2nd Japan Astronomy Olympiad National Finals

Problems

March 5, 2023 13:30–16:30

Instructions

1. Do not open this booklet until the signal to begin is given.
2. This booklet contains 24 pages in total. If you find any missing or misprinted pages, raise your hand and notify the invigilator.
3. Use only a black pencil or black mechanical pencil for your answers.
4. Write your candidate number in the designated field on the answer sheet and graph paper. Do not write your candidate number anywhere else.
5. Write all answers in the designated spaces on the answer sheet and graph paper.
6. Do not write any irrelevant characters, symbols, or marks in the answer fields of the answer sheet or graph paper.
7. The margins of this booklet may be used for rough work, but the booklet must not be torn.
8. Do not take the answer sheets or graph paper with you when you leave.
9. After the examination, take this question booklet, calculation paper, and draft graph paper with you.
10. No questions about the problems will be answered. If a significant assumption beyond those stated in the problem is required, make an appropriate assumption. If you believe a problem has a fatal flaw that makes it impossible to answer regardless of your knowledge or ability, briefly state the reason on the answer sheet. However, claims of impossibility due to insufficient knowledge on the part of the candidate will not be accepted and will be treated as a wrong answer. Do not write that a problem is unsolvable merely because you cannot solve it yourself.

Problem 1 Various Topics Related to the Solar System

Answer the following independent questions (Questions 1–3) related to the Solar System. For questions that have a space on the answer sheet for equations and reasoning, provide an explanation of how you arrived at your answer.

Question 1. Planetary surface temperature

The temperature of a planet's surface is one of the fundamental quantities characterising a planet. Answer the following questions about planetary temperatures.

(1) The solar constant is an important parameter when considering the temperatures of bodies in the Solar System. State the definition of the solar constant.

(2) Let S_0 denote the solar constant and A the albedo of the Earth. Express S_1, the value of the solar radiation energy received per unit area per unit time by the Earth averaged over the entire Earth's surface, in terms of S_0 and A. Give only the result.

(3) Averaged over the entire Earth, the energy brought by solar radiation and the energy lost by terrestrial radiation are in equilibrium. The surface temperature of the Earth in such an equilibrium state is called the radiative equilibrium temperature. Express the Earth's radiative equilibrium temperature T in terms of the Stefan–Boltzmann constant σ, S_0, and A. Also, taking $S_0 = 1.37\,\mathrm{kW/m^2}$, $A = 0.30$, and $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2/K^4}$, calculate the Earth's radiative equilibrium temperature to 3 significant figures. Give only the result.

(4) Compare the value of the radiative equilibrium temperature T found in (3) with the actual average surface temperature of the Earth, and discuss any discrepancy.

Question 2. Comets with elliptical orbits

Some comets have elliptical orbits and revolve periodically. Comets with orbital periods of 200 years or less are called short-period comets, and those with orbital periods exceeding 200 years are called long-period comets. Observations have shown that the orbits of short-period comets are concentrated near the ecliptic plane, while the orbits of long-period comets are distributed isotropically.

(1) Describe the shape of the region believed to be the source of short-period comets and long-period comets. Your answer must take into account the observational facts underlined above, and must include the name of the region considered to be the source of each type of comet.

(2) Calculate the semi-major axis of the orbit of a comet with an orbital period of 200 years, in astronomical units, to 2 significant figures. Also, state which planet in the Solar System has an orbital semi-major axis closest to the value you calculated.

(3) Comets generally tend to have larger eccentricities e than planets. Figure 1-1 shows the fractional distribution of orbital eccentricities for long-period comets and short-period comets separately (histograms normalised so that the total is 1). The eccentricities of long-period comets are extremely close to 1, such as $e = 0.999$; by contrast, short-period comets tend to have smaller eccentricities than long-period

comets, and few of them have eccentricities as large as $e = 0.999$. In this question we consider a short-period comet with the very high eccentricity $e = 0.999$ and investigate why such short-period comets do not exist.

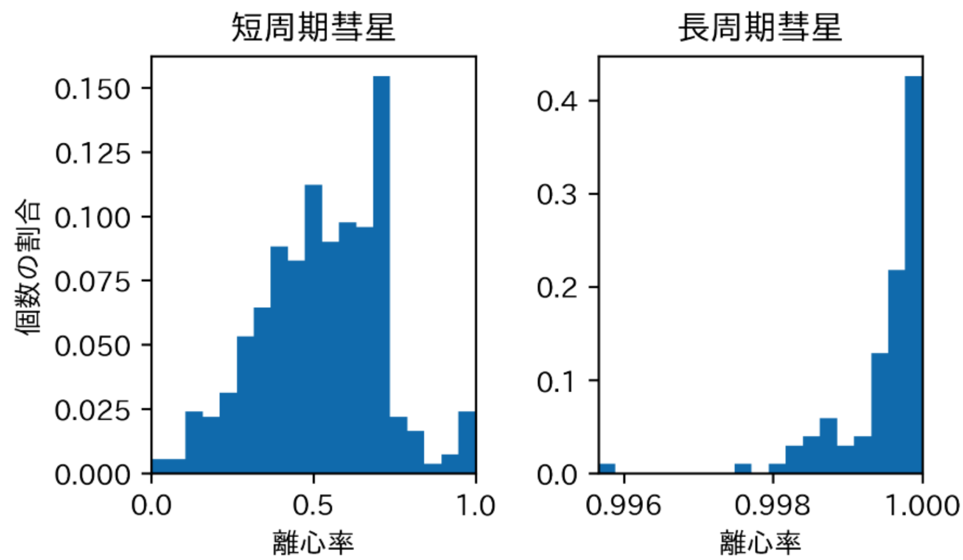


Figure 1-1. Fractional distribution of orbital eccentricities for short-period comets (left) and long-period comets (right). The horizontal axis spans from the minimum to the maximum value in each case, and the number of bins within the displayed range is equal for both. Short-period comet data from JPL Solar System Dynamics (`https://ssd.jpl.nasa.gov/`); long-period comet data from the CODE catalog (Królikowska & Dybczyński 2020, *Astronomy & Astrophysics*, 640, A97).

(a) Suppose a short-period comet with $e = 0.999$ has its aphelion at 5.20 au from the Sun. Find the perihelion distance of this comet in astronomical units to 2 significant figures. Show your equations and reasoning.

(b) Referring to the result of (3)(a), discuss why short-period comets with eccentricities extremely close to 1 do not exist.

Question 3. Meteorites and the chemical composition of Earth's interior

A meteorite is a small Solar System body that enters Earth's atmosphere and reaches the surface without burning up completely. Studying meteorites makes it possible to estimate the chemical composition of Earth's interior, contributing greatly to advances in Earth science. Answer the following questions about meteorites and the chemical composition of Earth's interior.

(1) The type of meteorite that allows estimation of the average chemical composition of the entire Earth is called a chondrite. Briefly describe the characteristics of chondrites and their origin.

(2) Meteorites are classified as "stony meteorites", "stony-iron meteorites", and "iron meteorites" according to their composition. Iron meteorites in particular are used to estimate the chemical composition of Earth's core. Briefly describe the origin of iron meteorites.

(3) Meteorites that reflect the bulk chemical composition of an entire planet exist only for terrestrial planets; no meteorites with a composition close to the bulk chemical composition of a giant (Jovian) planet exist. This is because the giant planets acquired so much mass that they were able to capture large amounts of gas. Explain why giant planets acquired much greater mass than terrestrial planets, with reference to the materials available for planet formation.

Problem 2 Extrasolar Planets

Answer Questions 1–3 below on extrasolar planets. For questions requiring numerical answers, give all answers to 2 significant figures. For questions that have a space on the answer sheet for equations and reasoning, provide an explanation of how you arrived at your answer. Diagrams may be used in your answers where helpful. Use the following symbols, constants, and relations as needed.

Speed of light c:	3.00×10^{8} m/s
π:	3.14
Astronomical unit (1 au):	1.50×10^{11} m
Light-year (ly) and parsec (pc):	$1\,\mathrm{ly} = 0.307\,\mathrm{pc}$
Solar mass $M_\odot$:	$M_\odot = 1.99 \times 10^{30}$ kg
Jupiter mass M_J:	$M_J = 9.55 \times 10^{-4}\,M_\odot$
Jupiter radius R_J:	$R_J = 1.00 \times 10^{-1}\,R_\odot$
Solar luminosity $L_\odot$:	3.83×10^{26} W
Jupiter's orbital semi-major axis:	5.20 au

Question 1. Exoplanet detection methods

(1) The **direct imaging method** directly observes light from an extrasolar planet.

(a) Fill in the blanks **(i)**, **(ii)**, **(iii)** in the following sentence with the appropriate word from each pair, and give the reason.

Systems where the planet's orbital semi-major axis is **{(i): large / small}**, the stellar luminosity is **{(ii): large / small}**, and the observer–star distance is **{(iii): near / far}** are advantageous for detection by the direct imaging method.

(b) When detecting an extrasolar planet by direct imaging, assuming that sufficient angular resolution can be achieved at any wavelength, at which wavelength band is it easiest to detect the planet? Choose the most appropriate option from ①–③ below and state your reason.

① Visible light ② Infrared ③ Ultraviolet

(2) The **astrometry method** detects extrasolar planets by directly observing changes in the position of the host star as it orbits the common centre of mass with its planet ("stellar wobble"). It is one of the oldest detection methods ever attempted.

(a) First, let us estimate the size of the stellar wobble. For simplicity, assume the Solar System contains only the Sun and Jupiter. Calculate the radius of the Sun's circular orbit around the Sun–Jupiter common centre of mass. Also, what is the parallax angle (in arcseconds) when this is observed from a distance of 10 pc from the Sun?

(b) To measure such a small parallax, ground-based observations are at a disadvantage compared with observations from a space telescope. Briefly explain the reason.

As shown above, one disadvantage of the astrometry method is the need to measure positions with extremely high precision; consequently, no extrasolar planets have yet been discovered by this method. On the other hand, one advantage over the related radial-velocity method is that the true mass of the planet can be obtained.

Hereafter, assume that planets move in circular orbits. The magnitude of the wobble does not depend on the orbital inclination angle i (the angle between the normal to the orbital plane and the line of sight, $0° \le i \le 90°$; corresponding to i in Figure 2-1). Indeed, if the orbital semi-major axis of the star (around the common centre of mass) is a, then when the orbital plane is perpendicular to the line of sight the stellar motion appears as a circle of radius a; when it is parallel to the line of sight the star appears to oscillate with amplitude a; and for any intermediate angle the star appears to trace an ellipse, but in all cases the semi-major axis of the apparent motion is a.

(c) In this question we estimate the mass of an extrasolar planet using the astrometry method. Star VB 10, at a distance of 19.3 light-years from Earth, was predicted to host exoplanet VB 10b because a wobble with a maximum amplitude of 4.42×10^{-3} arcseconds and a period of 2.72×10^{2} days was observed. Although the planet's existence is now ruled out, assuming the wobble is caused solely by VB 10b, how many times the mass of VB 10 is the mass of VB 10b? Choose the most appropriate answer from ①–④ below. Take the mass of VB 10 to be $7.79 \times 10^{-2}\, M_\odot$. You may use the approximation $\tan\theta \approx \theta$ for small angles θ.

① 7.8×10^{-1} ② 7.8×10^{-2} ③ 7.8×10^{-3} ④ 7.8×10^{-4}

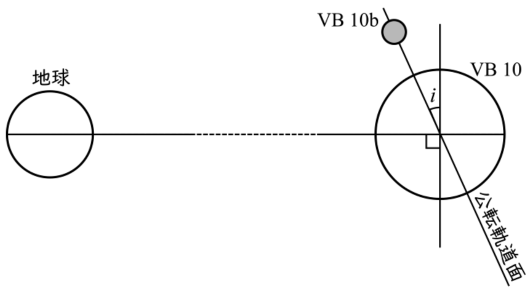


Figure 2-1. Geometry of Earth, VB 10, and VB 10b.

(3) The **radial-velocity method** detects extrasolar planets by observing changes in the position of the host star as it orbits the common centre of mass with its planet, through changes in the star's radial velocity. Because this radial-velocity variation is observed by exploiting the Doppler effect, it is also called the "Doppler method".

(a) Consider the binary system of star 51 Pegasi (hereafter 51 Peg) and planet 51 Pegasi b (hereafter 51 Peg b), both moving in circular orbits around their common centre of mass. The wavelength of the 656 nm spectral line of 51 Peg was observed as a function of time, yielding the results shown in Figure 2-2. Assuming an orbital inclination of 90° for 51 Peg b, find the orbital semi-major axis [au] and mass [M_J] of 51 Peg b. Note that from separate observations the mass of 51 Peg is known to be $1.11\, M_\odot$. Also, the mass of 51 Peg b is sufficiently small compared to that of 51 Peg and may be neglected.

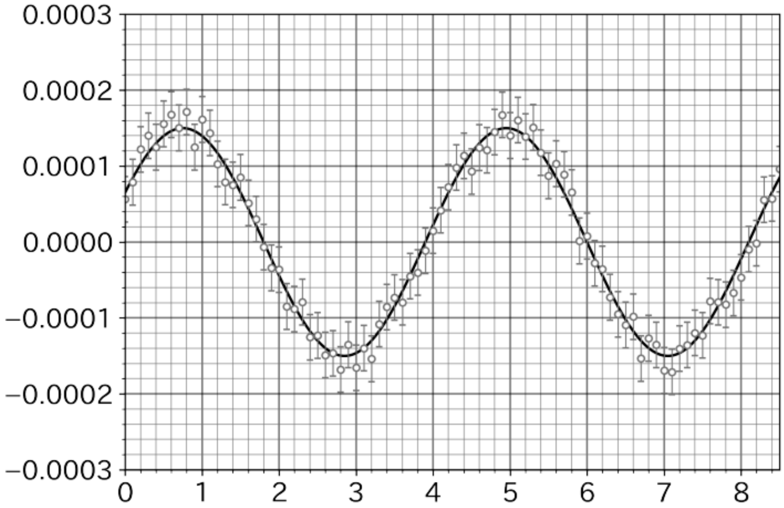


Figure 2-2. Time variation of the wavelength of the 656 nm spectral line of 51 Peg.

