Problem 1. Zodiac IOAA – J - 2022

In their apparent motions, the Moon and the other large planets in our solar system do not stray far from the plane of the ecliptic. Their apparent trajectories described on the celestial sphere remain contained in a region that extends symmetrically on both sides of the ecliptic, having a total width of about 18°. In the drawing in figure 1, in the plane of the ecliptic, the heliocentric orbit of the Earth is shown, and in the drawing in figure 2, the equivalent apparent geocentric orbit of the Sun, in relation to the Earth, is shown.

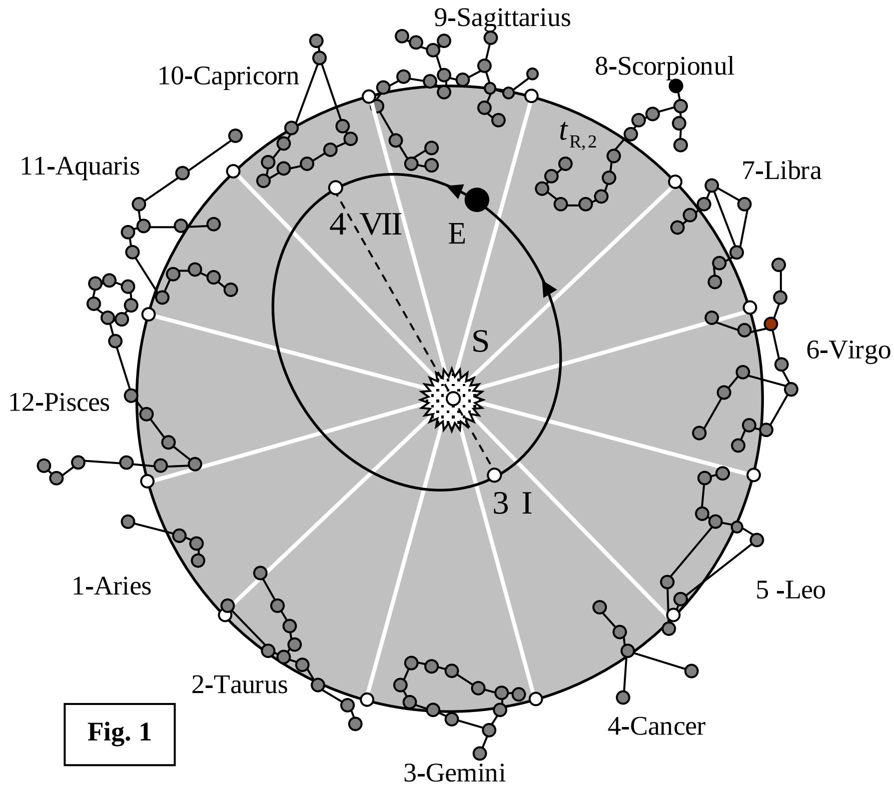

Fig. 1

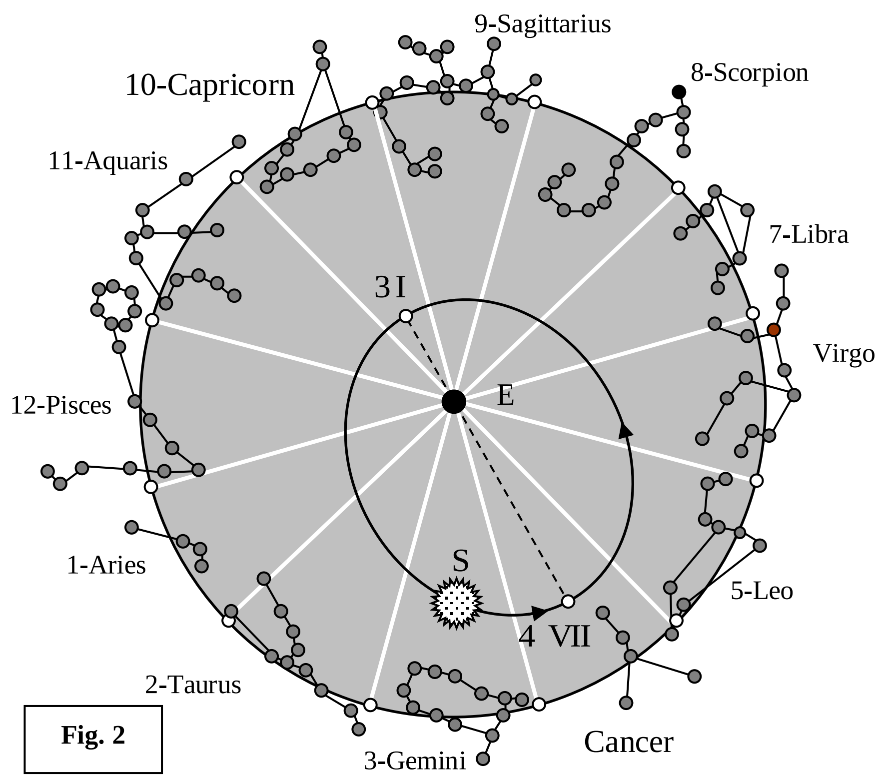

Fig. 2

The time intervals of the evolution of the true Sun in each of the 12 constellations of the Zodiac are as follows: **1)** Aries, 21 III - 20 IV; **2)** Taurus, 21 IV - 20 V; **3)** Gemini, 21 V - 20 VI; **4)** Cancer, 21 VI - 22 VII; **5)** Leo, 23 VII - 22 VIII; **6)** Virgo, 23 VIII - 22 IX; **7)** Libra, 23 IX - 22 X; **8)** Scorpio, 23 X - 21 XI; **9)** Sagittarius, 22 XI - 21 XII; **10)** Capricorn, 22 XII - 21 I; **11)** Aquarius, 22 I - 19 II; **12)** Pisces, 20 II - 21 III.

The International Olympics of Astronomy and Astrophysics for Juniors, Edition I, was to take place in SUCEAVA, in ROMANIA, on March 28 - April 3, 2020. The event, proposed by ROMANIA, was established at the International Olympics of Astronomy and Astrophysics, 13th Edition, held in Hungary, on August 2-10, 2019.

But, given the known conditions, OIAA for Juniors, Edition I, is taking place in Romania, only now, in November 2022!

a) *Identify*:

1. the sign of the OIAA development, 13th Edition, