Grid 4. The Terrestrial Satellite

A small satellite (a luminous sphere) that orbits the Earth, in an equatorial circular orbit at the altitude h, is photographed with a camera on the ground, the lens of which is a converging lens with focal length f. The photography is made when the satellite aligns with the camera and the center of the Earth.

Given: the mass of the Earth, M; the constant of gravitational attraction, K. During the photographic exposure the proper rotation of the Earth is neglected.

The length of the image on the photograph, after developing the photographic plate, placed in the focal plane of the objective lens, as indicated by the drawing in figure 1, if the exposure time of the camera was Δt, knowing that the radius of the Earth is R, is given by the expression:

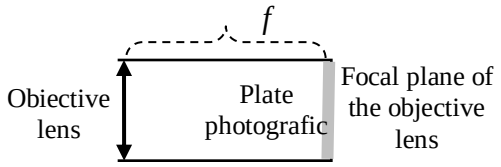

Fig. 1

a) $l = \dfrac{f}{h-f} \cdot \sqrt{K \dfrac{M}{R+h}} \cdot \Delta t$; b) $l = \dfrac{f}{h+f} \cdot \sqrt{K \dfrac{M}{R+h}} \cdot \Delta t$;

c) $l = \dfrac{f}{h-f} \cdot \sqrt{K \dfrac{M}{h-R}} \cdot \Delta t$; d) $l = \dfrac{f}{h-f} \cdot \sqrt{2K \dfrac{M}{R+h}} \cdot \Delta t$.

Solution

Sector AB, in the circular orbit of the satellite, represented in the drawing in Figure 2, traveled by the satellite during the time Δt, equal to the exposure time of the camera, has the length:

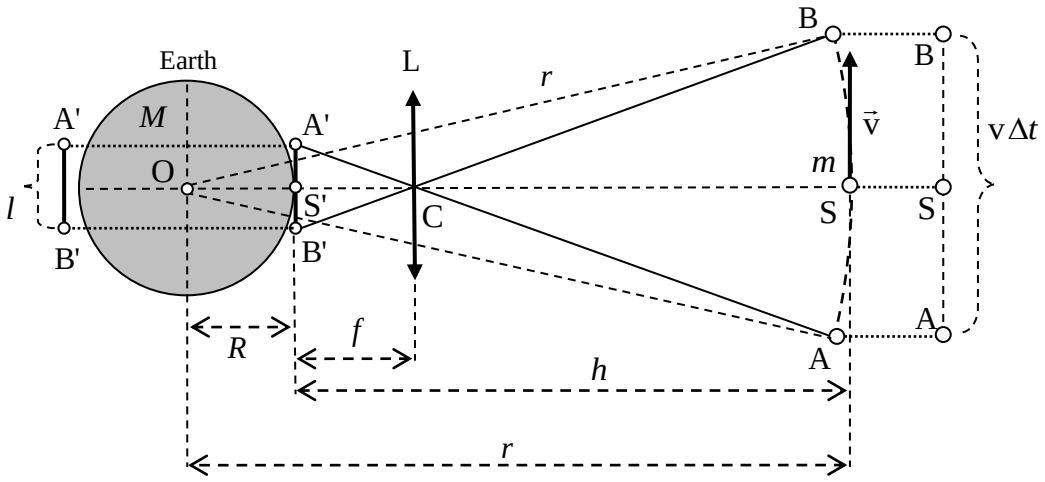

Fig. 1

$$AB = v_{rel} \cdot \Delta t,$$

where v – the speed of the satellite in its circular orbit, relative to the center of the Earth;

$$K\frac{mM}{r^2}=\frac{m\mathrm{v}^2}{r};$$

$$\mathrm{v}=\sqrt{K\frac{M}{r}}=\sqrt{K\frac{M}{R+h}}.$$

Because the distance between the satellite and the camera lens is very large, the image will be formed in the focal plane of the camera lens.

Since the circle sector AB is very short and has a very large radius ($r = R + h$), it can be assimilated with the line segment AB, so that:

$$\Delta(\mathrm{ACS})\sim\Delta(\mathrm{A'CS'});$$

$$\frac{\mathrm{A'S'}}{\mathrm{AS}}=\frac{\mathrm{CS'}}{\mathrm{CS}};\ \frac{\frac{l}{2}}{\frac{\mathrm{v}\cdot\Delta t}{2}}=\frac{f}{h-f},$$

where l – the length of the image on the photo;

$$l=\frac{f}{h-f}\cdot\mathrm{v}\cdot\Delta t;$$

$$l=\frac{f}{h-f}\cdot\sqrt{K\frac{M}{R+h}}\cdot\Delta t.$$