(b) In practice, the orbital inclination cannot be determined from the radial-velocity method alone. In (3)(a) you calculated the mass of the planet assuming an inclination of 90°. Is the mass calculated in this way larger or smaller than the range of masses the actual planet could take? Explain quantitatively.

(4) The **transit method** detects planets by capturing the slight dimming of a star caused when a planet passes in front of it (a transit), blocking part of the stellar disk. Because it observes a stellar eclipse caused by a planet, it is also called the "occultation method".

(a) A major feature of the transit method is that it gives the planet's radius, which cannot be obtained by other detection methods. Estimate the planet's radius [R_J] from the light curve in Figure 2-3. Note that from other observations the stellar radius is known to equal the solar radius. Assume that the star radiates uniformly.

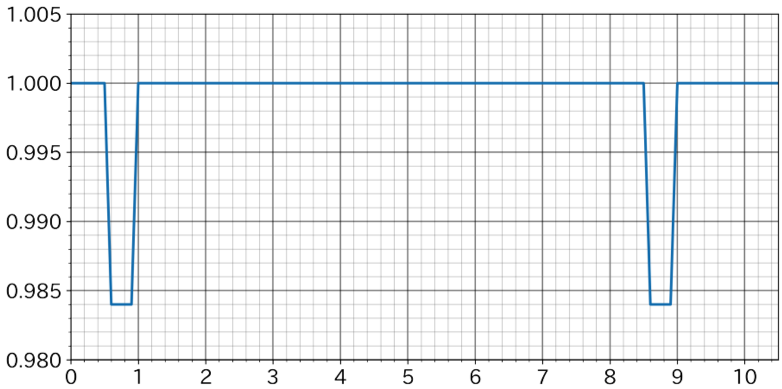


Figure 2-3. Light curve of a certain star.

(b) In practice, stars exhibit limb darkening and therefore do not radiate uniformly, so the light curve does not remain flat at the minimum during a transit. However, even when the star is assumed to radiate uniformly, there are cases where the light curve does not remain flat during the minimum of the transit, as shown in Figure 2-4. Explain why.

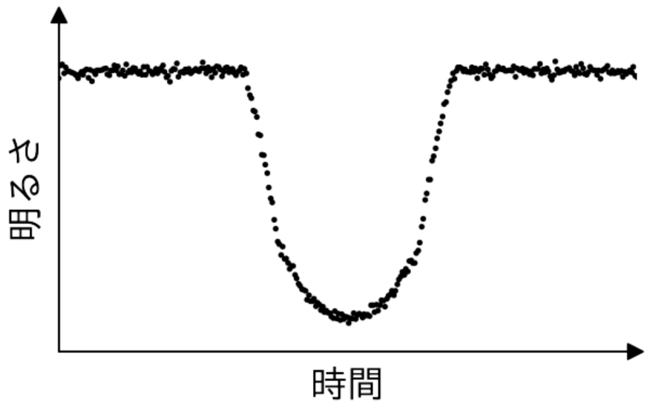


Figure 2-4. A light curve in which the flux does not remain constant during the transit minimum.

The stellar wobble detected by astrometry or the radial-velocity method occurs almost certainly whenever a planet is present. In contrast, the dimming detected by the transit method is extremely rare even when a planet is present. We now investigate the reason for this. Consider a hypothetical extrasolar star–planet system with stellar radius R, planetary orbital semi-major axis a, and orbital inclination i (see Figure 2-5). For simplicity, assume the planet moves in a circular orbit around the star.

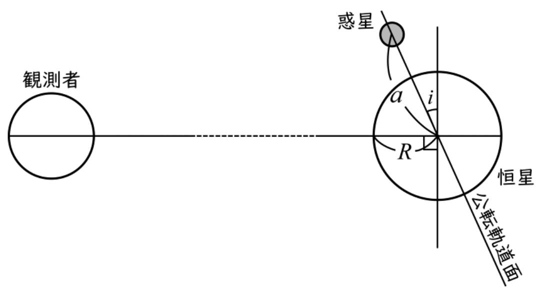


Figure 2-5. A hypothetical extrasolar star–planet system.

(c) One reason is that the planet must pass in front of the star as seen by the observer. Assuming a circular orbit, derive the relationship between R, a, and i required for an observer to observe a stellar transit. Also, from this result, express the probability that an observer would observe a transit from the star–planet system shown in Figure 2-5 in terms of R, a, and i. Use the fact that $\cos i$ follows a uniform probability distribution, since the orbital plane as seen from Earth points in a random direction.

(d) Even if the orbit is one that causes a transit, a long orbital period means that a single observation takes a long time, making observation impractical. To conclude from the transit method that a dimming event is caused by a planetary transit, one would ideally observe at least 3 dimming events. Assuming a circular orbit, an orbital inclination of 90°, and a stellar mass of $M\,[M_\odot]$, express the minimum time required to observe 3 transits in terms of a (in au). Be careful to state and use correct units. The transit duration may be neglected.

Question 2. Habitable zones

One purpose of extrasolar planet research is the discovery of life beyond Earth. Liquid water is essential for life. The annular region around a star where liquid water can exist is called the habitable zone (HZ), and it is known that planets with the potential to harbour life are found only within the HZ. In this question we study this topic. Assume that the HZ depends only on the energy received per unit time per unit area.

A star X has a mass of $\frac{1}{10}\,M_\odot$ and a luminosity of $\frac{1}{1000}\,L_\odot$, and has been found to host an orbiting extrasolar planet Y. The peak emission wavelength of star X is $1.0\,\mu$m. In the following sub-questions, the mass of exoplanet Y is sufficiently small compared with that of star X and may be neglected. The Sun's peak emission wavelength is $0.5\,\mu$m, and the Sun's surface temperature is 5800 K.

(1) State the spectral type of star X.

(2) By what factor does the radius of star X differ from the solar radius?

(3) Find the range of distances from star X that define its habitable zone, in astronomical units. The Sun's habitable zone extends from 1.0 au to 1.5 au from the Sun.

(4) Observing a transit of exoplanet Y yielded the light curve shown in Figure 2-6. Is exoplanet Y within the habitable zone calculated in (3)?

