3rd Japan Astronomy Olympiad
National Finals
Practical Problems

February 23, 2024 16:15–17:45

Instructions

1. Do not open this problem booklet until the start signal is given.
2. This booklet contains 16 pages in total. If you find any missing, disordered, or illegible pages, raise your hand and notify the proctor.
3. Use only a black pencil or black mechanical pencil for your answers.
4. Write your examinee number in the designated field on the answer sheet. Do not write it anywhere else.
5. Write all answers in the designated spaces on the answer sheet.
6. Do not write any unrelated characters, symbols, or marks in the answer spaces.
7. You may use the margins of this booklet as scratch paper, but do not tear any pages.
8. Do not take the answer sheet home.
9. After the exam, you may take this booklet and any calculation sheets home.
10. No questions about the problems will be answered. If you believe there is an error in a problem, write a note on your answer sheet; it will be considered during grading.

Problem 1

Answer the following independent sub-questions (Questions 1–3) about various topics related to observation and optical systems. For questions that have a space for equations and reasoning on the answer sheet, provide explanations showing how you arrived at your answer.

Question 1. Bright Stars in a Given Region

List all 7 stars that are currently brighter than magnitude 1.5 and located in the region with declination -20° to $+50^\circ$ and right ascension 4^h to 8^h (a region generally considered easy to observe from Japan in winter). Following the example given on the answer sheet, provide both the proper name and the Bayer designation for each star.

Question 2. Chromatic Aberration in Convex Lenses

Explain in about two lines what chromatic aberration (axial chromatic aberration) in a convex lens is, using **all** of the following terms:

Terms: refractive index, dispersion, focal length

Question 3. Foucault Test for Mirror Surface Accuracy

The Foucault test is one method used to verify the surface accuracy of a mirror used in a reflecting telescope. Answer the following questions.

(1) A perfect spherical mirror is placed in a dark room with no external light entering. As shown in Figure 1–1, a coordinate system is set up with the bottom of the spherical mirror as the origin: x and y coordinates in the plane perpendicular to the mirror's optical axis, and the z coordinate along the optical axis. A sufficiently small point light source is placed at a point A on the z-axis. As shown in Figure 1–1, the light emitted from A and reflected by the spherical mirror converges at point A′, which is on the z-axis very close to A.

A circular disk (occluder) is placed between A and A′. When the spherical mirror is observed from point O, which is on the z-axis on the positive-z side of A′, the entire mirror surface appears to glow due to the reflected light from the point source. Light emitted from the point source directly toward O in the positive-z direction is completely blocked by the circular disk and cannot be observed from O.

Next, an obstacle (such as a razor blade) is inserted from the $x \leq 0$ side in a direction perpendicular to the z-axis. The obstacle is thin enough compared to the distance between A and A′, and large enough to block light coming from the point source or the mirror. On the z-axis, let P_1 be a point closer to the spherical mirror than A′, and P_2 be a point closer to the observer than A′. As shown in Figure 1–2, when the obstacle is inserted from the negative-x direction into each of P_1, A′, and P_2 in turn, and the spherical mirror is observed from point O, indicate which part of the mirror surface appears as a shadow by shading the corresponding region black in the diagrams on the answer sheet.

Question 3 continues on the next page.

