Instructions

1. The theoretical competition will be 4 hours in duration and is marked out of a total of 180 points.

2. There are **Detailed Worksheets** for carrying out detailed work / rough work. On each of the **Detailed Worksheets**, please fill in

- Student Code
- Question No.
- Page no. and total number of pages.

3. Start each problem on a new page of the Detailed Worksheets. Please write only on the printed side of the sheet. Do not use the reverse side. If you have written something on any sheet which you do not want to be marked, cross it out.

4. There is a summary **Answer Sheet** with your student ID code for your final answers.

5. Please remember that the graders may not understand your language. As far as possible, write your solutions only using mathematical expressions and numbers. If it is necessary to explain something in words, please use short phrases (if possible in English).

6. You are not allowed to leave your exam desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Detailed Worksheets, etc.), please put up your hand to signal the invigilator.

7. The beginning and end of the competition will be indicated by a long sound of a bell. Additionally, there will be a short sound of a bell fifteen minutes before the end of the competition (before the final long sound of a bell).

8. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

9. A list of constants for this competition is given on the next page.

Fundamental Constants

Speed of light in a vacuum	$c = 2.998 \times 10^{8}\ \mathrm{m\ s^{-1}}$
Planck constant	$h = 6.626 \times 10^{-34}\ \mathrm{J\ s}$
Boltzmann constant	$k_B = 1.381 \times 10^{-23}\ \mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	$\sigma = 5.670 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Elementary charge	$e = 1.602 \times 10^{-19}\ \mathrm{C}$
Universal Gravitational constant	$G = 6.674 \times 10^{-11}\ \mathrm{N\ m^{2}\ kg^{-2}}$
Universal gas constant	$R = 8.315\ \mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	$N_A = 6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
Wien's displacement law	$\lambda_m T = 2.898 \times 10^{-3}\ \mathrm{m\ K}$
Mass of the electron	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of the proton	$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$
Mass of the neutron	$m_n = 1.67 \times 10^{-27}\ \mathrm{kg}$

Plane trigonometry

Sum of Sines	$\sin\ (A+B) = \sin\ A\ \cos\ B + \cos\ A\ \sin\ B$
Sum of Cosines	$\cos\ (A+B) = \cos\ A\ \cos\ B - \sin\ A\ \sin\ B$
Cosine theorem	$a^2 = b^2 + c^2 - 2bc\ \cos\ A$
Sine theorem	$\frac{\sin\ A}{a} = \frac{\sin\ B}{b} = \frac{\sin\ C}{c}$

Spherical trigonometry

Spherical Cosine theorem	$\cos\ a = \cos\ b\ \cos\ c + \sin\ b\ \sin\ c\ \cos\ A$
Spherical Sine theorem	$\frac{\sin\ A}{\sin\ a} = \frac{\sin\ B}{\sin\ b} = \frac{\sin\ C}{\sin\ c}$
Four parts formula	$\cot\ b\ \sin\ a = \cos\ a\ \cos\ C + \sin\ C\ \cot B$

Astronomical Data

1 parsec	$1\ \mathrm{pc} = 3.086 \times 10^{16}\ \mathrm{m} = 206\,265\ \mathrm{AU} = 3.262\ \mathrm{ly}$
1 Astronomical Unit (AU)	$1\ \mathrm{AU} = 1.496 \times 10^{11}\ \mathrm{m}$
1 Jansky	$1\ \mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$
Hubble constant	$H_0 = 70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$
Solar luminosity	$L_\odot = 3.826 \times 10^{26}\ \mathrm{W}$
Apparent mean angular diameter of Sun	$\theta_\odot = 32'$
Effective temperature of Sun	$T_{\mathrm{eff},\odot} = 5778\ \mathrm{K}$
Obliquity of the Ecliptic (Earth)	$\varepsilon = 23.5^\circ$
Inclination of the lunar orbit with respect to the Ecliptic	$05^\circ 08' 43''$
Apparent visual magnitude of full moon	-12.74
North Ecliptic Pole (J2000.0)	$(\alpha_E, \delta_E) = (18^{\mathrm{h}}00^{\mathrm{m}}00^{\mathrm{s}}, +66^\circ 33' 39'')$
North Galactic Pole (J2000.0)	$(\alpha_G, \delta_G) = (12^{\mathrm{h}}51^{\mathrm{m}}26^{\mathrm{s}}, +27^\circ 07' 42'')$
1 sidereal day	$23^{\mathrm{h}}56^{\mathrm{m}}04^{\mathrm{s}}$
1 tropical year	365.2422 solar days
1 sidereal year	365.2564 solar days
1 Lunar mean sidereal month	27.32166 days
1 Lunar mean synodic month	29.53059 days