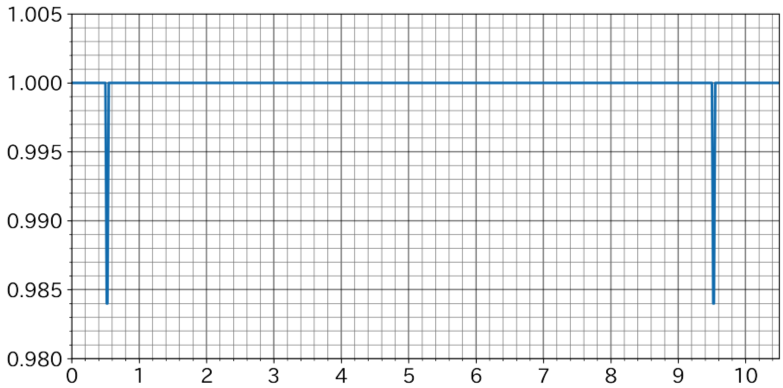


Figure 2-6. Light curve of star X.

Question 3. Synthesis

Answer the following questions based on the preceding problems.

(1) In recent years, much of extrasolar planet research has targeted red dwarfs — small-diameter, low-surface-temperature main-sequence stars that are relatively close to the Solar System. List as many reasons for this as you can and explain them qualitatively.

(2) Many Jupiter-sized planets in very close orbits around their host stars have been discovered (for example, 51 Pegasi b). Because they orbit so close to their star, these planets are hot and are called *hot Jupiters*. The large number of hot Jupiter detections is not because they are genuinely more common, but rather because observational conditions are especially favourable for them. List as many of these favourable conditions as you can and explain them qualitatively.

Problem 3 Galaxies and Star Formation

Note: For questions that have a space on the answer sheet for equations and reasoning, provide an explanation of how you arrived at your answer. In this problem, the effects of interstellar extinction are not considered.

Galaxies are systems in which large numbers of stars, interstellar matter (baryons), dark matter, and other components are bound together by their own gravity. Over the long history of the Universe, galaxies are thought to have grown by forming stars within themselves and by merging with other galaxies. The mass of stars formed per unit time within a galaxy is called the star formation rate (SFR); its unit is generally solar masses per year ($M_\odot$/yr). Various methods for determining the SFR have been proposed.

Question 1. Hα emission and star formation rate

One commonly used method for measuring the SFR of a galaxy employs the Hα emission line, one of the hydrogen recombination lines. Using $L_{\mathrm{H}\alpha}$, the luminosity of the Hα emission line (the energy radiated per unit time as Hα), the SFR can be estimated from the relation

$$\mathrm{SFR}\,[M_\odot/\mathrm{yr}] = 5.5 \times 10^{-35} \times L_{\mathrm{H}\alpha}\,[\mathrm{J/s}] \qquad \text{(eq. 1)}$$

(1) The Hα line is emitted mainly from regions where hydrogen has been ionised by intense energy radiation. The minimum energy required to ionise a neutral hydrogen atom is 2.18×10^{-18} J. Find, to 3 significant figures, the wavelength of a photon carrying this energy. Use the Planck constant $h = 6.63 \times 10^{-34}$ J s, the photon energy $E\,[\mathrm{J}] = h\nu$ where ν is the frequency [Hz], and the speed of light $c = 3.00 \times 10^8$ m/s.

(2) Select the spectral type of main-sequence star that is the primary source of the wavelength found in (1) in interstellar space, from the choices ①–④ below.

① O-type ② A-type ③ G-type ④ M-type

(3) Discuss why the relationship in (eq. 1) holds — i.e. why a stronger Hα emission line implies a larger SFR. You do not need to use specific numerical values or algebraic expressions (a qualitative argument is sufficient). Note that the SFR estimated from Hα reflects star formation activity on a timescale of approximately the last 100 million years.

Question 2. Spectroscopy of Galaxy A

Using a telescope and spectrograph, a spectrum of Galaxy A at a distance of 4.5×10^7 pc was obtained. The upper panel of Figure 3-1 shows the portion of the spectrum around the Hα line, and the lower panel shows an enlargement of only the wavelength range corresponding to the Hα line.

In Figure 3-1, the horizontal axis shows wavelength. The vertical axis shows a quantity called the energy flux density, which gives the energy received per unit time per unit area per unit wavelength. All effects of cosmological redshift, changes in energy flux density, atmospheric absorption, and interstellar extinction have been corrected.

(1) In observational astronomy, the spectral resolving power R is often used to quantify wavelength resolution during spectroscopic observations. R is defined as the ratio

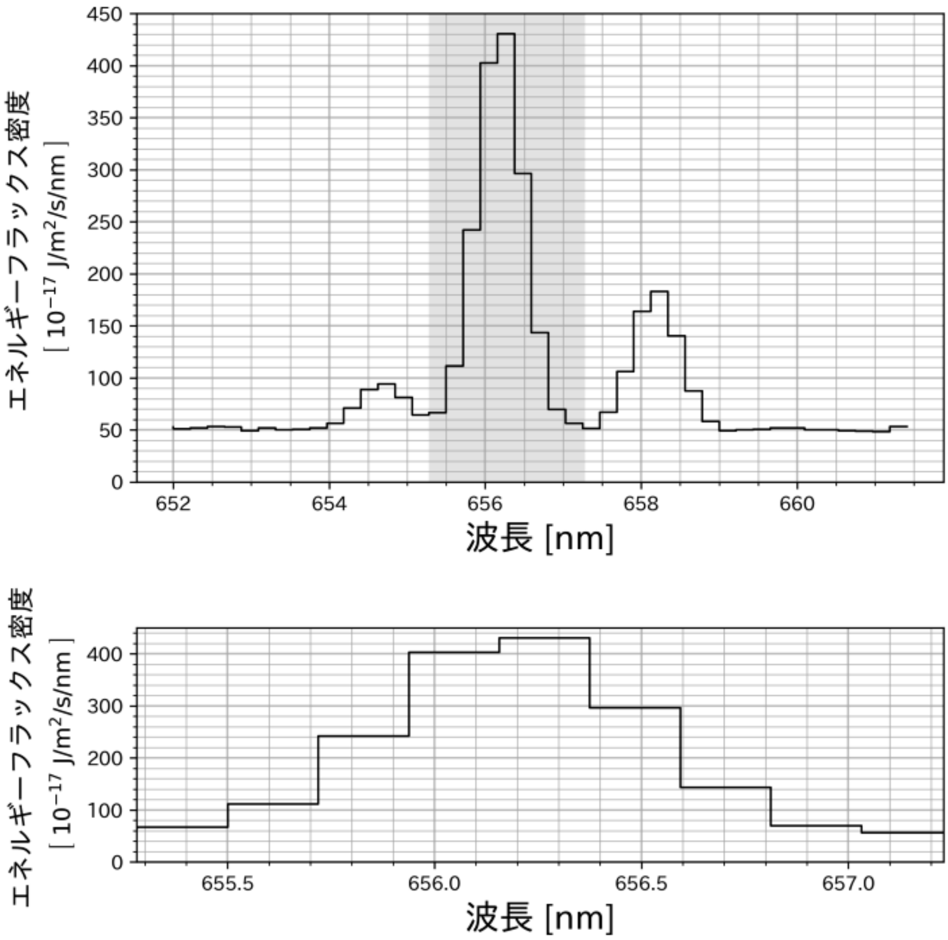


Figure 3-1 (upper panel). Spectrum of Galaxy A around the Hα line. The vertical axis shows energy flux density normalised in units of $10^{-17}\,\mathrm{J/m^2/s/nm}$. The grey shaded region corresponds to the wavelength range of the Hα line, enlarged in the lower panel. **(lower panel).** Enlargement of the grey-shaded Hα wavelength region from the upper panel.

of the wavelength λ to the wavelength resolution $\Delta\lambda$: $R = \lambda/\Delta\lambda$. Determine R for the spectrum in Figure 3-1 to 1 significant figure.

