3rd Japan Astronomy Olympiad National Finals
Practical Solutions

Note: The answers shown here are representative solutions. In actual grading, solutions other than those listed below may also have received credit.

Problem 1

Question 1.

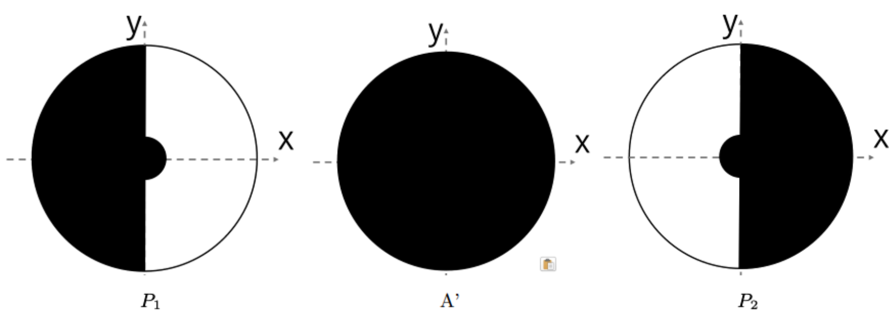
Star Name	Bayer Designation
Betelgeuse	α Ori
Rigel	β Ori
Sirius	α CMa
Procyon	α CMi
Capella	α Aur
Aldebaran	α Tau
Pollux	β Gem

Question 2.

The refractive index of a medium varies with wavelength; this phenomenon is called *dispersion*. Axial chromatic aberration refers to the colour fringing in an image caused by the fact that, for rays entering parallel to the optical axis, dispersion causes the focal length of a lens to vary with wavelength along the optical axis.

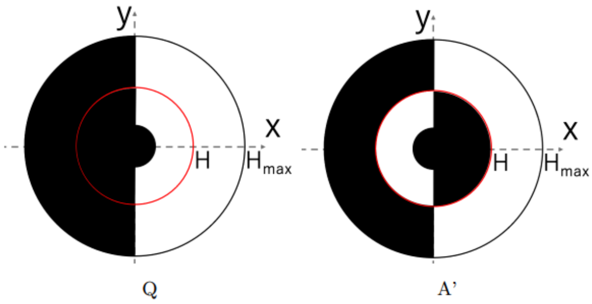
Question 3.

(1) (In this problem, figures with the x- and y-axes swapped were also accepted as correct.)



Phase diagrams at P_1, A', and P_2.

(2) (In this problem, figures in which the sign of the x-coordinate at A' is reversed were also accepted as correct.)



Phase diagrams at Q and A'.

Problem 2

Question 1.

(1) 1 day.

(2) The culmination altitude is 60° and remains constant throughout the year.

Question 2.

(1) Maximum value: 90°; Minimum value: 35°.

(2)

$$(P,\, Q) = (p\cos\alpha - q\sin\alpha,\ p\sin\alpha + q\cos\alpha).$$

(3)

$$(x,\, y,\, z) = \left(\frac{\sqrt{3}}{2}\cos t,\ \sin t,\ -\frac{1}{2}\cos t\right).$$

(4)

$$\boxed{A} = \frac{\sqrt{3}}{2}, \qquad \boxed{B} = 1 - \frac{\sqrt{3}}{2}.$$

(5)

$$\boxed{C} = -\tfrac{1}{2}\cos t, \qquad \boxed{D} = -\left(1 - \tfrac{\sqrt{3}}{2}\right)\cos t \sin t, \qquad \boxed{E} = 2 + \sqrt{3}.$$

(6) Sign table and quadrant identification:

	$0 < t < 90$	$90 < t < 180$	$180 < t < 270$	$270 < t < 360$
C	$-$	$+$	$+$	$-$
D	$-$	$+$	$-$	$+$
θ	(C)	(A)	(B)	(D)

(7)

$$\boxed{F} = \frac{\sqrt{3}}{2}, \qquad \boxed{G} = 1 + \frac{\sqrt{3}}{2}.$$

(8) Numerical table of azimuth θ and altitude r as a function of time t:

t (day)	θ (°)	r (°)	t (day)	θ (°)	r (°)
0	270	30.0	180	90	30.0
15	266	29.0	195	94	29.0
30	262	25.9	210	98	25.9
45	259	21.1	225	101	21.1
60	257	14.9	240	103	14.9
75	255	7.7	255	105	7.7
90	—	0	270	—	0
105	75	7.7	285	285	7.7
120	77	14.9	300	283	14.9
135	79	21.1	315	281	21.1
150	82	25.9	330	278	25.9
165	86	29.0	345	274	29.0

