Grid 3. The third cosmic speed

Given: $V_0 \approx 30\,\frac{\text{km}}{\text{s}}$, the speed of the Earth in its circular orbit around the Sun; $v_0 \approx 7{,}9\,\frac{\text{km}}{\text{s}}$, the speed of a terrestrial satellite orbiting the Earth in a very low circular orbit (the first cosmic speed).

It is known that: $\frac{M_{\text{Earth}}}{R_{\text{Earth}}} << \frac{M_{\text{Sun}}}{R_{\text{Earth–Sun}}}$.

The variation of the kinetic energy of the body, in relation to the Sun, is neglected during the evolution of the body from the surface of the Earth to the limit of the gravitational attraction of the Earth.

The approximate minimum escape velocity that must be imparted to a body, B, relative to the Earth, launched from the Earth so that it leaves the Solar System forever (the third cosmic velocity), is:

a) $v_B \approx 45{,}52\,\frac{\text{km}}{\text{s}}$; b) $v_B \approx 32{,}32\,\frac{\text{km}}{\text{s}}$;

c) $v_B \approx 22{,}22\,\frac{\text{km}}{\text{s}}$; **d)** $v_B \approx 42{,}42\,\frac{\text{km}}{\text{s}}$.

Solution

Let $\vec{v}_B$ the speed of body B at the time of its launch from Earth, in relation to the Sun, so that the body reaches the limit of the gravitational attraction of the Sun and there it is at rest in relation to the Sun. Using the details in Figure 1, in accordance with the law of conservation of mechanical energy, it follows:

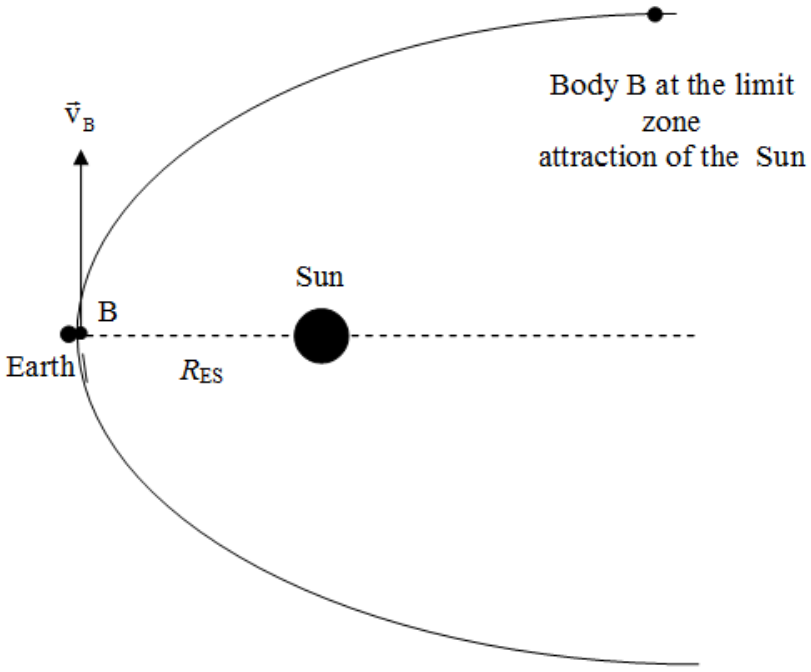

Fig. 1

$R_E << R_{ES}$;

$$\frac{m\mathrm{v}_\mathrm{B}^2}{2}-K\frac{mM_\mathrm{E}}{R_\mathrm{E}}-K\frac{mM_\mathrm{S}}{R_\mathrm{ES}}=0;$$

$$\frac{M_\mathrm{E}}{R_\mathrm{E}}<<\frac{M_\mathrm{S}}{R_\mathrm{ES}};$$

$$\frac{m\mathrm{v}_\mathrm{B}^2}{2}-K\frac{mM_\mathrm{S}}{R_\mathrm{ES}}=0;$$

$$\mathrm{v}_\mathrm{B}=\sqrt{2}\sqrt{K\frac{M_\mathrm{S}}{R_\mathrm{ES}}};$$

$$\sqrt{K\frac{M_\mathrm{S}}{R_\mathrm{ES}}}=\mathrm{v}_\mathrm{ES}=V_{\mathrm{orbital}}=V_0,$$

representing the orbital speed of the Earth on the circular trajectory around the Sun;

$$\mathrm{v}_\mathrm{B}=\sqrt{2}\ V_0,$$

representing the second cosmic speed in relation to the Sun (parabolic speed);

$$R_\mathrm{ES}\approx 1.5\cdot 10^8\ \mathrm{km};\ T_\mathrm{ES}=1\,\mathrm{year};$$

$$V_0=\frac{2\pi R_\mathrm{ES}}{T_\mathrm{S}}\approx 30\,\frac{\mathrm{km}}{\mathrm{s}};\ \mathrm{v}_\mathrm{B}\approx 42.42\,\frac{\mathrm{km}}{\mathrm{s}}.$$

Conclusion: the body launched from the Earth, reaches the limit of the gravitational attraction of the Sun, in relation to the Sun, on a parabolic trajectory, with the Sun in its focus.