Using the method discussed in Question 1, let us find the SFR of Galaxy A from the Hα luminosity. The emission line seen around $\lambda = 656.3\,\mathrm{nm}$ in Figure 3-1 is the Hα line. In this problem, assume that the Hα line extends only over the wavelength range shown in the lower panel of Figure 3-1. Also, within the wavelength band shown in the lower panel, assume that there is no contribution from emission lines other than the Hα line visible in the upper panel.

(2) Under the above assumptions, the energy flux density in the wavelength range shown in the lower panel of Figure 3-1 is the sum of the continuum flux density from the galaxy and the Hα line flux density. For simplicity, assume that the galaxy's continuum is constant across the wavelength range shown in the upper panel. By examining the energy flux density at wavelengths free from Hα or other emission lines, estimate the continuum flux density from the galaxy in the above wavelength range to 1 significant figure.

(3) Based on the spectrum shown in the lower panel of Figure 3-1, find the total energy flux (flux: energy received per unit time per unit area from the entire Hα line) to 1 significant figure. Subtract the contribution of the galaxy continuum using the continuum flux density estimated in (2). Note that the energy flux density is the energy received per unit time per unit area per unit wavelength.

(4) Using the Hα flux found in (3), calculate the Hα luminosity $L_{\mathrm{H}\alpha}$ to 1 significant figure. Note that luminosity is the energy emitted per unit time by the galaxy, whereas flux is the energy received per unit time per unit area. Use $1\,\mathrm{pc} = 3.08 \times 10^{16}\,\mathrm{m}$.

(5) Substituting the $L_{\mathrm{H}\alpha}$ found in (4) into (eq. 1), find the SFR of Galaxy A to 1 significant figure.

Question 3. Spectral energy distributions and star-formation histories

Another method for estimating a galaxy's SFR, besides the Hα approach, is to consider the galaxy's entire spectrum. By interpreting the galaxy's spectrum as the sum of spectra from individual stars, one can determine what mass of stars formed in each era to explain the observed galaxy continuum.

(1) From choices ①–④ below, select the visible-light spectrum closest to that of the Sun.

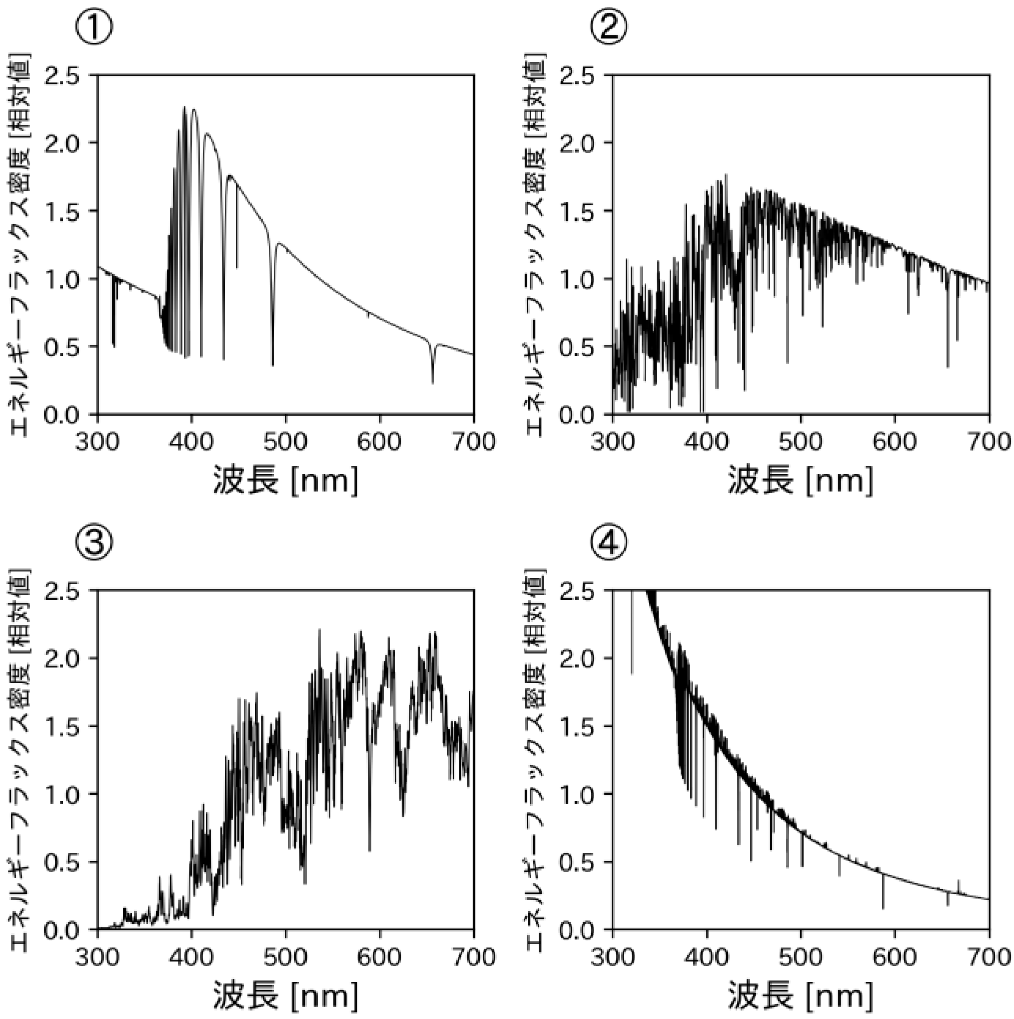
In general, it is thought that when stars are formed, the masses of the individual stars follow a certain distribution function called the initial mass function (IMF). A well-known example is the Salpeter (1955) IMF, in which the number N of stars formed with masses between M and $M + \Delta M$ is proportional to $M^{-2.35}\Delta M$.

(2) Assume that the mass–luminosity relation for main-sequence stars is such that luminosity is proportional to the cube of the mass. When a newly formed population of stars follows the Salpeter IMF and is observed as a whole, which stars — massive or low-mass — are expected to dominate the total luminosity? Give your reasoning.

