Grid 9. The Moon, seen on the way to the Moon

The Moon, whose linear diameter is $d = 3\,436\text{ km}$, at Earth in the distance $r_0 = 348\,000\text{ km}$, has the apparent magnitude $m_0 = -12.7$. The apparent magnitude of the Sun, seen from Earth, is $m_s = -26.84$.

The distance, r, from the Moon, where a cosmonaut is, on the way of his spaceship to the Moon, the brightness of the Moon should be the same as the brightness of the Sun seen from Earth, is:

a) $r \approx 675.25\text{ km}$; **b)** $r \approx 568.88\text{ km}$; c) $r \approx 763.23\text{ km}$; d) $r \approx 837.44\text{ km}$.

Solution

The approach of the cosmonaut to the Moon must be such that it determines a variation of the apparent magnitude of the Moon:

$$\Delta m = m - m_0 = m_S - m_0 = -26.84 - (-12.7) = -14.14,$$

so that the apparent magnitude of the Moon, viewed from this new distance, is:

$$m = m_0 + \Delta m = m_S = -26.84;$$

$$\Delta m = m_S - m_0 = -26.84 + 12.7 = -14.14.$$

Knowing that the luminous intensity, I, of a star, in particular of the Moon, is:

$$I = \frac{\phi}{\Omega} = \frac{L}{\Omega} = \frac{W}{t \cdot \Omega},$$

that is, the energy flow emitted by the star, the Moon, in the unit of solid angle, then:

$$\Omega = \frac{A}{\Delta^2};\; A = \Omega \cdot \Delta^2;\; \phi = I \cdot \Omega;\; E = \frac{\phi}{A} = \frac{I \cdot \Omega}{\Omega \cdot \Delta^2} = \frac{I}{\Delta^2};\; E = \frac{L}{4\pi\Delta^2}.$$

According to Pogson's formula, if E_0 and respectively E are the brightnesses of the Moon, corresponding to the initial (r_0) and respectively the final distances (r), shown in the drawing in figure 1, it results:

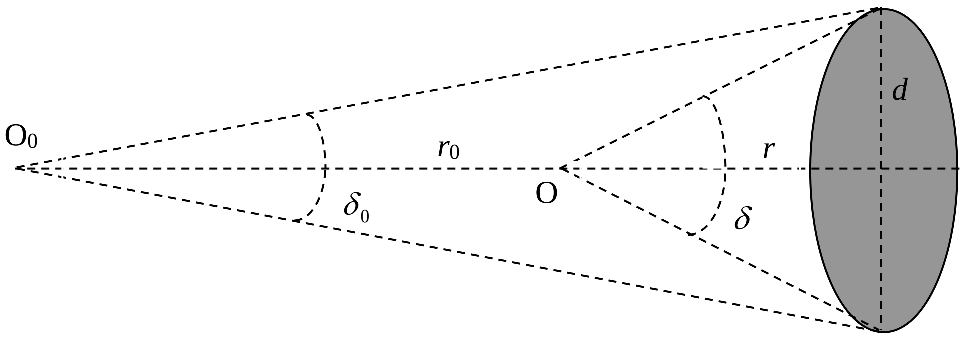

Fig. 1

$$\log \frac{E}{E_0} = -0.4 \cdot (m - m_0) = \log 10^{-0.4(m - m_0)};\; E_0 = \frac{I}{r_0^2};\; E = \frac{I}{r^2},$$

where I is the luminous intensity of the Moon;

$$\log \frac{r_0^2}{r^2} = \log 10^{-0.4(m - m_0)};$$

$$2\log \frac{r_0}{r} = -0.4 \cdot \Delta m;\; \log \frac{r_0}{r} = 0.2 \cdot 14.14 = 2.828 = \log 10^{2.828};$$

$$\frac{r_0}{r} = 10^{2.828};\; r = \frac{r_0}{10^{2.828}}$$

$$x = 10^{2.828};\quad \log x = 2.828;\quad x \approx 675;$$

$$r = \frac{r_0}{10^{2.828}};\ 10^{2.828} \approx 675;$$

$$r = \frac{r_0}{675} = \frac{384000\ \text{km}}{675} \approx 568.88\ \text{km},$$

this is the distance from which the Moon should be viewed, so that its brightness is the same as the brightness of the Sun, seen from Earth.