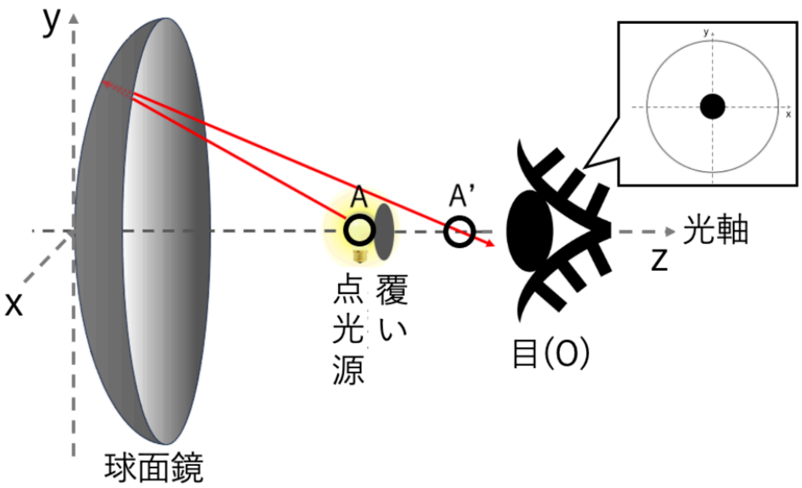

Figure 1–1: Relative positions of the spherical mirror, light source, and viewpoint. (In practice, the light source and occluder are infinitesimally small, so the positions of A and A′ are essentially the same.)

(2) A perfect paraboloidal mirror is placed in a dark room with no external light entering, with coordinates set up as in (1). A sufficiently small point light source is placed at point A on the z-axis. As shown in Figure 1–3, only light reflected from a point H on the mirror surface converges at point A′ on the z-axis very close to A. Light reflected from point H_{max} at the edge of the mirror converges at a point A″ behind A. (H is closer to the center of the mirror than H_{max}.)

A circular disk is placed between A and A′. When the paraboloidal mirror is observed from point O, which is on the z-axis on the positive-z side of A′, the entire mirror surface appears to glow due to the reflected light. As with the spherical mirror, light emitted from the point source directly toward O is completely blocked by the circular disk and cannot be observed from O.

Next, an obstacle is inserted from the $x \leq 0$ side in a direction perpendicular to the z-axis. The obstacle is thin enough compared to the AA′ distance, and large enough to block the light. Let Q be a point on the z-axis closer to the paraboloidal mirror than A′. As shown in Figure 1–3, when the obstacle is inserted from the negative-x direction into each of Q and A′ in turn, and the paraboloidal mirror is observed from point O, indicate which part of the mirror surface appears as a shadow by shading the corresponding region black in the diagrams on the answer sheet.

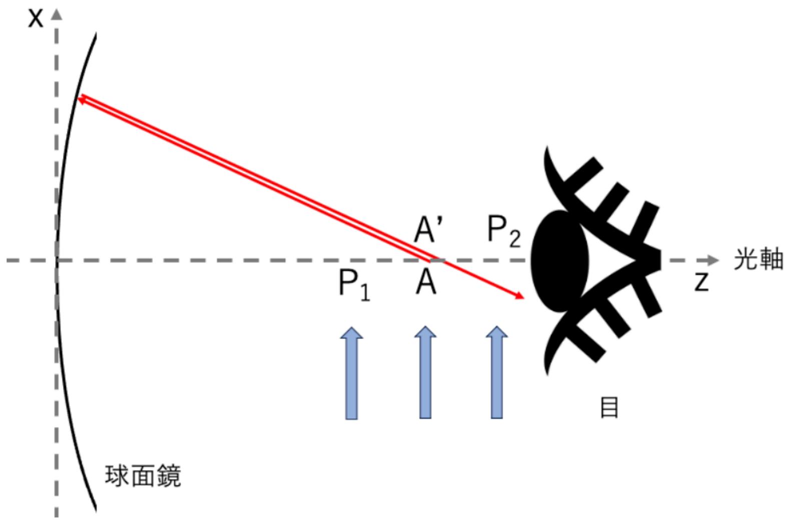

Figure 1–2: Relative positions of the spherical mirror and the observation point, viewed from the positive-y direction. (The occluder and light source are omitted.)

Problem 2

The **analemma** is the curve traced on the celestial sphere by connecting the positions of the Sun observed at a fixed time each day over the course of one year, from a given location on Earth. Answer the following questions about the analemma. The time units used in this problem are defined using the SI second (as defined by the caesium atomic clock), with:

60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 360 days = 1 year.

Use the formula sheet at the end of this booklet as needed.

