Grid 7. Balls Suspended inside a Terrestrial Satellite

An artificial satellite evolves around the Earth in a circular orbit with radius r, permanently maintaining the same orientation towards the Earth, as shown in the drawing in figure 1. Inside the satellite are suspended, by very light wires, four identical spherical balls, each with mass m, such that the balls (b) and (c) are symmetrical to the ball (a), the balls (a) and (d) are symetrical to the ball (c), and the difference of their distances to the center of the Earth is $\Delta r = 2d$.

Given: $M -$ the mass of the Earth; $K -$ gravitational attraction constant.

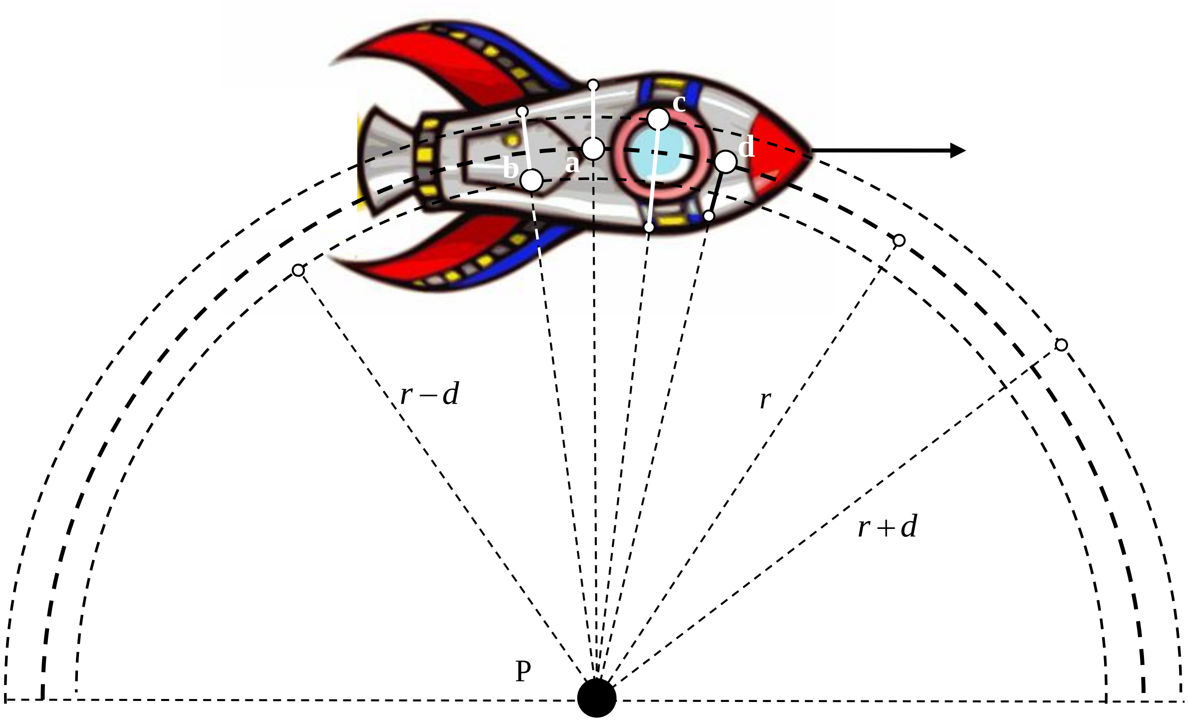

Fig. 1

The tension in each suspension wire, is:

a) $T_{(a)} = 0;\ T_{(b)} = 3K\dfrac{mMd}{r^3} > 0;\ T_{(c)} = 3K\dfrac{mMd}{r^3} > 0;\ T_{(d)} = 0;$

b) $T_{(a)} = K\dfrac{mMd}{r^3};\ T_{(b)} = K\dfrac{mMd}{r^3} > 0;\ T_{(c)} = K\dfrac{mMd}{r^3} > 0;\ T_{(d)} = K\dfrac{mMd}{r^3};$

