Grid 5. Andromeda Constellation Rotation

Our galaxy and the constellation Andromeda can be observed with the help of a telescope whose mirror has a diameter of D = 6 m.

The following are known: the Earth - Andromeda distance, $R = 1.42 \cdot 10^{11} R_0$, where R_0 is the radius of the Earth's circular orbit around the Sun; the mass of our Galaxy, $M_{\text{G}} = 2.5 \cdot 10^{11} M_0$, where M_0 is the mass of the Sun; $T_0 = 1$ terrestrial year, the period of rotation of the Earth around the Sun; the mass of the constellation Andromeda, $M_{\text{A}} = 3.6 \cdot 10^{11} M_0$.

Knowing that:

1) the photos were taken in visible light, with a wavelength $\lambda = 5 \cdot 10^{-7}$ m;

2) the angular distance between two objects for which they can be observed separately is $\varphi_0 = 1.22 \cdot \dfrac{\lambda}{D}$.

Using the photographs thus obtained, time required to highlight the rotational motions of our galaxy and the constellation Andromeda around their common center of mass, is:

a) 10^4 ani; **b)** 10^3 ani; c) 10^5 ani; d) 10^4 ani.

Solution

From the study of light diffraction it is known that the angular distance between two objects for which they can be observed separately is approximately. λ / D. It means that the change in the position of the constellation Andromeda can be observed if its angular displacement is:

$$\varphi_0 = 1.22 \cdot \frac{\lambda}{D} = 1.22 \cdot \frac{5 \cdot 10^{-7}}{6} = 10.16 \cdot 10^{-8} \text{radians.}$$

If T is the period of rotation of our Galaxy and the Andromeda Constellation, respectively, around their common center of mass, then the time required for the angular displacement φ_0 of the Andromeda constellation is:

$$\tau = \frac{\varphi_0}{\omega} = \frac{\varphi_0}{2\pi} T.$$

For two binary systems, using the generalized form of Kepler's third law, we can write that:

$$\left(\frac{T_1}{T_2} \right)^2 \left(\frac{M_1 + m_1}{M_2 + m_2} \right) = \left(\frac{a_1}{a_2} \right)^3,$$

out of which, for the galaxy - Andromeda and Sun - Earth systems, results in:

$$\left(\frac{T}{T_0} \right)^2 \left(\frac{M_{\text{G}} + M_{\text{A}}}{M_0 + M_{\text{P}}} \right) = \left(\frac{R}{R_0} \right)^3; \; M_{\text{P}} << M_0; \, T_0 = 1 \text{ terrestrial year;}$$

$$\left(\frac{T}{T_0} \right)^2 \left(\frac{M_{\text{G}} + M_{\text{A}}}{M_0} \right) = \left(\frac{R}{R_0} \right)^3;$$

$$T^2 = T_0^2 \cdot \left(\frac{R}{R_0} \right)^3 \cdot \left(\frac{M_0}{M_G + M_A} \right); \; \tau = \frac{\varphi_0}{2\pi} T; \; \varphi_0 = 1.22 \cdot \frac{\lambda}{D};$$

$$\tau = 1.22 \cdot \frac{\lambda}{2\pi D} T_0 \left(\frac{R}{R_0} \right)^{3/2} \left(\frac{M_0}{M_{\text{G}} + M_{\text{A}}} \right)^{1/2} \approx 10^3 \text{ years.}$$