

2nd Japan Astronomy Olympiad National Finals

Solutions

Problem 1

Question 1

- (1) The amount of solar radiation energy received per unit time and per unit area by a surface perpendicular to the solar radiation, at the top of Earth's atmosphere.

(2)

$$S_1 = \frac{1 - A}{4} S_0$$

(3)

$$T = \sqrt[4]{\frac{1 - A}{4\sigma} S_0} \approx 255 \text{ K } (-18^\circ\text{C})$$

- (4) The average surface temperature of the Earth is approximately 15°C , which is significantly higher than the radiative equilibrium temperature found in (3). This is because greenhouse gases in the atmosphere, such as water vapor and carbon dioxide, absorb very little of the solar radiation (which consists mainly of visible light), while they strongly absorb and re-emit the Earth's outgoing radiation (which consists mainly of infrared radiation).

Question 2

- (1) Since the orbits of short-period comets are concentrated in a single plane, the region thought to be their source — the Edgeworth–Kuiper Belt — is disk-shaped. In contrast, long-period comets arrive from all directions, so their source population, the Oort Cloud, is distributed in all directions, i.e., in a spherical shell.

- (2) By Kepler's third law, the semi-major axis is $200^{2/3} \approx 34$ au. This is approximately equal to the semi-major axis of Neptune's orbit (30 au).

(3) (a)

$$\frac{1 - 0.999}{1 + 0.999} \times 5.20 \text{ au} \approx 2.6 \times 10^{-3} \text{ au}$$

- (b) Short-period comets with eccentricities extremely close to 1 pass very close to the Sun (as found in (a)), so they receive an enormous amount of solar radiation at perihelion. Comets, being composed of ice, are therefore very likely to disintegrate. Furthermore, tidal disruption by the Sun is also highly probable. For these reasons, it is difficult to imagine that such periodic comets could survive.

Question 3

- (1) **Characteristic:** Contains spherical particles called chondrules.
Origin: Originates from bodies that have not undergone thermal differentiation.
- (2) Originates from the core of a body that has undergone thermal differentiation.
- (3) Beyond the snow line where the giant planets are located, water exists in solid form, making more solid material available for planet formation than in the inner region. In addition, planets with larger semi-major axes can sweep material from a larger area.

Problem 2

Question 1

- (1) (a) **Answer:** *v*: larger; *ro*: smaller; *ha*: closer.
Reason: Because it is necessary to separately detect the light from the star and the light from the planet.
- (b) **Answer:** ② Infrared.
Reason: In the infrared, the exoplanet appears brighter and the host star appears fainter.

- (2) (a)

$$5.2 \text{ au} \times \frac{M_J}{M_\odot + M_J} \approx 5.2 \text{ au} \times \frac{M_J}{M_\odot} = 5.2 \times 9.52 \times 10^{-4} \text{ au} \approx 5.0 \times 10^{-3} \text{ au}.$$

Since the parallax satisfies $\tan 1'' = \frac{1 \text{ au}}{1 \text{ pc}}$, we have $1'' \approx \frac{1 \text{ au}}{1 \text{ pc}}$. Therefore,

$$\frac{5.0 \times 10^{-3} \text{ au}}{10 \text{ pc}} = 5.0 \times 10^{-4} \times \frac{1 \text{ au}}{1 \text{ pc}} \approx 5.0 \times 10^{-4}''.$$

- (b) Because observations are affected by atmospheric turbulence.
- (c) Since $19.3 \text{ ly} \approx 5.93 \text{ pc}$, the semi-major axis of VB 10's wobble is $(5.93 \times 4.42 \times 10^{-3}) \text{ au} \approx 2.62 \times 10^{-2} \text{ au}$.