Question 1. Planet E_1: Circular Orbit, Upright Axis

Consider a hypothetical planet, Planet E_1, with the following properties:

- Planet E_1 orbits the Sun in a perfectly circular orbit.
- The orbital period of Planet E_1 is 1 year.
- The direction of revolution and rotation of Planet E_1, viewed from its north pole, is counterclockwise.
- The sidereal day of Planet E_1 is $\frac{360}{359}$ days.
- A certain location at latitude 30°N on Planet E_1 is called P_1.
- At some moment, the Sun culminates (transits the meridian) at P_1. This moment is defined as the start of a day (0:00), and the calendar on Planet E_1 is defined so that a new day begins every 24 hours thereafter. Within each day, time is expressed in hours (0 to 23), minutes (0 to 59), and seconds (0 to 59).

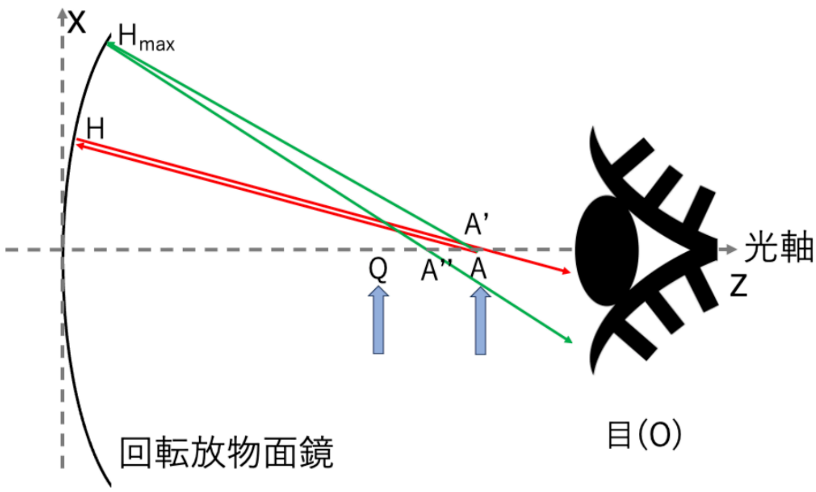

Figure 1–3: Relative positions of the paraboloidal mirror and the observation point, viewed from the positive-y direction. (The occluder and light source are omitted.)

- The rotation axis of Planet E_1 is perpendicular to the orbital plane.

A coordinate system on the celestial sphere is defined analogously to the equatorial coordinate system used on Earth, based on the equatorial plane of Planet E_1. In this system, the Sun as seen from Planet E_1 appears to travel once around the celestial equator over the course of one year, as shown by the red line in Figure 2–1.

(1) On Planet E_1, find the time from one solar culmination to the next, in units of days.

(2) Does the solar culmination altitude at latitude 30°N on Planet E_1 change throughout the year? Briefly describe in about one line, including the actual value(s) of the culmination altitude.

A hypothetical Sun that moves at uniform speed along the celestial equator, analogous to the Sun as seen from Planet E_1, is called the "**mean Sun**."

Question 2. Planet E_2: Circular Orbit, Tilted Axis

Next, consider Planet E_2, whose rotation axis is not perpendicular to its orbital plane. The settings for Planet E_2 are as follows. Planet E_1 does not affect the motion of Planet E_2 in any way.

- Planet E_2 orbits the Sun in a perfectly circular orbit.
- The orbital period of Planet E_2 is 1 year.
- The direction of revolution and rotation of Planet E_2, viewed from its north pole, is counterclockwise.
- The sidereal day of Planet E_2 is $\dfrac{360}{359}$ days.
- A certain location at latitude 30°N on Planet E_2 is called P_2.