c) $T_{(a)} = 3K\dfrac{mMd}{r^3} > 0;\ T_{(b)} = 0;\ T_{(c)} = 0;\ T_{(d)} = 3K\dfrac{mMd}{r^3} > 0;$

d) $T_{(a)} = 2K\dfrac{mMd}{r^3} > 0;\ T_{(b)} = 2K\dfrac{mMd}{r^3} > 0;\ T_{(c)} = 2K\dfrac{mMd}{r^3} > 0;\ T_{(d)} = 2K\dfrac{mMd}{r^3} > 0.$

Solution

The orientation of the spacecraft being constant, and it is, as a whole, a rigid solid body, means that all points belonging to the spacecraft move around the Earth at the same angular velocity, ω, whose expression is obtained as follows:

$$F_{\mathrm{ag(NC)}} = K\frac{m_{\mathrm{(NC)}}M}{r^2} = m_{\mathrm{(NC)}}\omega^2 r;\ \ \omega^2 = K\frac{M}{r^3}.$$

The centripetal accelerations of the three suspended balls are:

$$a_{\mathrm{cp(a)}} = \omega^2 r;\ \ a_{\mathrm{cp(b)}} = \omega^2(r-d);\ \ a_{\mathrm{cp(c)}} = \omega^2(r+d);\ \ a_{\mathrm{cp(d)}} = \omega^2 r = a_{\mathrm{cp(a)}}.$$

Assuming that all the suspension wires are tensioned, then, for each ball, the resultant of the gravitational pull force with the tension in the respective suspension wire is the centripetal force responsible for the circular motion of each ball.

Results that:

- f*or the ball* (a):

$$\vec{F}_{\mathrm{cp(a)}} = \vec{F}_{\mathrm{ag(a)}} + \vec{T}_{\mathrm{(a)}};\ \ F_{\mathrm{cp(a)}} = F_{\mathrm{ag(a)}} - T_{\mathrm{(a)}};\ \ T_{\mathrm{(a)}} = F_{\mathrm{ag(a)}} - F_{\mathrm{cp(a)}};$$

$$F_{\mathrm{ag(a)}} = K\frac{mM}{r^2};\ F_{\mathrm{cp(a)}} = ma_{\mathrm{cp(a)}} = m\omega^2 r;\ \omega^2 = K\frac{M}{r^3};\ F_{\mathrm{cp(a)}} = ma_{\mathrm{cp(a)}} = K\frac{mM}{r^2};$$

$$T_{\mathrm{(a)}} = K\frac{mM}{r^2} - K\frac{mM}{r^2};$$

$$T_{\mathrm{(a)}} = 0,$$

which means that the suspension wire of the ball (a) is not tensioned, so that the ball (a) is in a weightless state;

- *for the ball* (b):

$$\vec{F}_{\mathrm{cp(b)}} = \vec{F}_{\mathrm{ag(b)}} + \vec{T}_{\mathrm{(b)}};\ \ F_{\mathrm{cp(b)}} = F_{\mathrm{ag(b)}} - T_{\mathrm{(b)}};\ \ T_{\mathrm{(b)}} = F_{\mathrm{ag(b)}} - F_{\mathrm{cp(b)}};$$

$$F_{\mathrm{ag(b)}} = K\frac{mM}{(r-d)^2} = K\frac{mM}{r^2\left(1-\frac{d}{r}\right)^2} = K\frac{M}{r^2}\left(1-\frac{d}{r}\right)^{-2};$$

$$d << r;\ \left(1-\frac{d}{r}\right)^{-2} \approx 1 + 2\cdot\frac{d}{r};$$

$$F_{\mathrm{ag(b)}} = K\frac{M}{r^2}\left(1+2\cdot\frac{d}{r}\right);$$

$$F_{\mathrm{cp(b)}} = ma_{\mathrm{cp(b)}};\ \ a_{\mathrm{cp(b)}} = \omega^2(r-d);\ \ F_{\mathrm{cp(b)}} = m\omega^2(r-d);$$

$$\omega^2 = K\frac{M}{r^3};\ F_{\mathrm{cp(b)}} = K\frac{mM}{r^3}(r-d);$$

$$T_{\mathrm{(b)}} = K\frac{M}{r^2}\left(1+2\cdot\frac{d}{r}\right) - K\frac{mM}{r^3}(r-d) = K\frac{mM}{r^2} + 2K\frac{mMd}{r^3} - K\frac{mM}{r^2} + K\frac{mMd}{r^3};$$

$$T_{\mathrm{(b)}} = 3K\frac{mMd}{r^3} \neq 0;\ T_{\mathrm{(b)}} = 3K\frac{mMd}{r^3} > 0,$$

which means that the ball suspension wire (b) is tensioned so that the ball (b) is not weightless;

- *for the ball* (c)

$$\vec{F}_{\text{cp(c)}} = \vec{F}_{\text{ag(c)}} + \vec{T}_{\text{(c)}};\;\; F_{\text{cp(c)}} = F_{\text{ag(c)}} + T_{\text{(c)}};\;\; T_{\text{(c)}} = F_{\text{cp(c)}} - F_{\text{ag(c)}};$$

$$F_{\text{ag(c)}} = K\frac{mM}{(r+d)^2} = K\frac{mM}{r^2\left(1+\frac{d}{r}\right)^2} = K\frac{mM}{r^2}\left(1+\frac{d}{r}\right)^{-2};$$

$$d << r;\;\; \left(1+\frac{d}{r}\right)^{-2} \approx 1 - 2\cdot\frac{d}{r};$$

$$F_{\text{ag(c)}} = K\frac{mM}{r^2}\left(1-2\cdot\frac{d}{r}\right);$$

$$F_{\text{cp(c)}} = ma_{\text{cp(c)}};\;\; a_{\text{cp(c)}} = \omega^2(r+d);\;\; F_{\text{cp(c)}} = m\omega^2(r+d);$$

$$\omega^2 = K\frac{M}{r^3};\; F_{\text{cp(c)}} = K\frac{mM}{r^3}(r+d);$$

$$T_{\text{(c)}} = K\frac{mM}{r^3}(r+d) - K\frac{mM}{r^2}\left(1-2\cdot\frac{d}{r}\right) = K\frac{mM}{r^2} + K\frac{mMd}{r^3} - K\frac{mM}{r^2} + 2K\frac{mMd}{r^3};$$

$$T_{\text{(c)}} = -3K\frac{mMd}{r^3} \neq 0;\;\; T_{\text{(c)}} = 3K\frac{mMd}{r^3} > 0,$$

which means that the ball suspension wire (c) is tensioned so that the ball (c) is not weightless;

- *for the ball* (d)

$$\vec{F}_{\text{cp(d)}} = \vec{F}_{\text{ag(d)}} + \vec{T}_{\text{(d)}};\;\; F_{\text{cp(d)}} = F_{\text{ag(d)}} + T_{\text{(d)}};\;\; T_{\text{(d)}} = F_{\text{cp(d)}} - F_{\text{ag(d)}};$$

$$F_{\text{ag(d)}} = K\frac{mM}{r^2};\; F_{\text{cp(d)}} = ma_{\text{cp(d)}} = m\omega^2 r;\; \omega^2 = K\frac{M}{r^3};\; F_{\text{cp(d)}} = ma_{\text{cp(d)}} = K\frac{mM}{r^2};$$

$$T_{\text{(d)}} = K\frac{mM}{r^2} - K\frac{mM}{r^2};$$

$$T_{\text{(d)}} = 0,$$

which means that the suspension wire of the ball (d) is not tensioned, so that the ball (d) is in a weightless state.