(3) Galaxies are thought to have evolved by varying their SFR over time throughout cosmic history. In this question, we consider populations of stars formed at various epochs and ask how bright they appear today at various wavelengths. Stars are assumed to form according to the Salpeter IMF. For simplicity, consider only main-sequence stars of masses $0.1\,M_\odot$, $1\,M_\odot$, $10\,M_\odot$, and $100\,M_\odot$. Table 3-1 shows the number of main-sequence stars of each mass formed when a total of $10^6\,M_\odot$ of stars forms according to the Salpeter IMF.

Each star is assumed to shine at constant luminosity throughout its main-sequence lifetime, after which it contributes nothing further to the luminosity of the population. The main-sequence lifetimes and luminosities (relative values) of each mass in the various filter bands are given in Table 3-1, with the filter descriptions in Table 3-2. When stating luminosities in this question, normalise each result so that the V-band value equals 1.0.

(a) Calculate, to 2 significant figures, the luminosity of a freshly formed $10^6\,M_\odot$ stellar population observed in each filter, and enter the values in the table on the answer sheet.



Four candidate spectra (①–④). Horizontal axis: wavelength [nm]; vertical axis: relative energy flux density.

(b) Galaxy C formed a stellar population of $10^9\,M_\odot$ at 1.2×10^{10} years ago and another of $10^9\,M_\odot$ at 6×10^9 years ago. Calculate, to 2 significant figures, the luminosity of Galaxy C observed in each filter, and enter the values in the table on the answer sheet.

(c) Galaxy D formed a stellar population of $10^9\,M_\odot$ at 6×10^9 years ago and another of $10^9\,M_\odot$ at 5×10^6 years ago. Calculate, to 2 significant figures, the luminosity of Galaxy D observed in each filter, and enter the values in the table on the answer sheet.

(4) Figure 3-2 shows the spectra of two galaxies, E and F. Of the two galaxies, which is thought to have a larger fraction of stars created by more recent star formation? Briefly discuss the relationship between star-formation history and the continuum of a galaxy spectrum based on the results of (3), and then give your answer with reasoning.

Table 3-1. For each stellar mass: number formed when $10^6\,M_\odot$ of stars form according to the Salpeter IMF, main-sequence lifetime, and luminosity per star (relative values).

	Number formed per $10^6\,M_\odot$	Main-sequence lifetime	Luminosity per star (relative)				
			U	B	V	R	I
$0.1\,M_\odot$	5.8×10^6	3.2×10^{12} yr	1.30×10^{-5}	2.00×10^{-4}	7.00×10^{-4}	2.00×10^{-3}	1.23×10^{-2}
$1\,M_\odot$	2.5×10^5	1.0×10^{10} yr	0.238	0.637	1.00	1.16	1.23
$10\,M_\odot$	1.2×10^4	3.2×10^7 yr	0.690	1.54	1.93	2.01	1.93
$100\,M_\odot$	5.0×10^2	1.0×10^5 yr	1.29×10^2	1.02×10^2	72.3	55.7	38.5

Table 3-2. Filters used in this question.

Filter name	Primary transmission wavelength range [nm]
U band	300–430
B band	350–540
V band	460–640
R band	520–800
I band	660–960

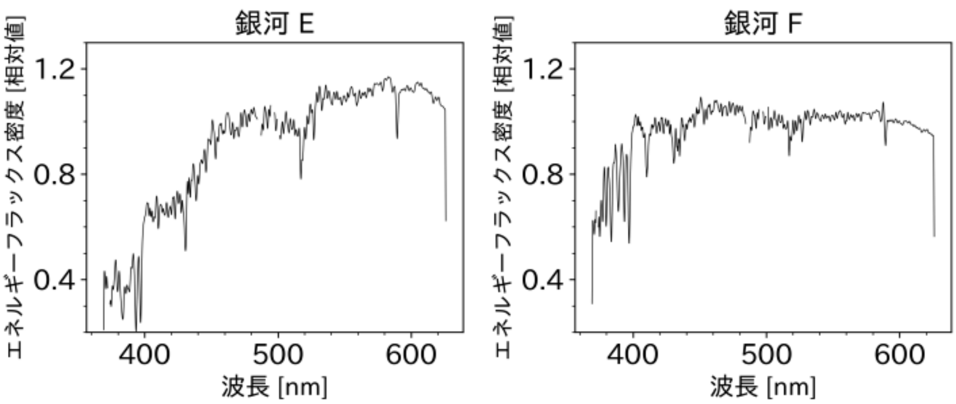


Figure 3-2. Spectra of galaxies E and F. The vertical axis shows energy flux density in relative units. Wavelength ranges containing emission lines have been excluded from the plot.

Problem 4 Stellar Clusters and Stellar Kinematics

Read the following text and answer the questions. For questions that have a space on the answer sheet for equations and reasoning, provide an explanation of how you arrived at your answer. Use the following constants and relations as needed.

Speed of light:	$c = 299792\,\mathrm{km/s}$
Conversion between light-years and parsecs:	$1\,\mathrm{ly} = 0.306601\,\mathrm{pc}$
π:	3.141593

Three sheets of graph paper are provided. Submit 2 sheets as instructed in the questions. The remaining sheet is a spare and may be used freely; it will not be collected. **All scatter plots and graphs must include the following elements. Missing elements may result in deductions or the answer not being graded.**

- Candidate number written in the designated field in the corner of the sheet.
- Tick marks and tick labels on both axes.
- Axis labels and units on both axes.
- Title of the figure.

An example scatter plot / graph is shown in Figure 4-0.

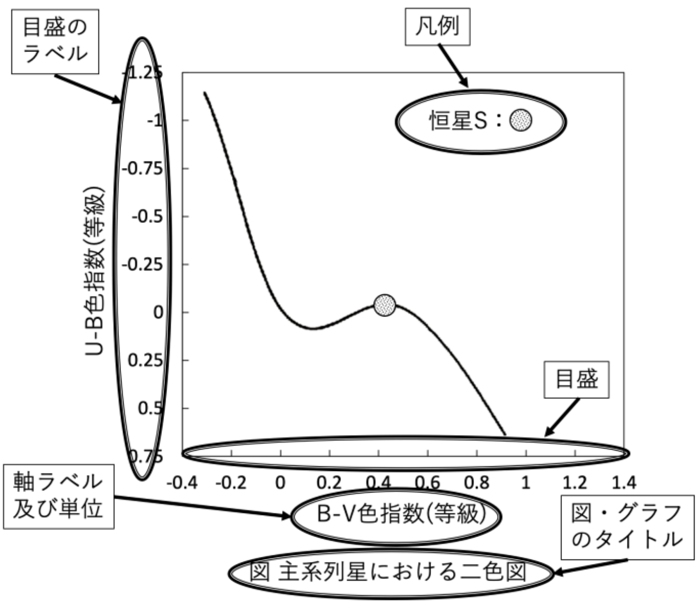


Figure 4-0. Example of a correctly formatted scatter plot / graph (colour-colour diagram for main-sequence stars). Elements indicated: tick label, legend, tick mark, axis label with units, figure title.

