2nd International Olympiad on Astronomy and Astrophysics Junior
Greece, Volos
24 – 30 September 2023
Theory

Instructions:

A) Do not touch envelopes until the start of the examination.

B) The theoretical examination lasts for 3.5 hours, has 12 questions in total and a total of 180 points.

C) Questions 1 to 10 are worth 10 points each.

D) Questions 11 and 12 are worth 40 points each.

E) For questions 9 and 10, you may only write the correct answer. No explanation is necessary. For all other questions detailed solution is necessary.

F) There are Answer Sheets for carrying out detailed work/rough work. On each Answer Sheet page, please fill in:

   a. Student Code (Country Code and student's number, example: GEO-3)

   b. Question Number

G) If you have written something on any sheet which you do not want to be evaluated, cross it out.

H) Use as many mathematical expressions as you think may help the evaluator to better understand your solutions. The evaluator may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible, in English).

I) You are not allowed to leave your work desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please draw the attention of the proctor.

J) The beginning and end of the examination will be indicated by the proctor. Additionally, the proctor will announce the remaining time every hour and fifteen minutes before the end of the examination.

K) At the end of the examination, you must stop writing immediately. Sort your Answer Sheets according to Question numbering and put everything back in the envelope.

L) Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the examination room.

M) A list of constants and useful relations are given on the next pages.

Useful Constants:

Fundamental constants			
Speed of light in vacuum	c	=	2.998×10^{8} m s^{-1}
Stefan-Boltzmann constant	σ	=	5.670×10^{-8} W m^{-2} K^{-4}
Universal gravitational constant	G	=	6.674×10^{-11} N m^{2} kg^{-2}
Universal gas constant	R	=	8.315 J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Wien's displacement law	$\lambda_m T$	=	2.898×10^{-3} m K
Astronomical data			
1 parsec	1 pc	=	3.086×10^{16} m
		=	206 265 au
		=	3.262 ly
1 astronomical unit (au)	1 au	=	1.496×10^{11} m
Hubble constant	H_0	=	70 km s^{-1} Mpc^{-1}
Solar luminosity	$L_\odot$	=	3.826×10^{26} W
Effective temperature of Sun	$T_{\mathrm{eff},\odot}$	=	5778 K
Obliquity of the ecliptic (Earth)	ε	=	23.5°
Inclination of the lunar orbit w.r.t. ecliptic		=	05°08’43”
Apparent visual magnitude of full moon		=	–12.74
1 sidereal day		=	$23^h56^m04^s$
1 tropical year		=	365.2422 solar days
1 sidereal year		=	365.2564 solar days
Solar magnitudes			
Apparent visual		=	–26.75
Absolute visual		=	+4.82
Apparent bolometric		=	–26.83
Absolute bolometric		=	+4.74