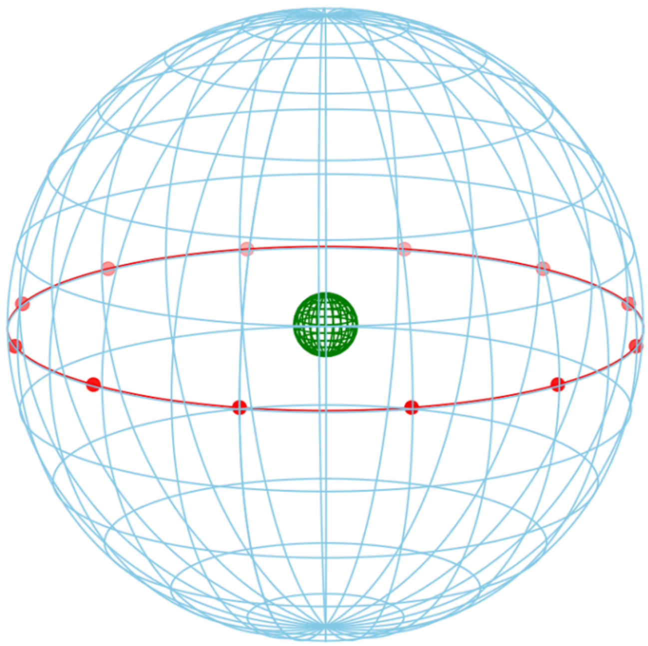

Figure 2–1: Path of the Sun on the celestial sphere as seen from Planet E_1. Red dots show the Sun's position every 30 days; blue lines show right ascension and declination grid lines.

- **The rotation axis of Planet E_2 is tilted 30° from the axis perpendicular to its orbital plane.**
- In the calendar of Planet E_2, **the start of the year (Day 1, 0:00) is defined as the moment when the Sun culminates at P_2 with the lowest culmination altitude over the entire year**. Thereafter, a new day begins every 24 hours. Within each day, time is expressed in hours (0 to 23), minutes (0 to 59), and seconds (0 to 59).

The solstices and equinoxes are defined as follows:

- The moment when the Sun culminates at P_2 with the *lowest* culmination altitude over the year (i.e., the start of the year) is the **winter solstice**; the moment with the *highest* culmination altitude is the **summer solstice**.
- The moment when the Sun crosses the celestial equator from south to north is the **vernal equinox**; from north to south is the **autumnal equinox**.
- The positions of the Sun on the celestial sphere at the winter solstice, vernal equinox, summer solstice, and autumnal equinox are called the **winter solstice point**, **vernal equinox point**, **summer solstice point**, and **autumnal equinox point**, respectively.

As seen from Planet E_2, the Sun moves along the orange curve in Figure 2–2 over the course of one year on the celestial sphere.

Consider the position of the Sun as seen from Planet E_2. When observed from P_2 at the same time every day throughout the year, the position of the true Sun does not necessarily coincide with that of the mean Sun.

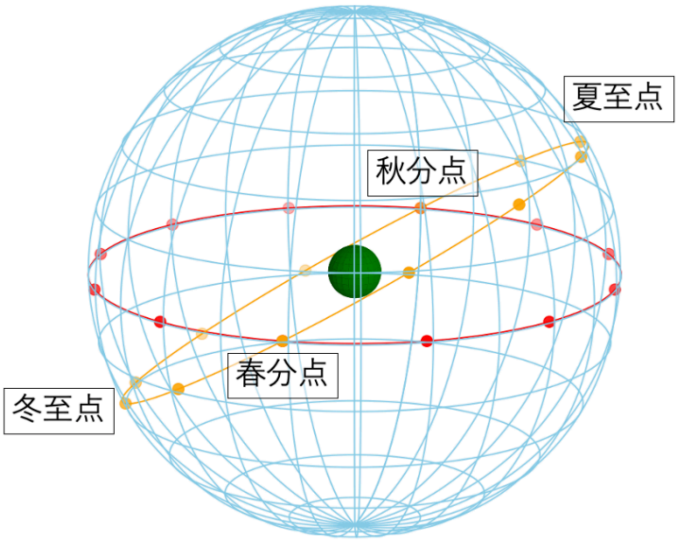

Figure 2–2: Path of the Sun on the celestial sphere as seen from Planet E_2. Red dots show the mean Sun's position every 30 days; orange dots show the true Sun's position every 30 days. Labeled points: 冬至点 = winter solstice point, 春分点 = vernal equinox point, 秋分点 = autumnal equinox point, 夏至点 = summer solstice point.

(1) Find the maximum and minimum values of the solar culmination altitude throughout the year at latitude 25°N on Planet E_2.

