Problem 2. The rocket that destroys the threatening asteroid

With the help of a rocket, R, launched from a spaceship, N, an asteroid, A, must be destroyed, approaching the Earth threateningly, coming right in the direction of the center of the Earth, as shown in the drawing in Figure 1. There are positions of the ship spacecraft, N and the asteroid A, respectively, at the time of the launch of the rocket R, to meet the asteroid A, when the speed of the spacecraft, N, relative to Earth, is constant,, oriented along the spacecraft line - asteroid, and the speed of asteroid A, relative to Earth, is considered constant, $\vec{v}_N$, oriented along the spacecraft line - asteroid, and the speed of asteroid A, relative to Earth, is considered constant, $\vec{v}_A$, its direction forming an angle α with the direction of the spacecraft, N.

During the flight of the rocket, from its launch to the impact with the asteroid, it is considered that the Earth, relative to the Sun, is at rest, and the movements of the rocket, R, the spacecraft, N, and the asteroid, A, are rectilinear and uniform movements.

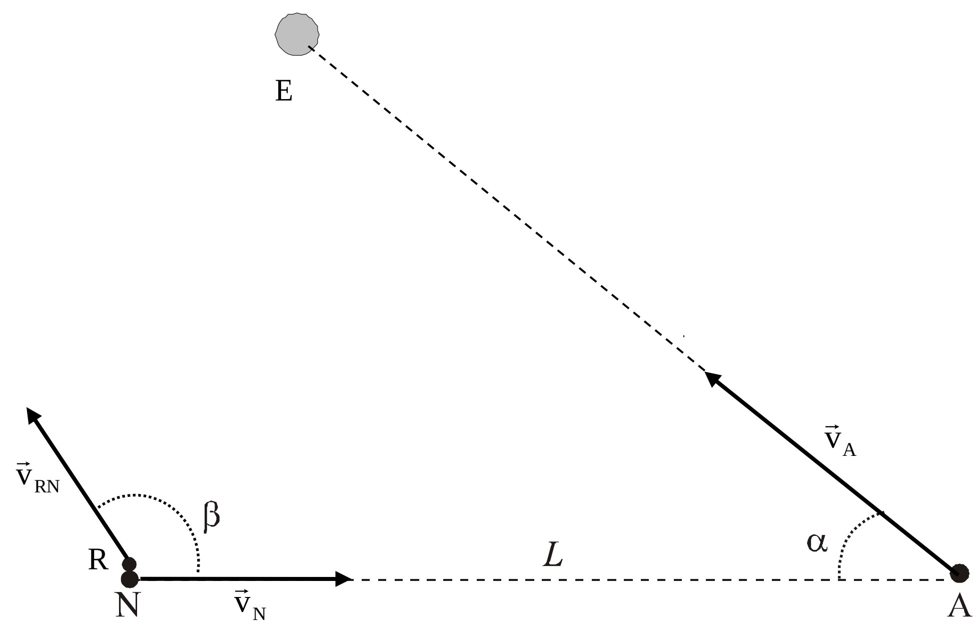

Fig. 1

Knowing that the direction of the rocket launching R was chosen in such a way that the duration of the rocket's movement, R, from launch to impact with asteroid A, is minimal, *determine* the angle β, at which the rocket R was launched, relative to the direction of movement spacecraft, N;

Solving

Reaching the target, asteroid A, by the rocket R, is done in a minimum time, if the direction of absolute displacement of the rocket R, ie the direction of movement NC, is perpendicular to the direction of absolute movement of asteroid A, ie the direction of movement AC, as indicated the drawing in Figure 2, which shows:

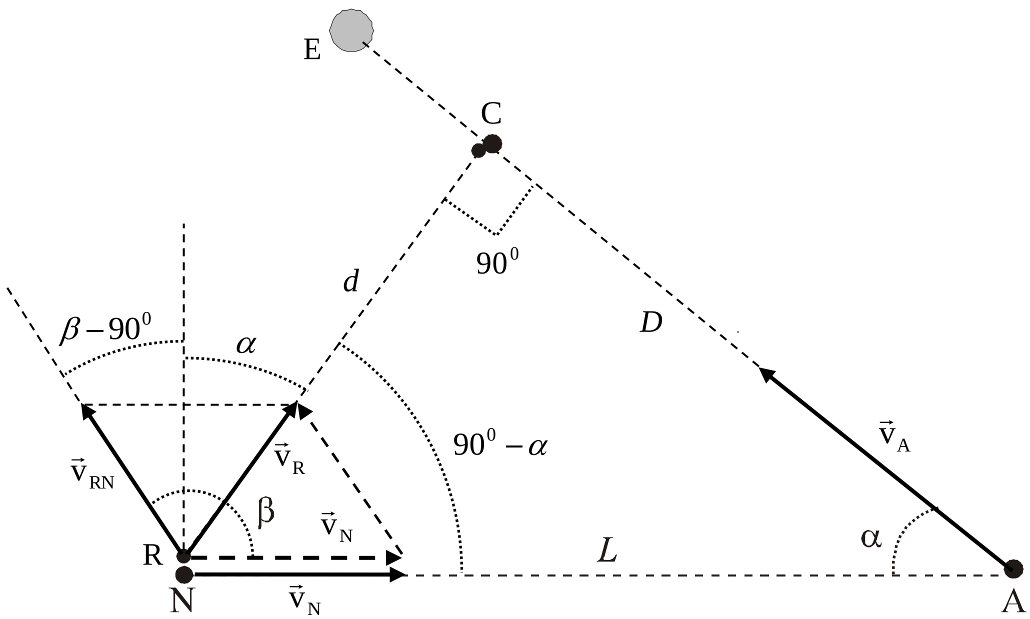

Fig. 2

$$D = v_A \cdot t_{min} = L \cdot \cos\alpha;$$
$$d = v_R \cdot t_{min} = L \cdot \sin\alpha,$$

where v_R is the absolute speed of the R rocket, from the moment of its launch;

$$t_{min} = \frac{L \cdot \cos\alpha}{v_A}; \; t_{min} = \frac{L \cdot \sin\alpha}{v_R};$$
$$\frac{L \cdot \cos\alpha}{v_A} = \frac{L \cdot \sin\alpha}{v_R}; \; \frac{\cos\alpha}{v_A} = \frac{\sin\alpha}{v_R};$$
$$v_R = v_A \cdot \tan\alpha,$$

where v_R is the absolute speed of the rocket, after launch;

$$\vec{v}_{RN} = \vec{v}_R - \vec{v}_A,$$

where $\vec{v}_{RN}$ is the relative speed of the rocket R, relative to the spacecraft N;

$$v_{RN}^2 = v_R^2 + v_A^2 - 2 \cdot v_R \cdot v_N \cdot \cos(90^0 - \alpha);$$
$$v_{RN}^2 = v_R^2 + v_A^2 - 2 \cdot v_R \cdot v_N \cdot \sin\alpha;$$
$$v_R \cdot \cos\alpha = v_{RN} \cdot \cos(\beta - 90^0),$$

where β is the angle at which the rocket R is launched from the spacecraft, N, relative to the flight direction of the spacecraft, N;

$$v_R \cdot \cos\alpha = v_{RN} \cdot \sin\beta;$$

$$\sin\beta = \frac{v_R}{v_{RN}} \cdot \cos\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$\sin\beta = \frac{v_A \cdot \tan\alpha}{v_{RN}} \cdot \cos\alpha = \frac{v_A}{v_{RN}} \cdot \frac{\sin\alpha}{\cos\alpha} \cdot \cos\alpha;$$

$$\sin\beta = \frac{v_A}{v_{RN}} \cdot \sin\alpha;$$

$$v_{RN}^2 = v_R^2 + v_A^2 - 2 \cdot v_R \cdot v_N \cdot \sin\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$v_{RN}^2 = v_A^2 \cdot \frac{\sin^2\alpha}{\cos^2\alpha} + v_A^2 - 2 \cdot v_A \cdot \frac{\sin\alpha}{\cos\alpha} \cdot v_N \cdot \sin\alpha;$$

$$v_{RN}^2 = v_A^2 \cdot \left(\frac{\sin^2\alpha}{\cos^2\alpha} + 1 \right) - 2 \cdot v_A \cdot \frac{\sin^2\alpha}{\cos\alpha} \cdot v_N;$$

$$v_{RN}^2 = v_A^2 \cdot \frac{1}{\cos^2\alpha} - 2 \cdot v_A \cdot v_N \cdot \frac{\sin^2\alpha}{\cos\alpha};$$

$$v_{RN} = v_A \cdot \sqrt{\frac{1}{\cos^2\alpha} - 2 \cdot \frac{v_N}{v_A} \cdot \frac{\sin^2\alpha}{\cos\alpha}};$$

$$v_{RN} = v_A \cdot \sqrt{\frac{1}{\cos^2\alpha} - 2 \cdot \frac{v_N}{v_A} \cdot \frac{\cos\alpha \cdot \sin^2\alpha}{\cos^2\alpha}};$$

$$v_{RN} = \frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha},$$

representing the relative speed of the rocket R, relative to the spacecraft N;

$$\sin\beta = \frac{v_R}{v_{RN}} \cdot \cos\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$v_{RN} = \frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha};$$

$$\sin\beta = \frac{v_A \cdot \frac{\sin\alpha}{\cos\alpha}}{\frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha}} \cdot \cos\alpha;$$

$$\sin\beta = \frac{\sin\alpha \cdot \cos\alpha}{\sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha}},$$

β being the angle at which the rocket R is launched from the spacecraft, N, relative to the direction of flight of the spacecraft, N, to destroy the asteroid.