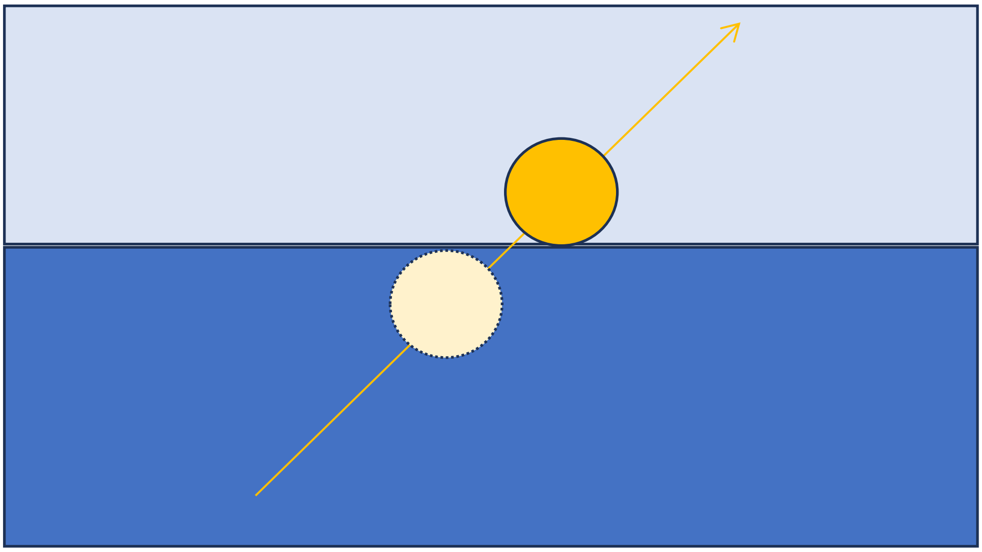
Solar System

Object	Mean radius (km)	Mass (kg)	Orbital Radius (au)
Sun	695 500	1.988×10^{30}	---
Mercury	2 440	3.301×10^{23}	0.387
Venus	6 052	4.867×10^{24}	0.723
Earth	6 378	5.972×10^{24}	1.000 000
Moon	1 737	7.346×10^{22}	0.002 572
Mars	3 390	6.417×10^{23}	1.524
Jupiter	69 911	1.898×10^{27}	5.203
Saturn	58 232	5.683×10^{26}	9.537
Uranus	25 362	8.681×10^{25}	19.189
Neptune	24 622	1.024×10^{26}	30.070

Question 1: Horizon crossing.

The Sun, at some location, crosses the horizon at sunrise on autumnal equinox day at an angle of 45°. The angular diameter of the Sun is 32 arcminutes. How long does it approximately take for the entire solar disk to cross the horizon?

You may neglect atmospheric effects.



Solution: The sun covers ~361 degrees on the celestial sphere in 24 hours, thus the velocity will be: v=361/(24x3600) degrees/sec. **[5 points]**

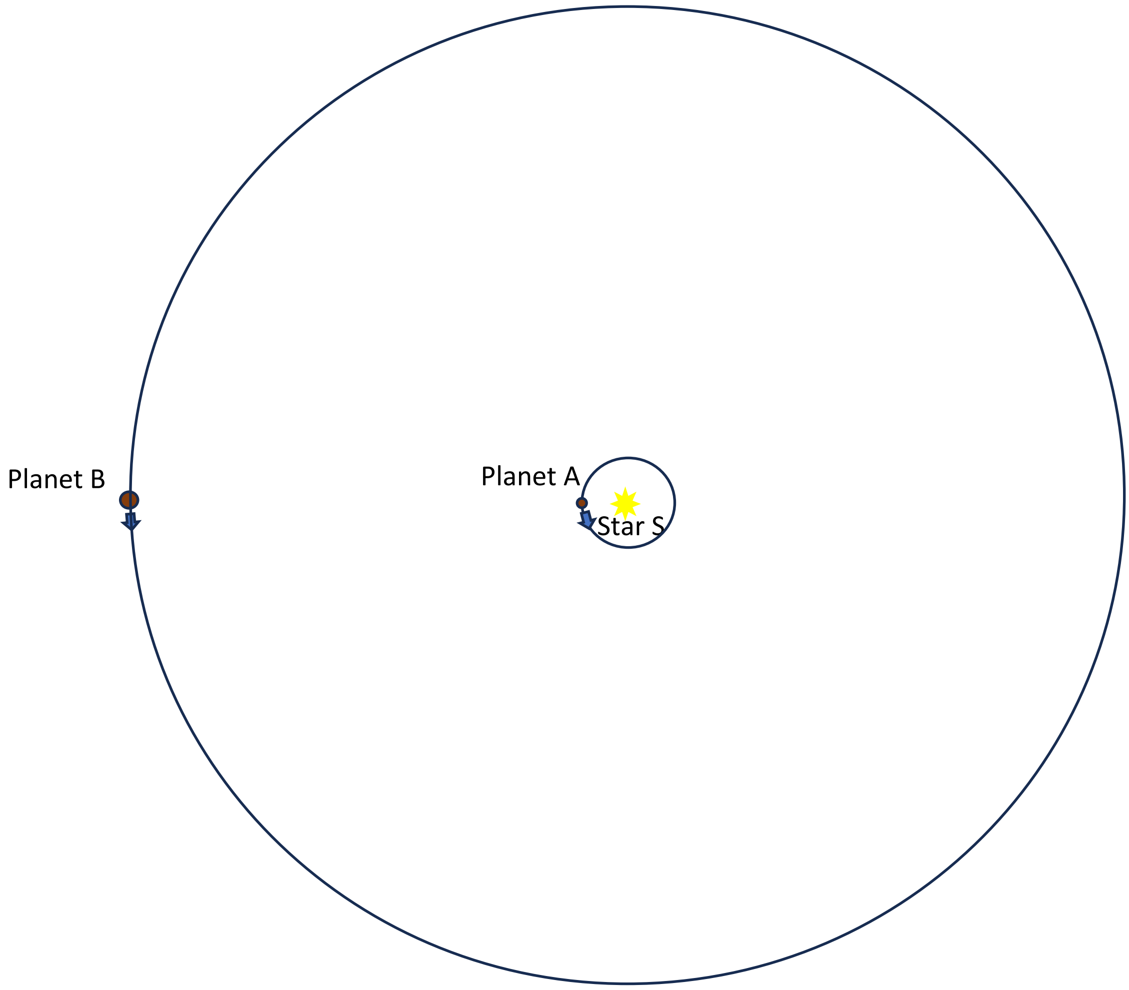
The Sun will need to move s=(32/60)/cos(π/4) degrees to cross the horizon. Therefore, the time it will need is: t =s/v=[(32/60)/cos(π/4)]/[361/(24x3600)]= 3 minutes. **[5 points]**