(9) Analemma diagram (figure ①):

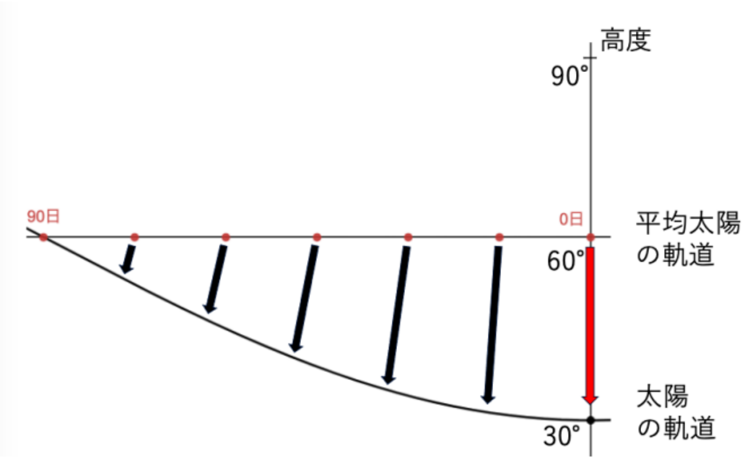


Figure 2–9 ①: Analemma in the altitude–azimuth plane. The horizontal axis represents the path of the mean Sun at altitude 60° (Day 90 on the left, Day 0 on the right); the vertical axis represents altitude. The true Sun's path is the lower black curve (altitude ≈ 30°), while the mean Sun's path is the horizontal line at altitude 60°. Short tick marks indicate the displacement of the true Sun from the mean Sun at 15-day intervals.

Analemma diagram (figure ②):

Question 3.

(1)

Day	Position of true Sun relative to mean Sun
Day 0	Same position
Day 90	East
Day 180	Same position
Day 270	West

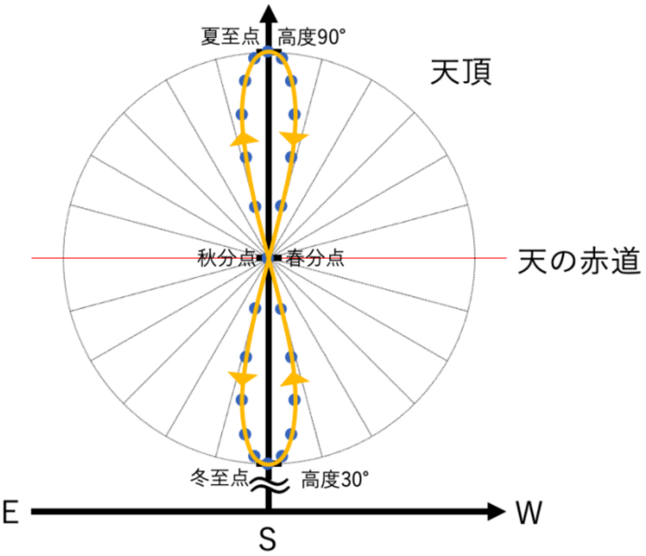
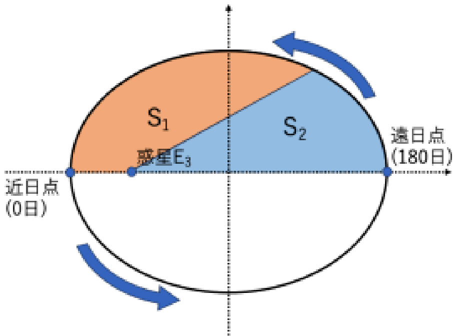


Figure 2–9 ②: Analemma as seen projected on the celestial sphere from the observer's zenith. Summer solstice is at the top (altitude 90°), winter solstice at the bottom (altitude 30°), and the celestial equator passes through the autumnal/vernal equinox points. East (E) is to the left and West (W) to the right.