Let d [au] and P [yr] be the semi-major axis and period of the wobble, a [au] the VB 10–VB 10b separation, and M [M_\odot] and kM [M_\odot] the masses of VB 10 and VB 10b respectively (so k is the quantity to be found). Then

$$d = \frac{kM}{kM + M} a, \quad \frac{a^3}{P^2} = kM + M.$$

Eliminating a from these two equations gives

$$\frac{(k+1)^2}{k^3} = \frac{P^2}{d^3 M} \approx 2.4 \times 10^3.$$

Among the choices, the one for which $\frac{(k+1)^2}{k^3}$ is closest to this value is ②.

- (3) (a) By the Doppler effect, $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$, so the maximum radial velocity of 51 Peg is

$$v = \frac{1.5 \times 10^{-4} \text{ nm}}{656 \text{ nm}} \times 3 \times 10^8 \text{ m/s} \approx 6.86 \times 10^1 \text{ m/s}.$$

From the graph, the wobble period is $P = 4.2$ days. With $v = \frac{2\pi d}{P}$, the wobble semi-major axis is $d = \frac{vP}{2\pi} \approx 2.64 \times 10^{-5} \text{ au}$.

Let a be the 51 Peg–51 Peg b separation and M , m be their respective masses. Then $d = \frac{m}{m+M}a$; since $m \ll M$, $d \approx \frac{m}{M}a$. Kepler's third law (neglecting the planet's mass) gives $\frac{(a/\text{au})^3}{(P/\text{yr})^2} = M/M_\odot$. Eliminating a and solving for m :

$$m = \left(\frac{Md^3}{P^2} \right)^{1/3}.$$

Substituting numerical values (with care for units):

$$m \approx 5.36 \times 10^{-4} M_{\odot} \approx 5.6 \times 10^{-1} M_J.$$

The semi-major axis is

$$a = \frac{M}{m} d = \frac{1.11}{5.36 \times 10^{-4}} \times 2.64 \times 10^{-5} \text{ au} \approx 5.5 \times 10^{-2} \text{ au}.$$

(b) Larger. The maximum radial velocity variation equals the orbital speed multiplied by $\sin i$. Since m is proportional to d and d is proportional to v , if the true mass is m_0 then $m = m_0 \sin i$, so the true mass is larger.

(4) (a) Let the quantity to be found be $x [R_J]$. Solving

$$\left(\frac{x [R_J]}{1 R_{\odot}} \right)^2 = \frac{1 - 0.984}{1}$$

gives $x [R_J] = 1.3 \times 10^{-1} R_{\odot} = 1.3 R_J$.

(b) During a partial transit, where the planetary disk has not fully moved inside the stellar disk, the area of the star occulted by the planet is not constant.

(c) For a transit to occur, the stellar disk must be crossed by the planet, so the condition on R (stellar radius), a (orbital semi-major axis), and i (inclination) is

$$\cos i \leq \frac{R}{a}.$$

Since $\cos i$ takes values uniformly, the transit probability is $\frac{R}{a}$.

(d) By Kepler's third law, the orbital period of the star is $P [\text{yr}] = \sqrt{\frac{a^3}{M}}$. Since at least 2 orbits are needed to observe 3 transits, the required time is

$$2\sqrt{\frac{a^3}{M}} \text{ years}.$$

Question 2

(1) By Wien's displacement law, the surface temperature of star X is

$$\frac{2.9 \times 10^{-3} \text{ m} \cdot \text{K}}{1.0 \mu\text{m}} = 2.9 \times 10^3 \text{ K},$$

so it is an M-type star.

(2) From the definition of luminosity and the Stefan–Boltzmann law, using the Stefan–Boltzmann constant σ :

$$L = 4\pi\sigma R^2 T^4,$$

so

$$\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}} \right)^2 \left(\frac{T}{T_{\odot}} \right)^4.$$

Substituting the given values:

$$\frac{R}{R_{\odot}} = \sqrt{\frac{10}{25}} \approx 1.3 \times 10^{-1}.$$

- (3) The energy incident per unit area is proportional to luminosity and inversely proportional to the square of the distance from the star. Letting d [au] be the lower limit of the habitable zone distance from star X:

$$1000 = \left(\frac{1}{d}\right)^2 \implies d = 3.2 \times 10^{-2} \text{ au.}$$

Similarly, the upper limit is 4.7×10^{-2} au.