Question 2: Planet orbits

Two planets, A and B, orbit a star S in circular, coplanar orbits, in the same direction. The orbital radius of planet A is 9 times smaller than the orbital radius of planet B. The orbital period of planet A is 10 Earth days. An astronomer on Earth realizes that at 10:00 am on 26/9/2023, an observer on planet B would observe a transit of planet A in front of star S. When is the next transit due to planet A to be seen by an observer on planet B?

(All dates/times are given on Earth times).

The masses of planets A and B are very small compared to the mass of star S.



The ratio of the orbits is 9, therefore, according to Kepler's law is the ratio of the squares of the periods is equal to the ratio of the cubes of the radii of the orbits: $(r_1/r_2)^3 = (P_1/P_2)^2$ **[3 points]**

Substituting the numbers, we obtain the period of planet 2:

$P_2=P_1(r_2/r_1)^{3/2} = 10\ 3^3=270$ days. **[2 points]**

The angular velocity is defined as $\omega= 2\pi/P$ and the orbital phase is $\varphi= \omega t$.

A transit will occur when $\varphi_1-\varphi_2=2k\pi$, with k being an integer. The first one will occur for k=1. Therefore:

$2\pi t\ (1/10-1/270) = 2\pi$, t = 10.3846 days = 10 days 9 hours and 14 minutes after the original one, thus on 6 October 2023, 7:14 pm. **[5 points]**

Question 3: Unresolved binary star

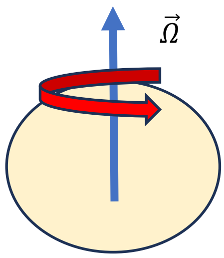
An unresolved binary star consists of two members whose apparent magnitudes are m_1=2.0 mag and m_2=4.5 mag. What is the combined apparent magnitude of this binary system?

The apparent magnitude of a star is defined as: $m = -2.5\log_{10}\left(\frac{F_x}{F_{x,0}}\right)$. **[4 points]**

We solve for the flux F_x and we add the fluxes corresponding to each star, obtaining the relation: $m_b = -2.5\log_{10}(10^{-0.4\,m_1} + 10^{-0.4\,m_2}) = 1.89 \approx 1.9$ mag. **[6 points]**

Question 4: Rapidly spinning neutron star.

A neutron star is a rapidly spinning compact star. A particular neutron star has a mass of 1.40 $M_\odot$ and its radius is 10.0 km. What is the shortest possible spin period for it, before it starts shedding its mass away?



The limit of mass shedding occurs when the centripetal/centrifugal acceleration equals the force of gravitational acceleration. This corresponds to:

$$\frac{GM}{r^2} = \frac{v^2}{r}$$

[4 points]

We find that the velocity, substituting the values for a neutron star, we find that the corresponding linear velocity on the equator due to rotation is 1.36×10^8m/s. We now need to evaluate the period: $\frac{2\pi}{P} r = v$, thus $P = \frac{2\pi r}{v} = 0.462$ ms **[6 points]**

Question 5: Moon phases

The first full moon for 2024 in Volos is on the 25th of January at 19:54. At what date and time will the last full moon for 2024 (visible from Volos) occur?

