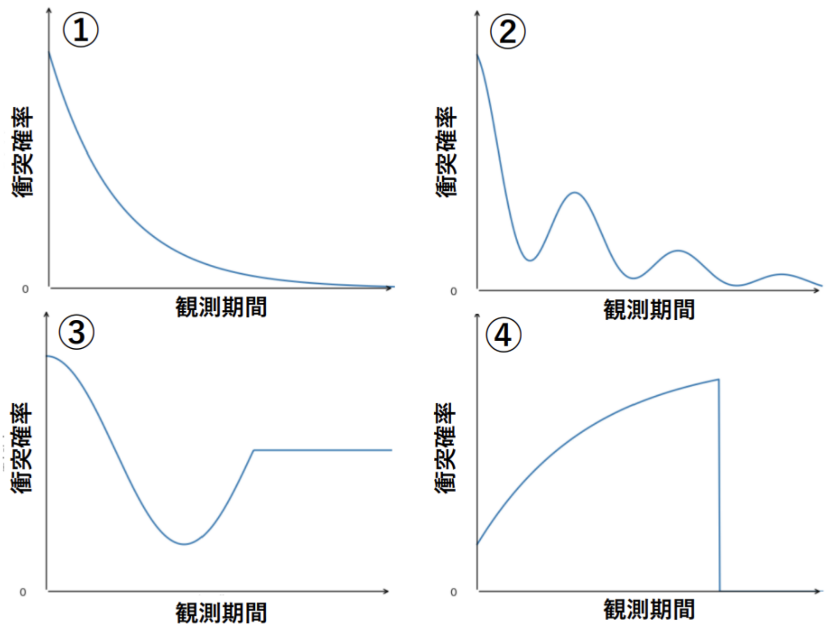

Figure 1–3: Four graphs labeled ①–④ showing collision probability vs. observation period. ① monotonically decreasing; ② oscillating with damped fluctuations (multiple ups and downs); ③ first decreasing then slightly rising (dip then plateau); ④ first rising then decreasing (peak then drop).

Question 4 (30 points)

The expansion history of the universe can be described as a balance between the decelerating effect of gravity from matter and the accelerating effect of dark energy. The dimensionless quantity describing the degree of cosmic expansion is called the scale factor $a(t)$. Normalizing the scale factor at the present time t_0 as $a(t_0) = 1$, the proper distance $D(t)$ between any two points at time t is expressed as $D(t) = a(t)D_0$, where D_0 is the present-day distance. The equation determining the acceleration of cosmic expansion is written approximately as:

$$\frac{\ddot{a}}{a} = -\frac{H_0^2}{2}\left(\Omega_m a^{-3} - 2\Omega_\Lambda\right)$$

where H_0 is the present Hubble constant, $\ddot{a}$ is the second time derivative (acceleration) of the scale factor, Ω_m is the present matter density parameter, and Ω_Λ is the present dark energy density parameter. Answer the following questions.

(1) Let $a_{\rm tr}$ be the scale factor at the moment when the cosmic expansion transitions from deceleration to acceleration. Express $a_{\rm tr}$ in terms of Ω_m and Ω_Λ. Also, using the present observed values $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, find the value of $a_{\rm tr}$ to 2 significant figures.

(2) Let $z_{\rm tr}$ be the redshift at which cosmic expansion began to accelerate. Using the result of (1), find the value of $z_{\rm tr}$ to 2 significant figures.

(3) At the epoch when galaxy formation peaked ($z \approx 2$), was the cosmic expansion accelerating or decelerating?

(4) In the early universe just after the Big Bang, when $a \ll 1$ (matter-dominated era), $a(t)$ is proportional to t^α. Find the value of α.

Problem 2 (90 points)

In 2025, the number of confirmed exoplanets exceeded 6000. One of the most powerful methods for detecting exoplanets is the *transit method*. The transit method detects the slight dimming that occurs when a planet passes in front of its host star, blocking a portion of the star's light.

Observations of a main-sequence star S with luminosity $L_S = 0.34\,L_\odot$ and effective temperature $T_{\rm eff} = 5.0 \times 10^3\,\rm K$ have confirmed the existence of a planet P orbiting S. Continued observation of the S system revealed that planet P transits in front of star S with a period of $T = 3.0$ days, causing a dimming each time.

In this problem, for simplicity, assume that planet P orbits in a circular orbit and that the normal to the circular orbit is exactly perpendicular to the line of sight from Earth. Also treat star S as a uniformly bright circular disk.