In astronomy, people have employed various methods to measure distances to celestial objects. Star cluster H is a cluster whose member stars are relatively bright and easy to observe, and its distance has been discussed since early times.

Table 4-1 shows data for 10 stars observed in the vicinity of star cluster H.

Table 4-1. Data for 10 stars in the vicinity of star cluster H (epoch J2000.0).

No.	R.A. α [h]	[m]	Dec. δ [°]	[′]	V mag [mag]	B mag [mag]	μ_α [$10^{-3\prime\prime}$/yr]	μ_δ [$10^{-3\prime\prime}$/yr]	v_r [km/s]
1	4	29	19	11	3.529	4.540	107.526	−36.200	38.4
2	4	20	15	38	3.642	4.650	115.460	−23.420	38.5
3	4	23	17	33	3.753	4.740	106.212	−27.692	37.6
4	4	38	12	31	4.262	4.390	104.695	−14.861	39.5
5	4	25	17	56	4.298	4.369	107.316	−31.000	39.4
6	4	39	15	55	4.665	4.842	83.170	−20.970	40.8
7	4	21	15	6	5.242	5.475	108.443	−21.171	36.2
8	4	31	13	43	5.395	5.655	106.565	−19.249	38.8
9	4	31	15	42	5.454	5.739	103.488	−24.354	39.2
10	4	23	16	47	5.625	5.957	106.971	−26.247	39.6

The proper motion is the apparent rate at which a star moves across the celestial sphere as seen from Earth; μ_α [″/yr] in Table 4-1 gives the velocity in right ascension and μ_δ [″/yr] in declination. The relationship between μ_α, μ_δ, and the star's actual velocity in the plane of the sky (tangential velocity) v_t [km/s], expressed using the star's distance d [pc], is

$$v_t = C\,d\sqrt{\mu_\alpha^2 + \mu_\delta^2},$$

where C is a proportionality constant.

Question 1.

Determine the proportionality constant C to 4 significant figures.

The radial velocity is the component of a star's velocity along the line of sight, indicating how fast the star is receding from the observer. Radial velocity is measured using the Doppler effect on light.

Question 2.

When a stationary observer measures a source receding at speed v_r, express the observed wavelength λ_o in terms of the rest wavelength λ_s, c, and v_r.

The radial velocity measured in this way is affected by the motion of the Solar System itself as it orbits the Galactic centre, as well as by Earth's orbital and rotational motion. The radial velocities in Table 4-1 and Table 4-3 have all been corrected for the motion of the Solar System and Earth.

It is known that some star clusters have all their member stars moving in the same direction; such clusters are called *moving clusters*. As seen from a distant observer, the proper motions of all stars belonging to a moving cluster converge to (or diverge from) a single point on the celestial sphere. This convergence (or divergence) point is called point X.

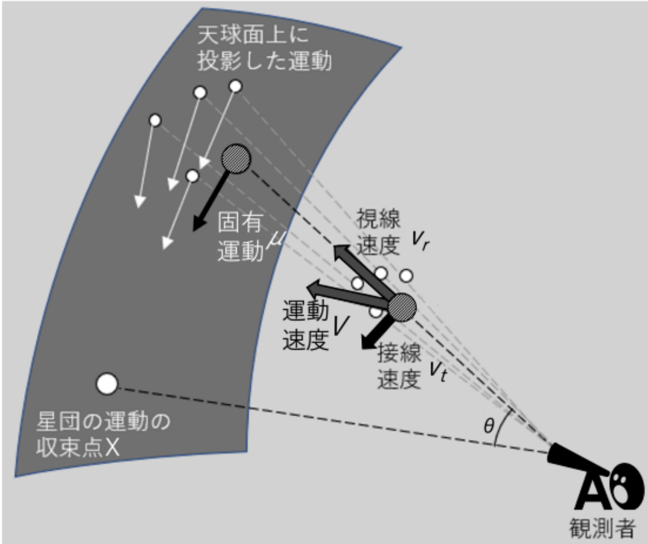


Figure 4-1. Schematic diagram of the projected motion of a star cluster on the celestial sphere. Illustrated are the proper motion, radial velocity v_r, tangential velocity v_t, total velocity V, convergence point X, and angle θ for a distant observer.

Question 3.

Following the procedure below, find the celestial coordinates of point X to the nearest arcminute [′]. Use graph paper sheet ① for your answer. Since graph paper sheet ① will be graded, you must write your candidate number in the designated field before submitting it.

Step 1. Set appropriate scales on graph paper sheet ① with right ascension on the vertical axis and declination on the horizontal axis. Plot the three stars No. 1, 8, and 9 from Table 4-1. Since we are considering a relatively small region of the celestial sphere in this problem, treating the equatorial coordinate system as a plane is not an issue; in subsequent questions, the length of one degree of right ascension is constant regardless of declination.

Step 2. For the three plotted stars (No. 1, 8, 9), draw a straight line in the direction of the proper motion of each star and identify the three intersection points. **If an intersection point falls outside the graph, return to Step 1.**

Step 3. Plot the remaining 7 stars from Table 4-1 on the graph paper.

Step 4. Find the average position of the three intersection points. (The average position is defined as the point with the mean right ascension and mean declination of the three intersection points.)

Step 5. Take the average position found in Step 4 as point X. In all subsequent questions, use this point X as the convergence point of cluster H.

Question 4.

Express the total velocity V and the tangential velocity v_t of each star in terms of v_r and θ, where θ is the angular distance of the star from point X. Also, for each of the 10 stars

(No. 1–10), calculate the following quantities to 3 significant figures and fill in the table on the answer sheet.

- Angular distance θ [°] of each star from point X.
- Total velocity V [km/s] of each star.
- Tangential velocity v_t [km/s] of each star.
- Distance d [pc] to each star.

Hereafter, assume that cluster H consists solely of the 10 stars No. 1–10.

Question 5.

Find the total velocity V [km/s] and the distance d [pc] to cluster H to 3 significant figures. The cluster's velocity and distance from the Solar System are taken to be the average of the velocities and distances of the individual member stars.

Question 6.

Find the radius R [pc] of cluster H to 2 significant figures. Show intermediate steps in your calculation. The apparent radius r [°] corresponding to R is defined as the distance from the centre O (the average right ascension and declination of the member stars) to the most distant member star from O.

We now consider the age of the cluster. The colour of the stars in a cluster is important for estimating its age. In astronomy, a quantity called the *colour index* is often used to describe the colour of a celestial body. When a body's magnitude is measured using various filters (Table 4-2), the colour index is defined as the magnitude measured with a shorter-wavelength filter minus the magnitude measured with a longer-wavelength filter. For example, the B−V colour index measured with the B-band and V-band filters is

$$B - V \text{ colour index} = B \text{ magnitude} - V \text{ magnitude}.$$

The V magnitude is often used as the visual magnitude because it is close to the magnitude perceived by the human eye.