The lunar month (from full moon to full moon) is 29.53 days. (29.5 days is acceptable).

[3 points]

12 lunar months correspond to 354.36 days, which gets us to 7/1/2025. Thus, we need to account for 11 lunar months: 324.83, which corresponds to the 15:49 on 15 December 2024.

If they use 29.5 they obtain 7:54 am on 15 December 2024.

[7 points]

Question 6: Solar Eclipse.

Assume that one day the Moon's apogee coincides with the Earth's perihelion. An observer, at an appropriate location on the Earth, observes the eclipse at zenith at its maximum. What fraction of the solar disk area is covered?

The maximum distance between the centers of the Earth and the Moon is 405,000 km, while the minimum distance between the Earth and the Sun is 147,100,000 km.

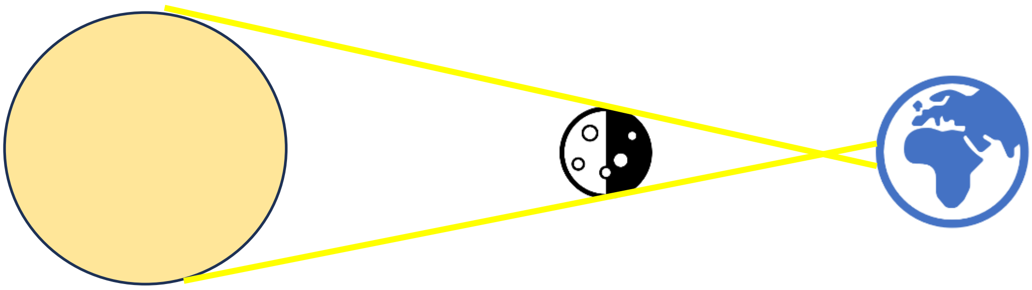


Figure not to scale.

The angular diameter of the Sun during perihelion is

$$d_1 = 2 \times \frac{695{,}500}{147{,}100{,}00 - 6{,}378} = 2 \times\ 0.004731 = 0.009463 \text{ rad,}$$

while the angular diameter of the moon during apogee is

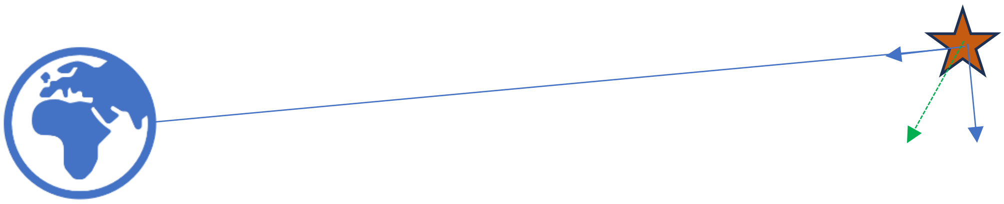
$$d_2 = 2 \times \frac{1{,}737}{405{,}000 - 6{,}378} = 2 \times 0.004358 = 0.008715 \text{ rad.}$$

[6 points]

The fraction of the area that is covered is the ratio of the squares of these values: $r = \left(\frac{d_2}{d_1}\right)^2 = 0.8482 = 84.82\%$. **[4 points]**

Question 7: Parallax and proper motion

The parallax of Barnard's star is 0.552 arcseconds and its proper motion is μ=10.31arcseconds/year. From its spectrum, an observer noticed that the absorption line H_α is seen at 656.044 nm instead of 656.281nm. What is the total speed of the Barnard's star relative to this observer?



Based on the information given we can extract the radial and the transverse velocity of Barnard's star. The radial velocity is evaluated through the doppler shift, due to the low values, we can use the non-relativistic formula: $v_r = \frac{\Delta\lambda}{\lambda} c = \frac{656.044-656.281}{656.281} \times 2.998 \times 10^5 = -108.27$ km/s.

[3 points]

For the tangential velocity, we need first to evaluate the radial distance. Starting from the parallax, we find that the distance is $d = \frac{1}{0.552} = 1.81\text{pc}$, as it moves 10.31 arcseconds/year, this corresponds to a tangential velocity of:

$v_t = \mu\, d = \frac{10.31}{206265} \times 1.81 \times 3.086 \times 10^{13} \times \frac{1}{365.2422 \times 24 \times 3600} = 88.55$ km/s. **[4 points]**

The speed can be evaluated through Pythagorean theorem:
$v = \sqrt{(108.27^2 + 88.55^2)} = 139.87$ km/s . **[3 points]**

Question 8: Telescope.

The focal length of a telescope is 900 mm and its aperture is 4 inches. We use an eyepiece whose focal length is 25 mm. What is the magnification *m* of this telescope and what is its focal ratio *f* ?

1 inch is 2.54 cm.

The magnification is the ratio of the focal length of the telescope over the focal length of the eyepiece, thus: $m = \frac{900}{25} = 36.$ **[5 points]**

The focal ratio f is the ratio of the focal length to the aperture, thus $\frac{900}{40 \times 2.54} = 8.9$. **[5 points]**

Question 9: Cosmology

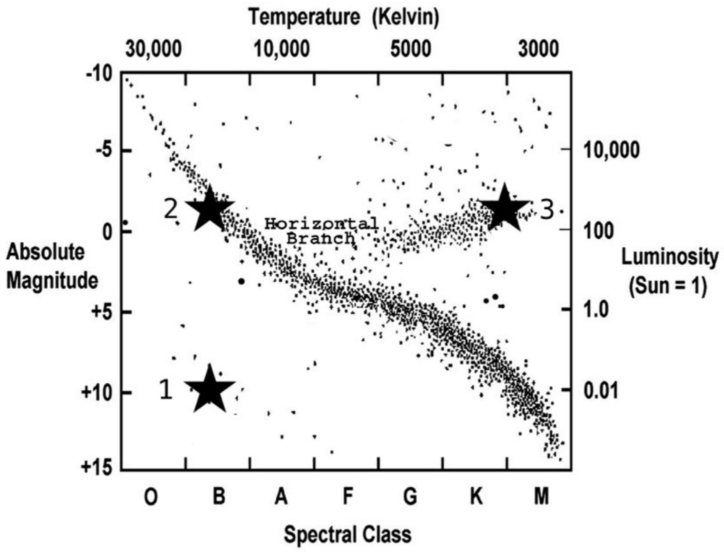
Arrange the following events in the order in which they occurred (from earliest to the most recent).

A) Formation of the first stars.
B) Inflationary era of the universe
C) Formation of helium nuclei
D) Formation of neutral hydrogen
E) Formation of oxygen atoms

The correct answer is: B, C, D, A, E

Question 10: H-R diagram.

The image below shows the H-R diagram and three stars: 1, 2 and 3. For each statement below write "TRUE" if the statement is definitely true. Write "FALSE" if the statement is definitely false. Write "MAYBE" if the information is insufficient to draw a conclusion.



A) Star 1 is a main sequence star.	FALSE	**[2 points]**
B) The radius of star 2 is larger than that of star 3.	FALSE	**[2 points]**
C) Star 2 is younger than the current age of the Sun.	TRUE	**[2 points]**
D) Star 3 is more massive than the Sun.	MAYBE	**[2 points]**
E) Star 3 is colder than the other two stars.	TRUE	**[2 points]**

Question 11: SIRIUS A and B

Sirius is the brightest star of the night sky during the winter of the northern hemisphere and the summer of the southern hemisphere. It has an apparent visual magnitude m_V=-1.47 mag. Sirius B is the binary companion of Sirius A, its magnitude is m_V=8.5 mag. Their orbital period is 50 years and the parallax of the system is 350 milliarcseconds, while their angular separation is 7 arcseconds.

You can assume that Sirius A has a surface temperature of T=10,000 K. The temperature of Sirius B is estimated to be 25,000 K. Both stars have a bolometric correction equal to -0.4 mag. An absolute bolometric magnitude of zero corresponds to a luminosity of 3×10^{28} W. The orbit is assumed to be circular, and the line of sight is perpendicular to the orbital plane of the system.

A) Calculate the luminosity and radius of Sirius A and B. **[12 Points]**

B) What is the orbital separation of the system in meters? **[8 Points]**

C) What is the total mass of the system? **[8 Points]**

D) Assuming that the mass of Sirius A is twice the mass of Sirius B, what is the mean density of Sirius B? **[6 Points]**

E) In the H-R diagram shown in question 10, which star number (1, 2, 3) will be the closest to star Sirius B? **[6 Points]**

A) We will use the relation between the apparent and absolute magnitude of the star:
$m - M = 5\log(d) - 5$.
The distance can be evaluated using the parallax: $d = \frac{1}{0.350} = 2.86$ pc. Combining this with the above relation we obtain the absolute visual magnitude of Sirius A:
$M_V = -1.47 - 5\log 2.86 + 5 = 1.25$mag. Taking into account the bolometric correction, the absolute bolometric magnitude is: $M_B = 1.25 - 0.4 = 0.85$mag.
Regarding Sirius B, we repeat the same process, we obtain: $M_B = 0.85 + (8.5 + 1.47) = 10.82$.
To obtain the luminosity we use the following relation:
$M_V = -2.5\log\frac{L}{L_0}$
$L = L_0 \times 10^{-0.4\,M_V}$

Substituting for Sirius A and B, we obtain, for Sirius A $L_A = 1.37 \times 10^{28}$ W,
Sirius B $L_B = 1.41 \times 10^{24}$ W.

Next we apply Stephan-Boltzmann law to get the radius: $L = 4\,\pi\,R^2\sigma\,T^4$, the radius of Sirius A is $R_A = 1.39 \times 10^9$m.

Whereas the radius of Sirius B is: $R_B = 2.25 \times 10^6$ m

B) Let us convert the angular separation to radians: $\theta_1 = \frac{7}{206265} = 3.39 \times 10^{-5}$ rad. Based on the distance found in the previous question, the separation is
$s_1 = 3.39 \times 10^{-5} \times 2.86 = 9.71 \times 10^{-5}$ pc which converted into meters is: 3.00×10^{12}m.

C) We can evaluate the total mass from the relation: $M_{tot} = a^3/P^2$ where a is the separation in Astronomical Units, P is the period in years and the total mass is given solar masses. Their separation converted to AU is: 20 AU, thus the total mass is:
$M_{tot} = \frac{20.0^3}{50^2} = 3.20\ M_{\odot}$.

D) The mass of Sirius B is $\frac{1}{3}3.20 = 1.07M_{\odot}$. To evaluate the mean density we need to find its radius. As we have already evaluated the luminosity, we now need to evaluate the radius Using part A of this question, the radius is: $R = 2.25 \times 10^6$ m. The mean density is: $\bar{\rho} = 4.4 \times 10^{10} kg/m^3$.

E) Based on the above results (mass radius and density), Sirius B has a radius comparable to the radius of Earth, while its mass is close to the mass of the Sun. Therefore, it is a white dwarf, closer to 1.

Question 12: SATELLITES

The term geostationary (GEO) refers to a satellite whose orbit is on the equatorial plane, its orbit is circular, and its orbital period is equal to Earth's period of revolution around its axis.

For this problem, all orbits around Earth are considered circular. An important characteristic of the orbit of a satellite is the elevation angle of the satellite, which is defined as the angle at which the satellite is visible above the local horizon for an observer on Earth.

A) What is the maximum latitude the geostationary satellite is visible at? **[12 Points]**

Geostationary satellites, because of their distance from the center of the Earth, observe only a fraction of the Earth's surface. If a satellite needs to view a larger fraction of the Earth, it must change its orbit using rockets.

B) The satellite moves from its location and observes the Earth having an angular diameter of 10°. What is the distance of the satellite from the center of the Earth? **[10 Points]**

C) How much does the total (kinetic and potential) energy of the satellite need to change in order to move from the geostationary orbit to the one described in question B? The satellite has a mass of 1000 kg. **[10 Points]**

D) What is the ratio of the gravitational acceleration g on the Earth's surface compared to the gravitational acceleration at the location described in question (B). **[8 Points]**