- (4) From the light curve, the orbital period of exoplanet Y is 9 days. Let a [au] be its semi-major axis; by Kepler's third law:

$$\frac{a^3}{(9/365)^2} = \frac{1}{10} \implies a = 3.9 \times 10^{-2} \text{ au.}$$

This lies within the habitable zone found in (3), so liquid water can exist.

Question 3

- (1)
- Because the red dwarf host star has a small mass, the semi-major axis of its orbit around the common centre of mass with the planet is large, so the amplitude of the star's radial velocity is also large.
 - Because the red dwarf's radius is small, the fractional dimming during a transit is large and an occultation is easier to observe.
 - Because the habitable zone is close to the star, planets in the habitable zone have short periods, making transits easier to observe.
- (2)
- Because the planet's mass is large, the radial velocity variation of the star is large.
 - Because the planet's radius is large, the fractional dimming during a transit is large and an occultation is easier to observe.
 - Because the orbital period is short (a few days), a transit can be observed in a short observation campaign.
 - The probability of the orbit being oriented to produce transits is high.

Problem 3

Question 1

(1) 91.2 nm

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.18 \times 10^{-18}} \approx 91.2 \text{ nm}$$

(2) ① Wien's displacement law:

$$\frac{2.9 \times 10^{-3}}{9.12 \times 10^{-8}} \approx 3.2 \times 10^4 \text{ K}$$

(3) Main-sequence stars capable of emitting radiation at the wavelength found in (2), which carries enough energy to ionise hydrogen, are limited to massive stars with high surface temperatures. Such main-sequence stars generally have short lifetimes, so H α emission can be used to trace regions where star formation has occurred very recently.

Question 2

(1) $R \approx 3 \times 10^3$

$$R = \frac{656}{0.22} \approx 3 \times 10^3$$

(2) $5 \times 10^{-16} \text{ J m}^{-2} \text{ s}^{-1} \text{ nm}^{-1}$

(3) $3 \times 10^{-15} \text{ J m}^{-2} \text{ s}^{-1}$

(4) $L \approx 7 \times 10^{34} \text{ J s}^{-1}$

$$L = 4\pi d^2 = 4\pi \times (4.5 \times 10^7 \times 3.1 \times 10^{16})^2 \approx 7.3 \times 10^{34} \text{ J s}^{-1}$$

(5) $\text{SFR} \approx 4 M_{\odot} \text{ yr}^{-1}$

$$\text{SFR} = 5.5 \times 10^{-35} \times L = 5.5 \times 10^{-35} \times 7.3 \times 10^{34} \approx 4.0 M_{\odot} \text{ yr}^{-1}$$

Question 3

(1) ②

(2) In the Salpeter IMF, the number of stars formed decreases in proportion to mass^{-2.35}. On the other hand, from the mass–luminosity relation, luminosity is proportional to mass³, so more massive stars contribute more to the total luminosity.

(3) The results are shown in the table below.

Sub-question	Luminosity in each band (normalised so $V = 1.0$)				
	U	B	V	R	I
(a)	0.42	0.73	1.0	1.1	1.3
(b)	0.23	0.63	1.0	1.2	1.7
(c)	0.24	0.63	1.0	1.2	1.5

- (4) From the results of (3), as more time elapses since the stellar population formed, it becomes brighter at longer wavelengths; when recent star formation occurs, it becomes brighter at shorter wavelengths. Therefore, galaxy F, which is brighter at shorter wavelengths, is inferred to contain more stars formed in more recent star-formation episodes.