Question 1

(1) Find the radius R_S of star S in units of the solar radius $R_\odot$ to 2 significant figures.

(2) Assuming the mass–luminosity relation $L \propto M^4$ for main-sequence stars, find the orbital semi-major axis a of planet P in au to 2 significant figures.

(3) Express the transit depth Δ[mag] in terms of the radius ratio $R_P/R_S = k$ between star S and planet P. Ignore any reflection or emission from planet P.

In reality, the brightness of the S system includes a small contribution from the planet. Because of this, a dimming can also occur when the planet passes behind the star; this is called the *secondary eclipse*. Continuous mid-infrared observations of the S system centered on wavelength $8\,\mu$m revealed, as shown in Figure 2–1, not only a transit at central time $t = 0.5$ days but also a secondary eclipse at central time $t = 2.0$ days. The vertical axis of the figure is normalized so that the flux F_0 at time $t = 2.0$ days equals 1.

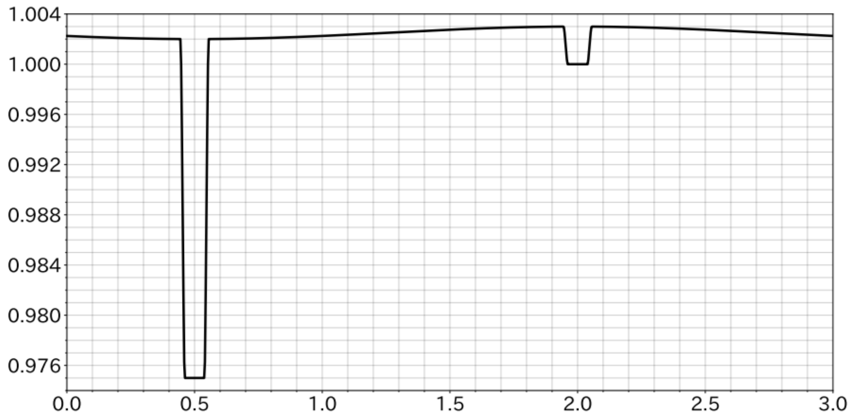

Figure 2–1: Light curve of the S system over one period. Horizontal axis: time (days), 0 to 3; Vertical axis: normalized flux $[F_0]$, approximately 0.976 to 1.004. A deep transit dip is seen near $t = 0.5$ days, and a shallower secondary eclipse near $t = 2.0$ days. The out-of-eclipse flux varies continuously and is slightly above 1.000 outside of the secondary eclipse.

Question 2

(1) By reading Figure 2–1, express the following fluxes in units of F_0.

(1-1) The flux from star S observed when planet P is absent

(1-2) The flux from the night hemisphere of planet P (the hemisphere facing away from the star)

(1-3) The flux from the day hemisphere of planet P (the hemisphere facing toward the star)

(2) Find the radius R_P of planet P in units of the Jupiter radius R_J to 2 significant figures.

Question 3

In the S system, the secondary eclipse observed and the continuous brightness variation during times without a transit or secondary eclipse are due to planet-derived radiation. Planet-derived radiation consists of reflected starlight from the planet and thermal emission from the planet's surface temperature.

(1) Estimate the contribution from reflected starlight from the planet. Using the results of Question 1(2) and Question 2(2), find the upper limit of the reflected light component from planet P in units of F_0 to 1 significant figure. Assume that the reflected light from planet P spreads isotropically over a hemisphere of π sr.

(2) The result above suggests that the observed secondary eclipse and brightness variation are primarily due to thermal emission from the planet's surface. Explain in about two lines why infrared radiation is more suitable than visible light for detecting such thermal-emission secondary eclipses.

(3) Use the observational results to estimate the surface temperature distribution of planet P. As the simplest model, assume that the temperature of the dayside and nightside hemispheres are constant at T_day and T_night respectively, and that the planet emits as a perfect blackbody.

(3-1) Show that at wavelength λ, the ratio of the blackbody radiation intensity F_P from the hemisphere of a planet with surface temperature T_P to the blackbody radiation intensity F_S of the star is:

$$\frac{F_\mathrm{P}(\lambda)}{F_\mathrm{S}(\lambda)} = \frac{e^{hc/(\lambda k_\mathrm{B} T_\mathrm{eff})} - 1}{e^{hc/(\lambda k_\mathrm{B} T_\mathrm{P})} - 1} \times k^2$$

You may use the fact that the blackbody radiation intensity per unit wavelength per unit solid angle at temperature T and wavelength λ is:

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_\mathrm{B} T)} - 1}$$

(3-2) Find T_day and T_night to 2 significant figures.