(2) To concretely calculate the trajectory traced by the Sun on the celestial sphere as seen from P_2, consider rotation in two-dimensional coordinates. In a 2D Cartesian coordinate system, consider rotating the point (p, q) counterclockwise by angle α about the origin to arrive at the point (P, Q), as shown in Figure 2–3. Express the coordinates (P, Q) after the rotation in terms of p, q, and α.

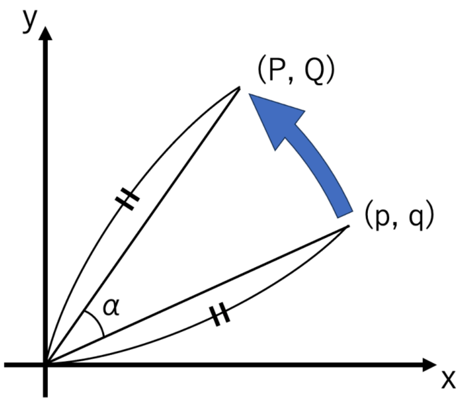

Figure 2–3: Rotation in a 2D Cartesian coordinate system.

(3) To express the position of the Sun on the celestial sphere, define coordinates as shown in Figure 2–4. With the radius of this celestial sphere set to 1, the position of the mean Sun on day t can be expressed as $(\cos t, \sin t, 0)$. Apply the formula found in (2) with $\alpha = -30°$ to the x-axis and z-axis, and express the actual position (x, y, z) of the Sun in terms of t.

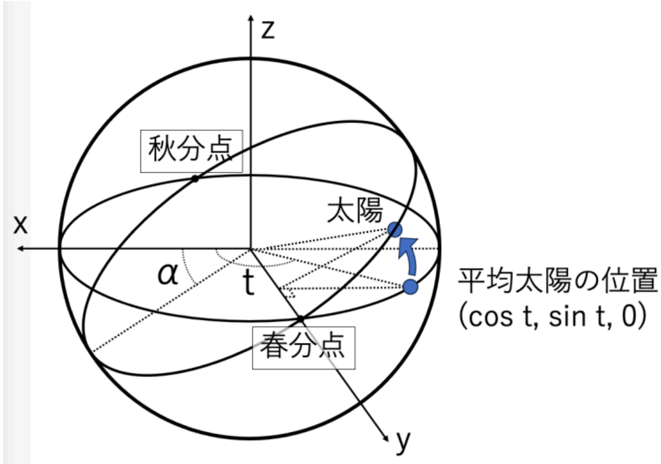

Figure 2–4: Positional relationship between the Sun and the mean Sun on the celestial sphere as seen from Planet E_2. The z-axis points toward the north celestial pole; the x–y plane is the celestial equator. 秋分点 = autumnal equinox point, 春分点 = vernal equinox point. The mean Sun is at $(\cos t, \sin t, 0)$; the true Sun is displaced by angle α.

(4) Answer the numbers that fill in the blanks $\boxed{\text{A}}$ and $\boxed{\text{B}}$ in the following passage about the angular distance between the Sun and the mean Sun on the celestial sphere.

Consider the sector shape shown in Figure 2–5 connecting the mean Sun, the Sun, and the center of the celestial sphere. Let the central angle of this sector be r [deg] ($0° \leq r \leq 360°$). By focusing on the triangle formed by the center of the celestial sphere, the Sun, and the mean Sun, one can show that:

$$\cos r = \boxed{\text{A}} + \boxed{\text{B}} \sin^2 t.$$

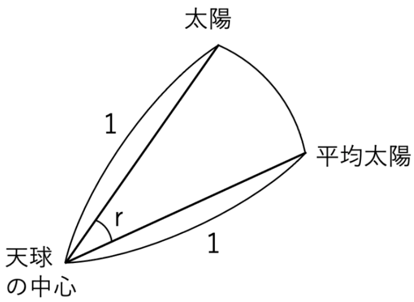

Figure 2–5: Sector with center at the center of the celestial sphere, connecting the Sun and the mean Sun.

(5) Answer the numbers that fill in the blanks $\boxed{\text{C}}$, $\boxed{\text{D}}$, $\boxed{\text{E}}$, and the term that fills in the underlined part (a) in the following passage about the north–south and east–west displacements of the Sun relative to the mean Sun.