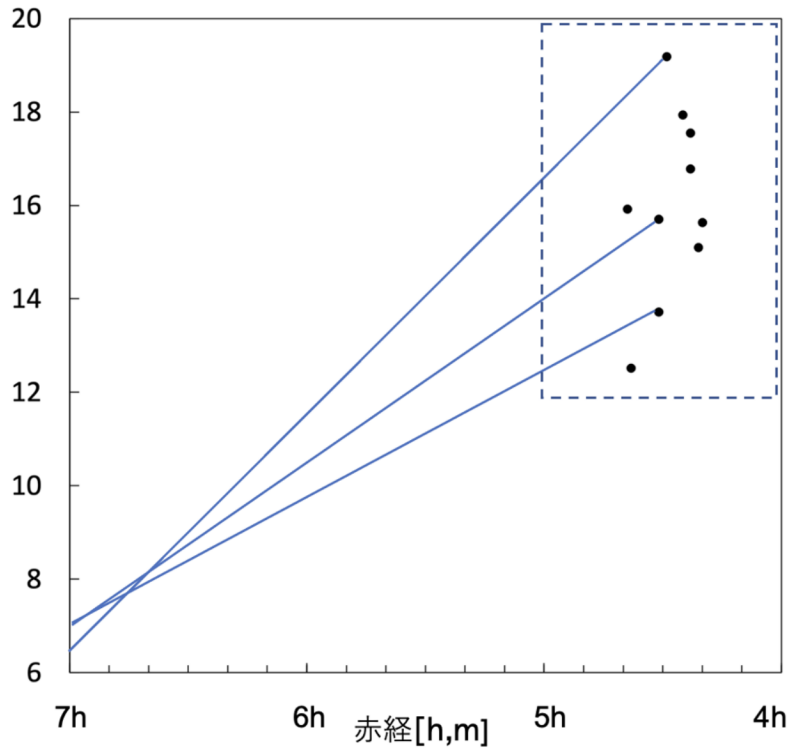
Problem 4

Question 1 $C = 4.740$

Question 2

$$\lambda_o = \frac{c + v_r}{c} \lambda_s$$

Question 3 From the figure below: $6^{\text{h}} 48^{\text{m}}, 7^{\circ} 35'$.



Convergent-point diagram used to determine the right ascension and declination.

Question 4

$$V = \frac{v_r}{\cos \theta}, \quad v_t = V \sin \theta = v_r \tan \theta$$

No.	θ [°]	V [km/s]	v_t [km/s]	d [pc]
1	36.6	47.9	28.6	53.2
2	37.9	48.7	29.9	53.6
3	37.6	47.4	28.9	55.7
4	32.9	47.0	25.5	51.0
5	37.2	49.5	29.9	56.6
6	33.3	48.8	26.8	66.0
7	37.5	45.6	27.8	53.1
8	34.8	47.2	27.0	52.6
9	35.2	48.0	27.6	54.9
10	37.4	49.9	30.3	58.1

Question 5 $V = 48.0 \text{ km/s}$, $d = 55.4 \text{ pc}$

Question 6 Averaging the right ascensions and declinations of Nos. 1–10 gives right ascension $4^{\text{h}} 28^{\text{m}} 0^{\text{s}}$ and declination $16^{\circ} 0' 12''$. This position is point O.

Next, the angular distance from O to each star is computed. Since $4^{\text{h}} 28^{\text{m}} 0^{\text{s}}$ and $16^{\circ} 0' 12''$ correspond to 67.000° and 16.003° respectively, if the right ascension and declination of each star are α and δ (in degrees), then

$$\theta = \sqrt{(\alpha - 67.000^{\circ})^2 + (\delta - 16.003^{\circ})^2}.$$

No.	θ [°]	No.	θ [°]	No.	θ [°]
1	3.19	5	2.07	9	0.81
2	2.03	6	2.75	10	1.47
3	1.99	7	1.97		
4	4.29	8	2.41		