(4) Verify the validity of the above result. Using the result of Question 2(2), and assuming an albedo of 0.3 for planet P, find the equilibrium temperature T_eq of the planet to 2 significant figures.

Problem 3 (90 points)

Solid dust particles floating in space experience gravitational attraction from massive objects such as stars, and when exposed to stellar radiation, they also experience “radiation pressure” due to the momentum carried by absorbed or reflected photons. In particular, the radiation pressure (force per unit area) p acting on a perfectly absorbing body due to radiation is:

$$p = I/c$$

where c is the speed of light and I is the radiation energy passing through a unit area per unit time. In this problem, we consider the orbital evolution of dust particles via radiation pressure and gravity.

Question 1

Consider a dust particle of mass m at distance r from a star of mass M and luminosity L. The particle is spherical with radius s and density ρ. Ignore any reaction force from re-emission.

(1) Express the gravitational force F_g and the radiation pressure repulsion F_{rad} acting on the particle in terms of G, M, m, r and L, s, r, c respectively.

(2) Show that the ratio $\beta = F_{\mathrm{rad}}/F_g$ can be expressed using M, L, s, ρ, G, c, and is a constant independent of distance r.

(3) Explain why the net force on the dust particle can be treated as gravity from a star whose effective mass appears to be $M_{\mathrm{eff}} = M(1-\beta)$.

(4) What properties does a particle with $\beta > 1$ have?

Question 2

In the x-y plane, two stars A and B, each of mass M, move back and forth in a straight line with maximum separation $2R$ about their common center of mass O. This can be regarded as motion on an elliptical orbit with eccentricity $e = 1$.

(1) Find the time t_{star} for star A to travel from the maximum separation point to the center of mass O.

(2) Consider the combined force F_{net} acting on a particle at height H on the z-axis, received from both stars. Show that for $H \gg R$, this force can be approximated by the gravitational attraction of a body of mass $2M_{\mathrm{eff}} = 2M(1-\beta)$ at the center of mass O, by approximating to first order in R/H.

(3) A dust particle is released from rest at height H on the z-axis ($H \gg R$). Express the time t_{part} for this particle to fall to the center of mass O in terms of G, M, β, H.

(4) Express the minimum height H for the particle to collide with a star at the center of mass O (i.e., satisfying $t_{\mathrm{part}} = t_{\mathrm{star}}$) in terms of R and β. Also, describe with physical reasoning how H changes as β approaches 1.

Question 3

Consider a dust particle orbiting in a circular orbit of radius a in the ecliptic plane of the solar system.

(1) Due to the orbital velocity of the particle, in the particle's rest frame, starlight appears to be incident at an angle α from the direction of the star. As a result, the radiation pressure acts as a drag force in the direction opposite to the particle's motion. Express this force F_{PR} in terms of the particle's orbital velocity v, the radiation force F_{rad}, and c.

(2) The angular momentum $L = m\sqrt{2GM(1-\beta)a}$ of the particle decreases due to F_{PR}. Using $\frac{dL}{dt} = -F_{\mathrm{PR}} \cdot a$, find the rate of change of orbital radius $\frac{da}{dt}$, and show that it takes the form $\frac{da}{dt} = -\frac{k}{a}$ (where k is a constant).

(3) Express the time for a particle starting at $a = a_0$ to fall into the star in terms of a_0, β, G, M, c. Also, calculate the lifetime of a particle with $\beta = 0.5$ falling into the Sun from $a_0 = 1.0$ au, in years, to 1 significant figure. You may use without proof that the solution to the ODE $\frac{da}{dt} = -\frac{k}{a}$ is $a(t) = \sqrt{(a(0))^2 - 2kt}$.

(4) Comparing the result of (3) with the age of the solar system, explain why dust still exists in the solar system today by naming two supply sources.

Problem 4 (90 points)

Galaxies are not uniformly distributed in the universe but instead have a clumpy distribution. A system in which roughly 100 or more galaxies are gravitationally bound together is called a *galaxy cluster*. It is known that the intergalactic space within a galaxy cluster contains high-temperature ionized gas (ICM: intracluster medium) at temperatures of 10^7–10^8 K. The ICM primarily radiates X-rays through a process called *thermal bremsstrahlung*. By observing the ICM in X-rays, it is possible to estimate the properties of the ICM and even the total mass of the galaxy cluster.