Let θ [deg] be the angle between the plane containing the sector found in (4) and the plane of the celestial equator. The celestial sphere surface near the Sun as seen from P_2 looks as shown in Figure 2–6. Use this figure to consider the value of $\tan \theta$.

The north–south displacement of the Sun relative to the mean Sun can be approximated by the value of the z-coordinate in (3), so it equals $\boxed{\text{C}}$.

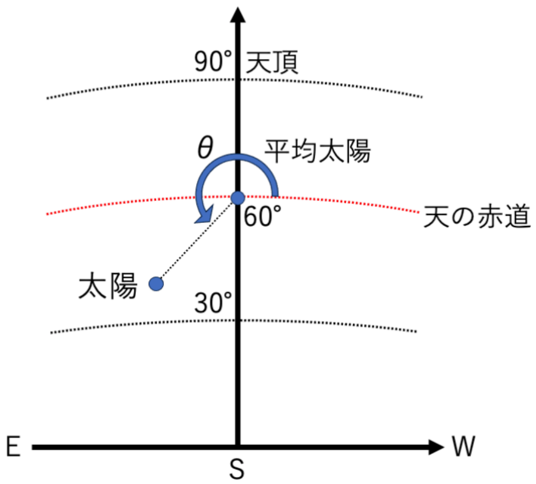

Figure 2–6: A portion of the celestial sphere surface. The vertical axis shows altitude (天頂 = zenith at 90°; 平均太陽 = mean Sun on the celestial equator 天の赤道 at 60°; 太陽 = true Sun at 30°). The horizontal axis shows directions E (east), S (south), W (west).

Next, consider the east–west displacement of the Sun relative to the mean Sun. Figure 2–7 shows the celestial sphere viewed from directly above (from a point at infinity in the direction of the north celestial pole). The positive direction is taken as clockwise (westward). The east–west displacement expressed as an hour angle is called (a) ________________. Expressed as an angle, this can be approximated by the length s along the black dotted line in Figure 2–7, and equals $\boxed{\text{D}}$. Therefore:

$$\tan\theta = \frac{\boxed{\text{C}}}{\boxed{\text{D}}} = \boxed{\text{E}} \times \frac{1}{\sin t}.$$

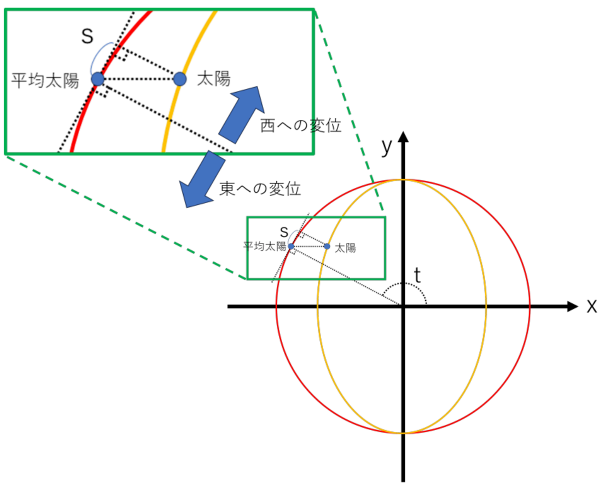

Figure 2–7: View of the celestial sphere from directly above (north celestial pole direction). The mean Sun (平均太陽) and true Sun (太陽) are shown, with eastward (東への変位) and westward (西への変位) displacements indicated. The arc s along the dotted line approximates the east–west angular separation.

(6) From the result of (5):

$$\theta = \arctan\left(\frac{\boxed{\text{C}}}{\boxed{\text{D}}}\right) = \arctan\left(\boxed{\text{E}} \times \frac{1}{\sin t}\right).$$

Fill in the table on the answer sheet with the signs of $\boxed{\text{C}}$ and $\boxed{\text{D}}$, and the range of θ. For the $\boxed{\text{C}}$ and $\boxed{\text{D}}$ columns, write the sign (positive or negative); for the θ column, select one of the following options (A)–(D) for each row.