Reason: By Kepler's Second Law, the area swept out by the line joining planet E_3 to the Sun in a fixed time interval is constant. Therefore, at aphelion and perihelion there is no difference between the true Sun and the mean Sun. On the other hand, for example on Day 270, the area S_1 (shown in orange in the figure) equals the area S_2 (shown in blue). This indicates that, as seen from planet E_3, the true Sun lies to the *west* of the mean Sun. The same argument applies on Day 90 (where the true Sun is to the east).



Schematic of equal-area sectors S_1 and S_2 illustrating the east/west displacement. Planet E_3 is at perihelion (Day 0, left) and aphelion (Day 180, right).

(2)

For $V = 0$: from (1), on Day 90 the true Sun is displaced to the east of the mean Sun. The direction of displacement of the true Sun relative to the mean Sun therefore evolves as shown in the figure below.

By similarly considering $V = 90, 180, 270$, the analemmas for each value of V are expected to take the forms shown below.

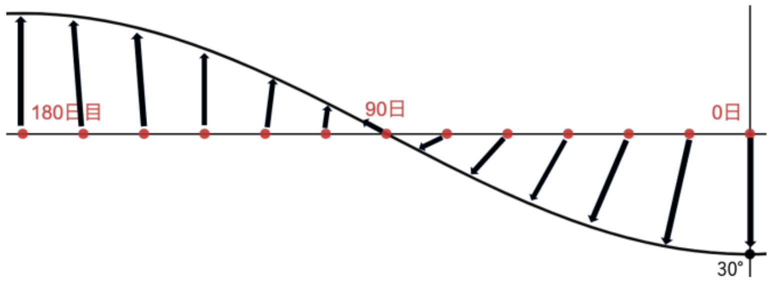


Figure ($V = 0$ case): Direction of displacement of the true Sun relative to the mean Sun. Horizontal axis = path of mean Sun (altitude 60°), Day 90 at left, Day 0 at right; vertical axis = altitude; lower black curve = path of true Sun (altitude ≈ 30°).

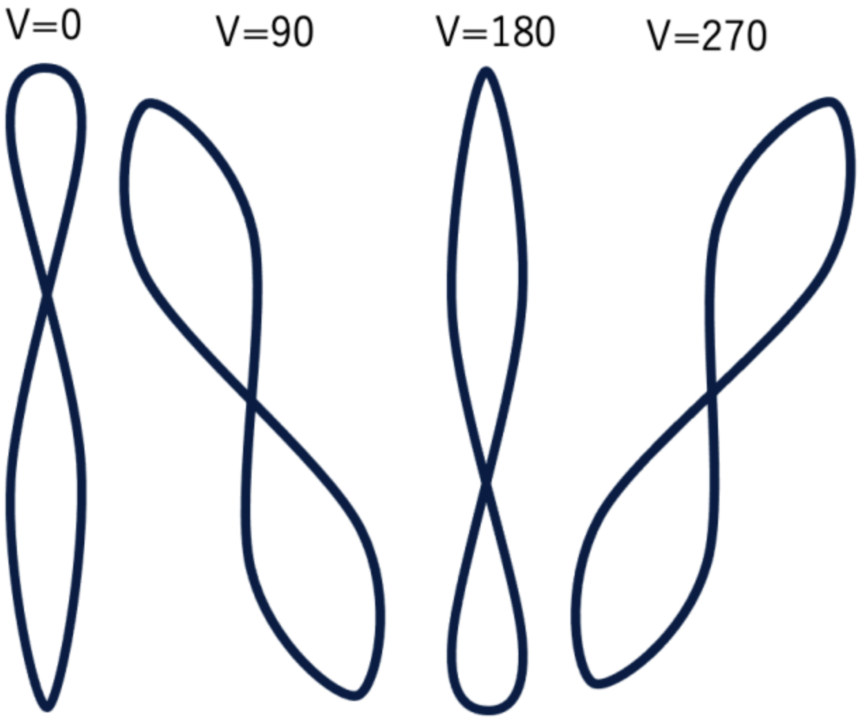


Figure: Expected analemma shapes for $V = 0, 90, 180, 270$. Each panel shows the figure-eight (or deformed figure-eight) traced by the true Sun over one year.

Furthermore, when the eccentricity is small the orbit approaches a circle, and the analemma approaches the shape found in Question 2. When the eccentricity is large—taking $V = 0$ as an example—the displacement direction never moves eastward during the interval from Day 0 to Day 180, so the shape is expected to become a teardrop rather than a figure-eight.