Question 1

X-ray telescopes used to observe the ICM are all launched into the upper atmosphere or space. Briefly state in about one line why there are no ground-based X-ray telescopes.

Question 2

Thermal bremsstrahlung is the radiation process that occurs when an electron passes near a proton in a thermally equilibrated plasma and undergoes deceleration. Assuming that the electrons in the plasma follow a Boltzmann energy distribution, the energy emitted by thermal bremsstrahlung per unit frequency, unit volume, and unit time is a function of electron number density n_e, temperature T, and frequency ν:

$$\varepsilon(n_e, T, \nu) = 6.3\times10^{-48} \left(\frac{n_e}{10^{-3}\,\mathrm{cm^{-3}}}\right)^2 \left(\frac{k_\mathrm{B}T}{1.6\times10^{-8}\,\mathrm{erg}}\right)^{1/2} e^{-h\nu/k_\mathrm{B}T}\times\tilde{g}\ \ \mathrm{erg\ s^{-1}cm^{-3}Hz^{-1}}$$

where $\tilde{g}$ is the Gaunt factor (a quantum mechanical correction to the classical result), which is at most of order 1 (use $\tilde{g} \approx 1$ hereafter). Referring to the above formula, compare the frequency dependence of the thermal bremsstrahlung spectrum at low frequencies ($h\nu < k_\mathrm{B}T$) and at high energies ($h\nu > k_\mathrm{B}T$), and show that the spectrum changes significantly around $\nu = \nu_0 = k_\mathrm{B}T/h$.

Question 3

X-ray observations of a certain galaxy cluster revealed thermal bremsstrahlung from the ICM spread over a region of radius 16 arcmin.

(1) The Fe XXV emission line from this galaxy cluster (from 24-times ionized iron, rest-frame frequency 1.62×10^{18} Hz) was observed at a frequency of 1.54×10^{18} Hz. Find the redshift of this galaxy cluster and its distance to 1 significant figure. Use a Hubble constant of $H = 70\,\mathrm{km\,s^{-1}Mpc^{-1}}$.

(2) Using the distance found in (1), find the radius R of this galaxy cluster's ICM in Mpc to 1 significant figure.

(3) The characteristic frequency at which the observed spectrum bends was 1.0×10^{18} Hz everywhere in the cluster. Find the temperature of the ICM to 1 significant figure.

From analysis of the thermal bremsstrahlung spectrum, the electron density distribution can also be estimated. For this galaxy cluster, the electron density as a function of radius r from the center is:

$$n_e(r) = \begin{cases} n_0, & (r \leq r_0) \\ n_0 \left(\dfrac{r}{r_0}\right)^{-2}, & (r > r_0) \end{cases}$$

where $r_0 = 0.1$ Mpc and n_0 is a constant.

(4) The ICM is optically thin to electromagnetic radiation in the frequency range under discussion. The surface brightness I_ν (energy received per unit solid angle, unit frequency, unit area, unit time) corresponds to integrating $\varepsilon/4\pi$ along the line of sight:

$$I_\nu(\nu) = \frac{1}{4\pi} \int \varepsilon(n_e, T, \nu)\, dS$$

where S is the distance along the line of sight. Given that the surface brightness toward the cluster center is $I_\nu(\nu_0) = 1.2 \times 10^{-23}$ erg s^{-1} cm^{-2} Hz^{-1} sr^{-1}, find n_0 in units of cm^{-3} to 1 significant figure. Treat the ICM as a perfect sphere of the radius found in Question 3(3), and consider only the path that passes straight through the cluster center.

(5) Consider a unit-area column of gas in the ICM, extending radially outward from the center, for the thin shell between radii r and $r + dr$. Let the pressures on the inner and outer faces of this shell be P and $P + dP$, respectively. Show that when the force from the pressure difference across the shell and the gravitational force on the shell are in equilibrium (hydrostatic equilibrium):

$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2}$$

where $\rho(r)$ is the mass density at radius r and $M(r)$ is the total mass within radius r.

(6) Treat the hot gas as an ideal gas. For a fully ionized gas where the number densities of protons and electrons are equal, $P = 2n_e(r)k_{\mathrm{B}}T$. Also, for simplicity, use $\rho(r) = m_p n_e(r)$ where m_p is the proton mass. Express $M(r)$ for $r > r_0$ using T, r, and physical constants only. Also find $M(R)$ in units of $M_\odot$ to 1 significant figure.