(A) $0^\circ < \theta < 90^\circ$

(B) $90^\circ < \theta < 180^\circ$

(C) $180^\circ < \theta < 270^\circ$

(D) $270^\circ < \theta < 360^\circ$

(7) Find the numbers that fill in the blanks $\boxed{\text{F}}$ and $\boxed{\text{G}}$.

Consider the relationship between r and θ. Using the results of (4) and (5), one can show:

$$r = \pm \arccos\left(\boxed{\text{F}} + \boxed{\text{G}} \times \frac{1}{\tan^2 t}\right).$$

(8) Fill in the table on the answer sheet for t, θ, and r, following the example given. When using a calculator, be careful about the difference between radian (rad) and degree (deg) notation for angles.

(9) Using the table created in (8), answer the following questions.

① On the figure in the answer space, indicate with black arrows the displacement of the Sun relative to the mean Sun every 15 days, paying attention to direction. Day 0 is indicated with a red arrow. (Note: this sub-question is worth 0 points.)

② Using the table from (8) and the diagram from ① as references, draw the analemma as seen from P_2 at 0:00 every day. On the figure in the answer space, show:

- The position of the Sun every 15 days
- The expected shape of the analemma
- The direction the Sun moves along the analemma (indicate with arrows)
- The vernal equinox point, summer solstice point, autumnal equinox point, and winter solstice point (clearly indicate which point corresponds to which)

The Sun as seen from Planet E_2, discussed in Question 2, is called the "**ideal Sun**."

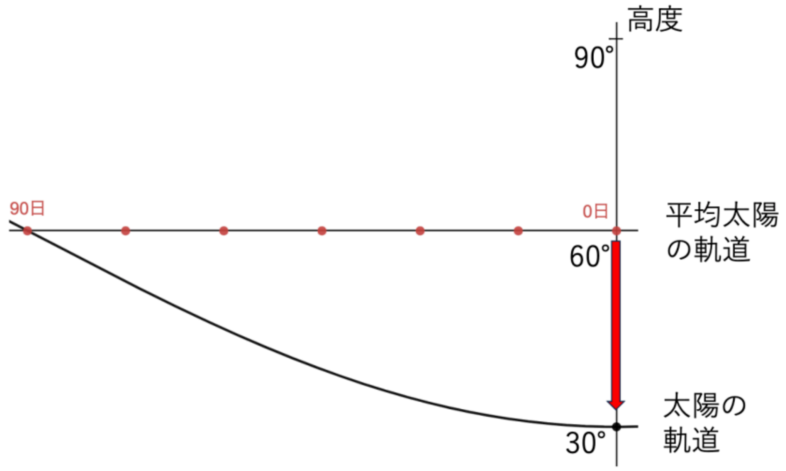

Figure 2–8: Direction of displacement of the Sun relative to the mean Sun. (Write answers in the figure on the answer sheet.) The horizontal axis shows the path of the mean Sun at altitude 60°, with Day 90 on the left and Day 0 on the right. The vertical axis shows altitude. The path of the true Sun is shown by the black curve at altitude around 30°.

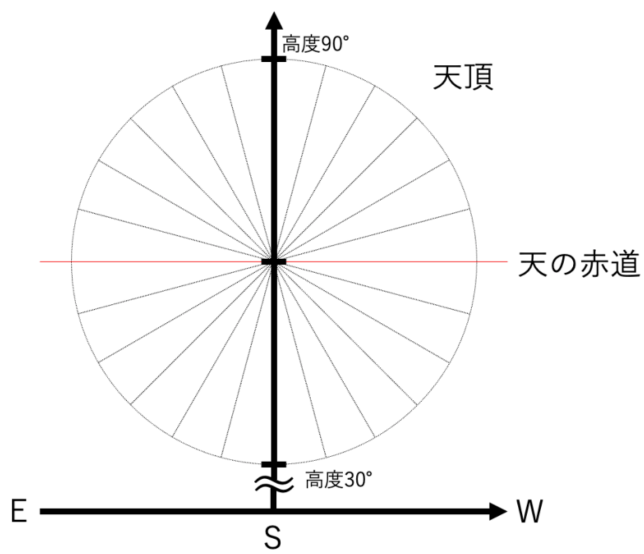

Figure 2–9: Coordinate system for drawing the analemma. (Write answers in the figure on the answer sheet.) The outer circle represents the horizon; the vertical axis points to the zenith (90° altitude), the horizontal axis points E and W; the celestial equator (天の赤道) is shown as the red circle at altitude 60°; the horizontal dashed line at 30° altitude is also marked. S is at the bottom.