Solar magnitudes

Apparent visual	$= -26.75$
Absolute visual	$= +4.82$
Apparent bolometric	$= -26.83$
Absolute bolometric	$= +4.74$

Solar System

Object	Mean radius (km)	Mass (kg)	Semi-major axis (AU)	Eccentricity	Albedo
Sun	695 500	1.988×10^{30}	---	---	---
Mercury	2 440	3.301×10^{23}	0.387	0.206	0.088
Venus	6 052	4.867×10^{24}	0.723	0.007	0.76
Earth	6 378	5.972×10^{24}	1.000	0.016710	0.31
Moon	1 737	7.346×10^{22}	3.844×10^{5} km	0.054900	0.11
Mars	3 390	6.417×10^{23}	1.524	0.093	0.25
Jupiter	69 911	1.898×10^{27}	5.203	0.048	0.51
Saturn	58 232	5.683×10^{26}	9.537	0.054	0.34
Uranus	25 362	8.681×10^{25}	19.189	0.047	0.30
Neptune	24 622	1.024×10^{26}	30.070	0.009	0.29

Information about the host city

- Latitude of Kathmandu City, $\phi_{kathmandu} = 27.7172^{o}N$
- Time Zone for Nepal = UTC + 5:45:00

T1 (15 marks)

A student used a telescope of diameter (D) 100cm, to observe a distant star whose absolute magnitude is -0.5. If we consider the limiting magnitude of the eye as 6 and the diameter of the pupil (d) to be 7mm, then answer the following questions:

1. If no interstellar medium (ISM) is present, find the maximum distance to which student can detect this star.
2. In reality, an ISM of extinction factor 0.05 mag/Kpc is present between the star and the Earth. Find the maximum distance to which the student can detect the star.

Note: When an equation cannot be solved analytically, we use iterative methods to find an approximate numerical solution. As an example, suppose you want to find out the value of $x = x_{fin}$, for which $x_{fin} = f(x_{fin})$, we start with some value $x = x_0$ and proceed as follows:

- Step 1: Find $f(x_0)$ and call it x_1.
- Step 2: Find $f(x_1)$ and call it x_2.
- Step 3: Find $f(x_2)$ and call it x_3.

Keep repeating this iteration process till x_n is almost same as x_{n-1}. That is your desired x_{fin} value.

T2 (15 points)

Starting from the law of conservation of energy, derive an expression for the total mechanical energy of a body of mass m on an elliptical orbit, in terms of G, the mass of the object being orbited (M) and the semi-major axis of the orbit a. Use your derivation to then find the ratio of the speeds at perihelion and aphelion, for an object that has an orbit with eccentricity e.

T3 (15 points)