Question 4

The mass found in Question 3(6) is the dynamically determined mass, so it includes not only the stellar mass in galaxies and the gas mass in the ICM, but also the dark matter within the galaxy cluster. In this question, we estimate the baryonic mass and subtract it from the result of Question 3(6) to estimate the total dark matter mass in the cluster.

(1) For this galaxy cluster, find the gas mass in units of $M_\odot$ to 1 significant figure by spatially integrating the gas mass density $\rho(r)$ from the center to $r = R$.

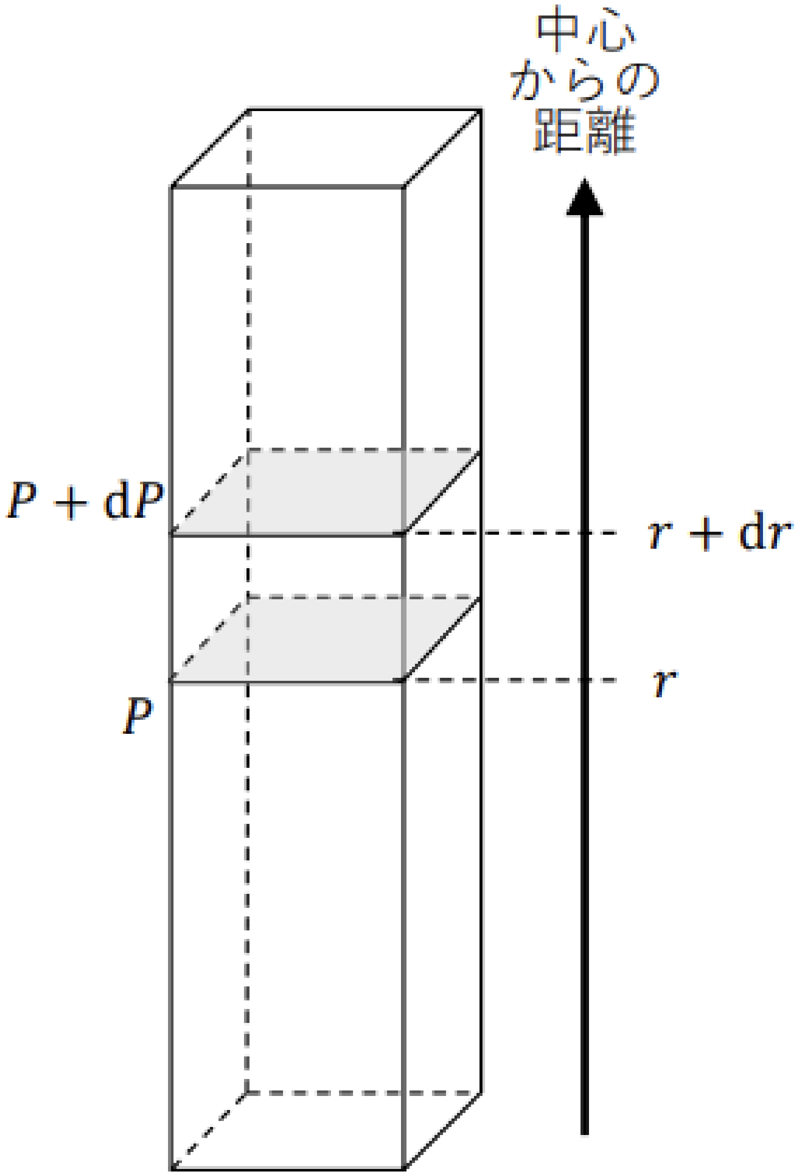

Figure 4–1: Schematic diagram of hydrostatic equilibrium.

(2) The combined apparent magnitude of all galaxies in this cluster is 9.5 mag. Given that the absolute magnitude of the Sun is 4.5 mag, and assuming all stars in these galaxies have the same luminosity and mass as the Sun, find the total stellar mass within this galaxy cluster in units of $M_\odot$ to 1 significant figure. Ignore the effects of interstellar extinction, and assume all galaxy brightness comes from stars.

(3) By subtracting the masses found in Question 4(1) and (2) from the mass found in Question 3(6), find the dark matter mass within this galaxy cluster in units of $M_\odot$ to 1 significant figure. Also find the fraction of the total cluster mass that is dark matter to 1 significant figure.