A) We need first to evaluate the radius of the geostationary satellite. The radius of the orbit is found through the relation, using the sidereal period $T = 23h\ 56m\ 4s$:

$$T^2 = \frac{4\pi^2 R_S^3}{GM_E} \Rightarrow R_S = 4.2163 \times 10^7\ m = 42163\ km$$

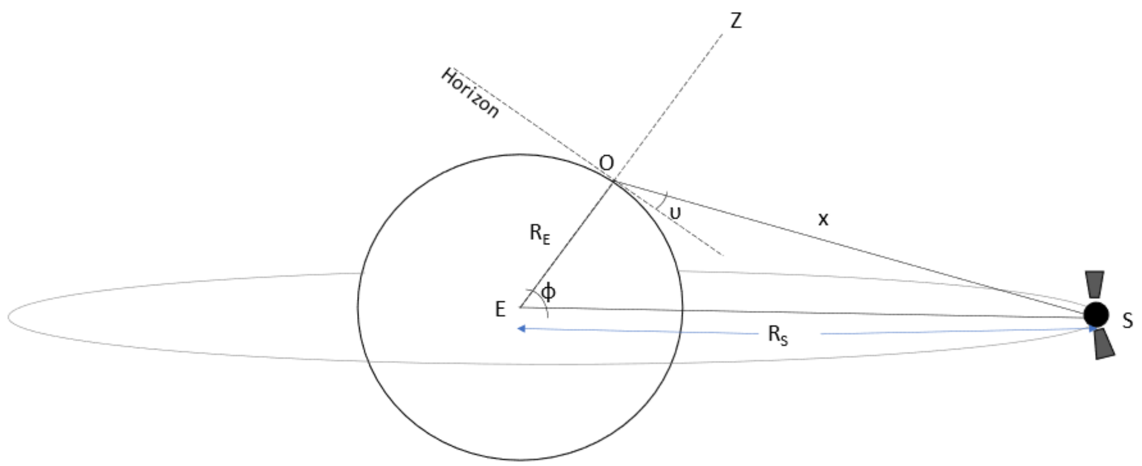


Figure 1.

Naming all the points and lines in Figure 1:

E: center of Earth, S: satellite, O: observer on Earth, Z: observer's zenith

φ: observer's latitude , υ: elevation angle

EO = R_E: Earth's radius, ES = R_S: Satellite's orbital radius, OS = x: observer – satellite distance

From the EOS:

The ends of the range of latitudes the satellite is visible is when υ = 0.

Using the sine law: $\frac{\sin(\widehat{EOS})}{R_S} = \frac{\sin(\widehat{OSE})}{R_E} \Rightarrow \frac{\sin(90°)}{R_S} = \frac{\sin\left(180°-(90°+\varphi)\right)}{R_E} \Longrightarrow \varphi = \cos^{-1}\left(\frac{R_E}{R_S}\right)$

$$\therefore \varphi \cong 81.3°$$

The maximum latitudes the satellite is visible is 81.3° N and 81.3° S .

B) We need to find the location where angle $O\hat{S}E = 5^o$, while $\upsilon = 0$. This means that the triangle SOE is orthogonal, with angle $E\hat{O}S = 90^o$. From the definition of the sin $O\hat{S}E = \frac{R_E}{R_S} = \sin 5^o = 0.0872$ we obtain $R_S = 6378/0.0872 = 73179$ km.

C) The total energy of a satellite is given by the following relation:
$E_{tot} = \frac{1}{2}\, m\, v^2 - G\frac{M_E m}{r}$

We obtain the velocity from the centripetal force relation:
$\frac{v^2}{r} = G\frac{M}{r^2}$, therefore the relation for the total energy becomes:

$$E_{tot} = -\frac{1}{2} G \frac{M_E m}{r}.$$

We substitute the physical quantities for the two locations, and we obtain:

$E_{tot,1} = -4.727 \times 10^9$ J, $E_{tot,2} = -2.723 \times 10^9$J, thus energy has increased by 2.00×10^9J.

D) The gravitational acceleration is inversely proportional to the square of the radius: $g = GM/r^2$, thus the ration of the gravitational acceleration is the inverse of the ratio of the squares. Therefore:

$$\frac{g_E}{g_B} = \left(\frac{r_B}{r_E}\right)^2 = \left(\frac{73179000}{6378000}\right)^2 = 131.6.$$