Therefore, among Nos. 1–10, the star farthest from the centre O is No. 4, with angular distance $\theta = 4.29^{\circ}$. Since the mean distance is 55.4 pc:

$$R = 55.4 \times 4.29 \times \frac{2\pi}{360} \approx 4.1 \text{ pc}.$$

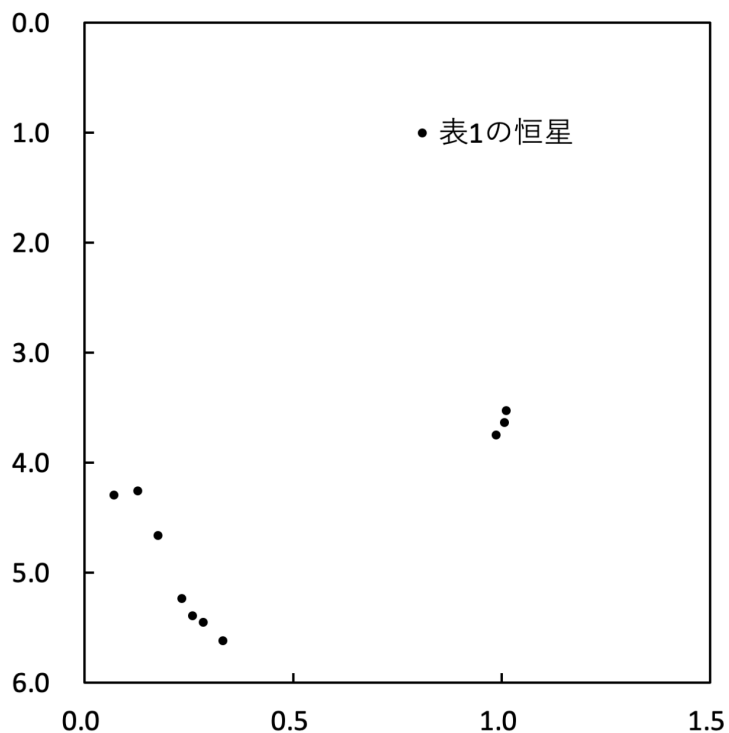
Question 7 $\alpha \text{ CMa}$

$\alpha \text{ CMa}$ (also known as Sirius) is a representative blue A-type main-sequence star, while $\alpha \text{ Ori}$ (Betelgeuse) is a representative red supergiant. Therefore, $\alpha \text{ CMa}$ has a small B-magnitude and a large V-magnitude, whereas $\alpha \text{ Ori}$ has a large B-magnitude and a small V-magnitude. Consequently, the B–V colour index of $\alpha \text{ CMa}$ is lower than that of $\alpha \text{ Ori}$.

Question 8 ⑧

As starlight passes through interstellar matter, it is attenuated, so the U, B, and V magnitudes all increase. In particular, shorter wavelengths (U) are more strongly affected by extinction, so their magnitudes increase by a larger amount; longer wavelengths (V) are less affected, so their magnitudes increase by a smaller amount. As a result, both the U–B and B–V colour indices increase, meaning the star moves toward the lower right in the two-colour diagram. The option matching this is ⑧.

Question 9 See figure



Colour-magnitude diagram (V magnitude vs. B-V colour index) for the stars in Table 1.

Question 10 No. 5

Question 11 10^9 yr

Question 12 $V = 53$ km/s

Question 13 Cluster H exists at a distance of 55.4 pc from us with a radius of 4.1 pc, but star A, which is at a distance of 20.5 pc, does not lie within this volume. Furthermore, although their speeds are similar, star A's proper motion is not directed toward the convergent point X of cluster H, which further indicates that star A does not belong to cluster H. In addition, their ages differ greatly and the types of stars inferred from the two-colour diagram are different (i.e., their colours differ), so their origins are also considered to be different. From the above, we conclude that star A does not belong to cluster H, and that star A and cluster H do not share the same origin.