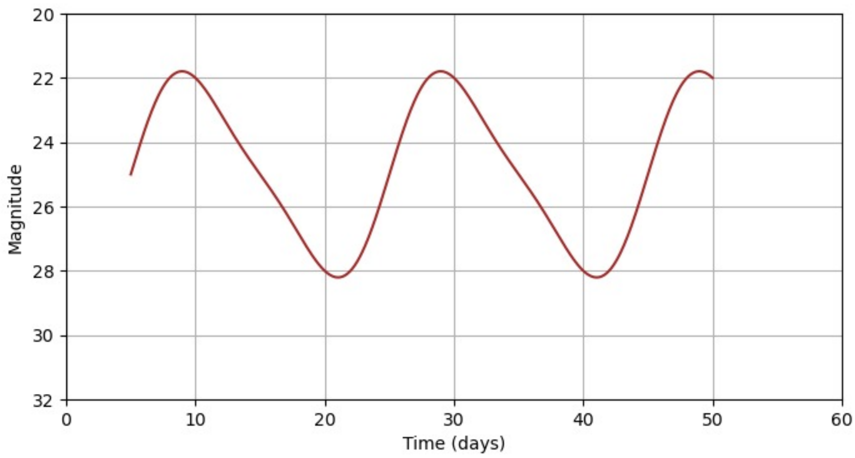
Nepal has a unique timezone of UTC +5:45, with the meridian for the country passing through the peak of Mt. Gaurishankar in the Himalayas.

1. Knowing that Mt. Gaurishankar is about 105km due East from Kathmandu, estimate Kathmandu's longitude.
2. Binod, who stays in Kathmandu, sees a bright star at meridian at exactly 8:35 pm as per his watch. Calculate Kathmandu's local time for this star's meridian crossing.
3. Binod wants to tell about this bright star to his friend Ravi who lives in Ranchi, a city in India that has the same longitude as Kathmandu, but a timezone of UTC +5:30. What is the difference in local time and time in the watch for Ravi? What time will the same star cross the meridian as per Ravi's watch?

T4 (15 marks)

In Cosmology, Cepheid variables are called standard candles as they can be used to estimate distances. Edwin Hubble used these standard candles to settle the Shapley-Curtis debate by measuring the distance to the Andromeda Galaxy. Cepheid variables are variable stars whose brightness vary periodically. Henrietta Leavitt discovered the relation between the periodicity and average luminosity of these stars, and this relation can be used to derive the equation linking Absolute magnitude and Period. For Cepheid variables, Luminosity(L) varies with Period(P) as $L \propto P^{2.5}$, where P is in days.

1. Find the relation between Absolute Magnitude (M) and P for this type of variable stars.
2. You might notice that it is a linear equation between M and a function of P. If the unknown constatnt of the given equation is -1.5 mag, then find the distance to the given Cepheid Variable.



T5 (15 points)

Nepal has ambitions to launch a geostationary satellite of its own. If the satellite is to maintain a geostationary orbit such that, when viewed from Kathmandu, it is at the prime meridian. What would be its altitude when observed from Kathmandu? You can neglect refraction due to the atmosphere and other smaller effects.

T6 (15 points)

The festival of Dashain is one of the most important celebrations in the Nepali calendar. The final day of the festival coincides with a Waxing Gibbous Moon that is exactly 10 days old. The festival always happens in a fixed month of lunar calendar, which has 12 months. Answer the following questions regarding the estimated timing of the Dashain festival.

1. In 2024, the final day is scheduled to fall on the 12th of October. Assuming that the Moon is exactly 10 days old at 11 am, predict on which date the final day of Dashain occurred in 2023.
2. Predict on which day *Kojagrat Purnima*, which is the Full Moon immediately after Dashain, will fall in 2025.

T7 (15 points)

The Cosmic Horseshoe is a gravitationally lensed galaxy at $z = 2.379$. It has a magnification factor ($\mu = 30$). The magnification that occurs due to gravitational lensing is analogous to that provided by an optical telescope, but is due to the gravitational effects of massive bodies. Answer some questions about this galaxy to help a researcher. [Note: AB magnitudes are converted analogously to the magnitude scale that you are familiar with. The average flux density of an object is the Flux (F) of that object divided by the bandwidth $\Delta\nu$ of the detector.]