Question 3. Planet E_3: Elliptical Orbit

Finally, consider the case where the orbital path is not circular. The settings for Planet E_3 are as follows. Planets E_1 and E_2 do not affect the motion of Planet E_3 in any way.

- Planet E_3 orbits the Sun in a perfectly elliptical orbit with the Sun at one focus.
- The orbital period of Planet E_3 is 1 year.
- The direction of revolution and rotation of Planet E_3, viewed from its north pole, is counterclockwise.
- The sidereal day of Planet E_3 is $\frac{360}{359}$ days.
- A certain location at latitude 30°N on Planet E_3 is called P_3.
- In the calendar of Planet E_3, the start of the year (Day 1, 0:00) is defined as the moment when the Sun culminates at P_3 with the lowest culmination altitude over the entire year. Thereafter, a new day begins every 24 hours. Within each day, time is expressed in hours (0 to 23), minutes (0 to 59), and seconds (0 to 59).
- **The position of the Sun when it is closest to Planet E_3 over the year is called the perihelion, and the position when it is farthest is called the aphelion. The moment of perihelion passage is called Day V.**
- The moment when the Sun culminates at P_3 with the lowest culmination altitude over the year (i.e., the start of the year) is the **winter solstice**; the moment with the highest culmination altitude is the **summer solstice**.
- The moment when the Sun crosses the celestial equator from south to north is the **vernal equinox**; from north to south is the **autumnal equinox**.
- The positions of the Sun on the celestial sphere at the winter solstice, vernal equinox, summer solstice, and autumnal equinox are called the winter solstice point, vernal equinox point, summer solstice point, and autumnal equinox point, respectively.

(1) Figure 2–10 shows the relative motion of the Sun as seen from Planet E_3 (the elliptical orbit) when $V = 0$. On days 0 (perihelion), 90, 180 (aphelion), and 270, state whether the Sun is to the east, at the same position, or to the west of the ideal Sun on the orbital path on the celestial sphere, and provide the reason. In your reasoning, you **must** use the phrase "Kepler's second law." You may use the figure printed on the answer sheet.

(2) Describe what shape the analemma traced by the Sun as seen from Planet E_3 would take for $V = 0$, 90, 180, and 270 days. Also, discuss how the magnitude of the eccentricity affects the shape of the analemma. Use the diagram drawn in Question 2 (9) ① or the figures on the answer sheet as needed.

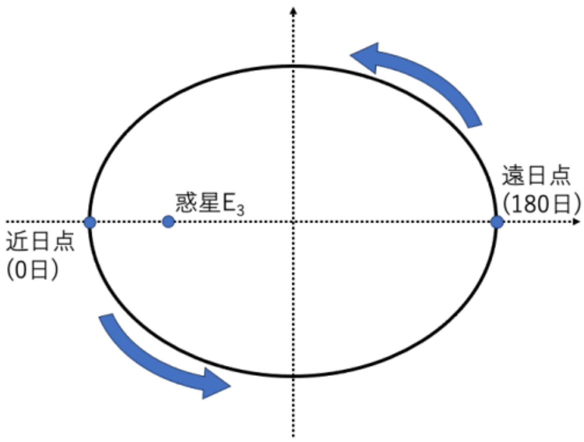

Figure 2–10: Orbit of the Sun as seen with Planet E_3 at the center. The perihelion (近日点) is on the left (Day 0) and the aphelion (遠日点) is on the right (Day 180). The blue arrows indicate the direction of orbital motion (counterclockwise when viewed from above).