Table 4-2. Filters used for colour index measurements.

Filter	Mean transmission wavelength [nm]	Transmitted light	Example data for α UMi [mag]
U band	365	Ultraviolet	3.00
B band	440	Blue (visible)	2.62
V band	520	Yellow–green (visible)	2.02
R band	760	Red (visible)	1.53
I band	800	Infrared	1.22

Question 7.

Which of α CMa and α Ori has the smaller B−V colour index? Answer with reference to the relationship between the B−V colour index and the colour of a star as perceived by the human eye.

It is known that starlight is absorbed and scattered by interstellar matter, causing extinction. In particular, shorter-wavelength light is more easily scattered and absorbed, so the colour of a star affected by interstellar extinction appears redder than its true colour. This phenomenon is called *interstellar reddening*. One method for correcting interstellar reddening is the *colour-colour diagram* (two-colour diagram), in which the B−V colour index is plotted on the horizontal axis and the U−B colour index on the vertical axis.

Question 8.

Figure 4-2 shows a colour-colour diagram for typical main-sequence stars after correction for interstellar extinction. Main-sequence star S is plotted with a circle on the diagram. If the effect of interstellar matter were *not* corrected, where would star S be plotted on the diagram? Choose the most appropriate symbol from those in the figure and give your reason.

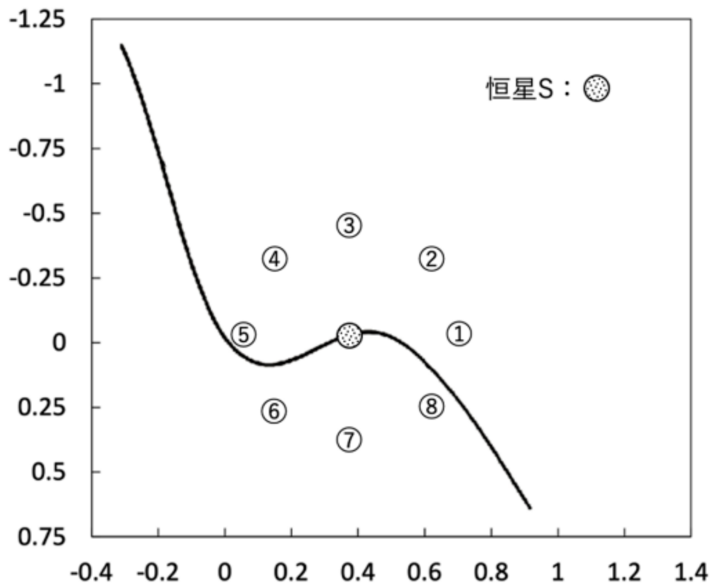


Figure 4-2. Colour-colour diagram for main-sequence stars. The horizontal axis shows the B−V colour index (magnitudes) and the vertical axis shows the U−B colour index (magnitudes). Symbols ①–⑧ mark candidate positions for the uncorrected position of star S.

The B and V magnitudes shown in Tables 4-1 and 4-2 have already been corrected for interstellar extinction.

A plot of the member stars of a cluster with B−V colour index on the horizontal axis and V magnitude on the vertical axis is called a *colour-magnitude (CM) diagram*, and is frequently used to study the characteristics of the cluster. In a CM diagram, stars with a smaller B−V colour index and smaller V magnitude are located towards the upper left.

A CM diagram constructed for a cluster can be used in essentially the same way as an HR diagram.

Question 9.

On graph paper sheet ②, set appropriate scales with B−V colour index on the horizontal axis and V magnitude on the vertical axis, plot the 10 stars from Table 4-1, and produce a CM diagram. Graph paper sheet ② must be submitted with your candidate number written in the designated field.

Let us actually use the CM diagram to determine the age of cluster H. In the HR diagram, the evolutionary track of a star depends on its mass. Lines showing the distribution of stars of the same age are called *isochrones*, and it is known that isochrones have characteristic shapes that vary with age. The time evolution of the isochrones can also be seen in the CM diagram. Therefore, by constructing a CM diagram from observational data and identifying the position of the *turnoff point* (the point where stars deviate from the main sequence and move towards the upper right), and comparing it with known isochrones, the approximate age of the cluster can be determined. Figure 4-3 shows theoretically computed isochrones in the CM diagram (note that the vertical axis gives the V magnitude converted to absolute magnitude).

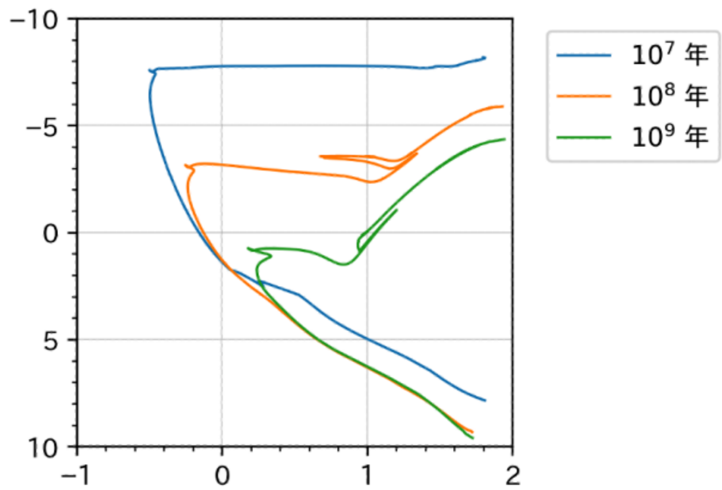


Figure 4-3. Isochrones in the CM diagram showing the relationship between B−V colour index, absolute V magnitude, and stellar age (10^7 yr: blue, 10^8 yr: orange, 10^9 yr: green).

Question 10.

Identify the star in cluster H corresponding to the turnoff point. In this question, the turnoff point is defined as the CM-diagram position of the bluest star in cluster H.

Question 11.

From the legend of Figure 4-3, select the age of cluster H that is considered the closest match.

A star, star A, was observed in the region of cluster H, yielding the data shown in Table 4-3.

Table 4-3. Data for star A in the region of cluster H (epoch J2000.0).

No.	R.A. α [h]	[m]	Dec. δ [°]	[′]	V mag [mag]	B mag [mag]
A	4	36	16	31	0.860	2.400

No.	μ_α [$10^{-3\prime\prime}$/yr]	μ_δ [$10^{-3\prime\prime}$/yr]	v_r [km/s]	Annual parallax [$10^{-3\prime\prime}$]	Age [100 Myr]
A	63.450	−188.940	49.0	48.9	66

Question 12.

Using the radial velocity and proper motion from Table 4-3, find the three-dimensional space velocity V [km/s] of star A to 3 significant figures. Show intermediate steps in your calculation.

Question 13.

Does star A currently belong to cluster H? Is the origin of star A the same as that of cluster H? Answer with reasoning.