1. At what wavelength (λ_{obs}) would you expect to detect the H_α emission line emitted by this galaxy? [Note: $\lambda_{rest} = 656.3$nm.]
2. Source plane reconstruction is the technique by which the lensing effects on the galaxy are estimated, in order to get an understanding of the galaxy's morphology. We use the technique and now need to determine the object's physical properties. Using the *Hubble Space Telescope (HST)*, the F606W band AB magnitude is found to be 20.5. What is the object's magnitude after correcting for lensing?
3. One can convert from AB magnitudes to flux density by using the fact that a source at 0 AB magnitude has 3631 Jy of flux density. What is (a) the apparent flux density of the Cosmic Horseshoe and (b) what is the magnification corrected flux density?
4. The Cosmic Horseshoe is observed to have its F606W UV continuum emission spread across an area of 13arcsec^2. What is its apparent UV continuum emission surface density, in units of mag arcsec^{-2}?
5. If the angular scale at the Horseshoe's redshift is 8 kpc arcsec^{-1}, what is the radius of the galaxy, assuming that its a face-on disk?

T8 (15 points)

As we saw in the group competition, Nepal got the chance to name a star in the constellation of Leo. The name given to that star is Sagarmatha. The coordinates of this star are $\alpha = 11^h 35^m 52^s$, $\delta = -4^o 45' 21''$. Given that the Autumnal equinox for the year 2024 happened on Sep 22 at 6:28 pm Nepal time, and neglecting the effects of atmospheric refraction,

1. Find the rise time of Sagarmatha as seen from Kathmandu on 6^{th} October 2024. (Note: The rise time here refers to local time in Kathmandu)

2. We want to find Sagarmatha through a telescope. Since Sagarmatha is too faint, we first point to ϕ Leo ($\alpha = 11^h 16^m 38^s$, $\delta = -3^o 38' 58''$). Find angular separation between ϕ Leo and Sagarmatha.

T9 (15 points)

Suppose you are travelling on a spacecraft that encounters a uniformly spherically distributed gas cloud with radius R, and volume density ρ. At your moment of approach, you decelarate till you are at rest with respect to the gas cloud and then turn off your engines. You may neglect gas friction in this problem.

*Note: Gravitational Potential inside the uniform solid sphere is given by

$$V\left(r\right) = -\frac{GM}{R^3}\left(\frac{3R^2}{2} - \frac{r^2}{2}\right)$$

where r is the distance from the center of the gas cloud. You may also find the following fact useful. For an equation of the form $a = -k^2 r$, where k is some constant and a is acceleration, the time period of the oscillation is given by: $T = \frac{2\pi}{k}$.

1. Once inside the gas cloud, how long will it take you to return to your original position if you are in free fall inside the cloud?
2. Start from the law of conservation of energy. Suppose you are tired of being inside the gas cloud after investigating it for a while and want to escape it (to infinity). What is your required speed if you are initially located in the center of the cloud and are at rest?

T10 (15 points)

Spectroscopic binary star systems are those systems in which individual stars of the system can't be resolved visually due to their low angular separation. But we can detect that the system is binary due to shifts in their spectral line caused by Doppler shift. In double lined spectroscopic binaries, we can detect two spectra, one for each star. From the change in wavelength of the spectrum over a period of time, we can find the radial velocity curve.

Suppose an astronomer observed a hypothetical spectroscopic binary system having a circular orbit as shown in Figure 1. The plane of the orbit makes an angle θ w.r.t the plane of the sky. Star 1 (mass m_1) is located on the outer orbit and Star 2 (mass m_2) is located in the inner orbit. The observed radial velocity graph is shown in Figure 2.

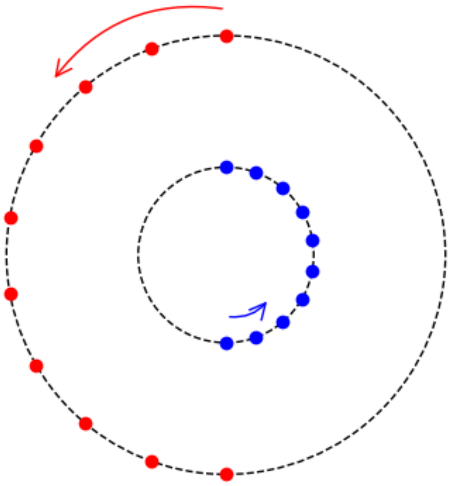


Figure 1. A top-view of the binary system.

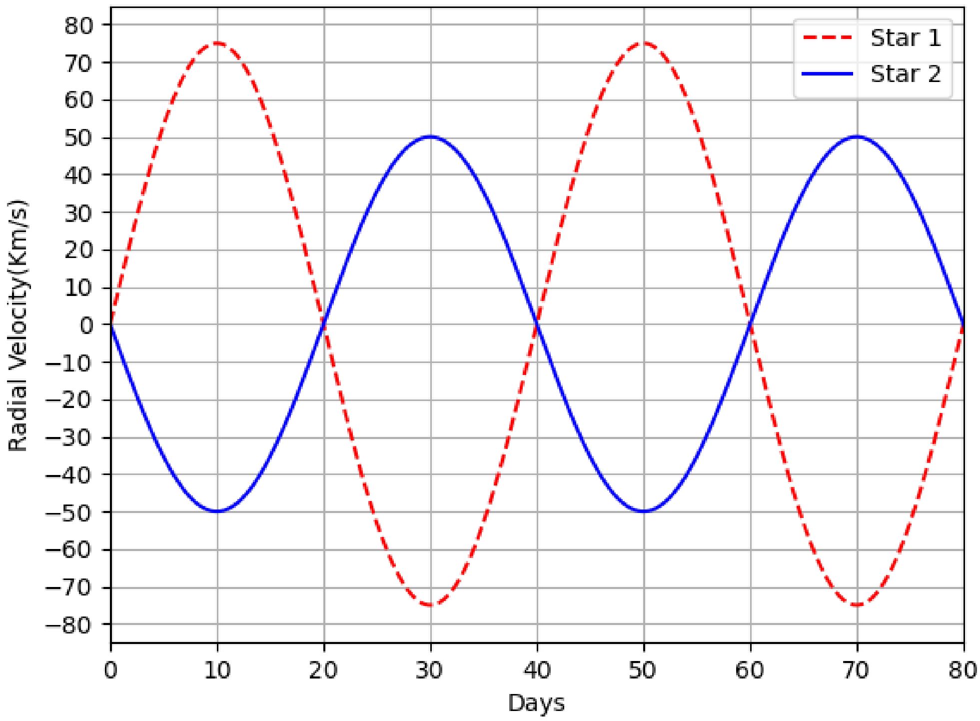


Figure 2. Radial Velocity Plots for the Binary Stars

1. The binary mass function is an expression which depends upon only the observable quantities such as radial velocity and period. Find the binary mass function for both star 1 and star 2.
2. Use your expression in (1) to estimate the masses for the stars. Is your estimate exact, a lower-limit or an upper-limit?

T11 (15 points)

In a classical approximation to get a feel for expanding universe, one may assume that the universe is uniformly filled cloud with small particles, and that the critical density (ρ_c) is the density of the cloud at which the expansion of the universe (the Hubble-Lemaitre law) stops at infinite time.

1. Derive a classical expression for (ρ_c) of the universe.
2. Find the value of critical density in SI units.
3. If the universe was filled with tennis balls of mass 0.145 kg each distributed uniformly (no other mass) and the mass density of this universe equal to the critical density then how many tennis balls would be included in a sphere of size of the Sun?

T12 (15 marks)

Interstellar dust is heated up by hot ionizing radiation from young blue stars. They then re-emit the UV light. A dust cloud in a dusty galaxy was recently studied by a researcher. Make the simplifying approximation that you can treat the dust cloud as a blackbody. Help him to answer the following questions:

1. Find the wavelength where the dust emission peaks (λ_{max}) if it has a temperature of $T = 50\,\mathrm{K}$.
2. The researcher determines that the dust has a total luminosity (L_{dust}) = $1.6 \times 10^{11} L_\odot$. Estimate the size of the dust cloud, assuming a spherical distribution.
3. If the cosmological redshift of the galaxy (z) is 0.01, estimate the angular size (θ) of the dust cloud.
4. What aperture telescope is minimally needed if we want to resolve the dust cloud?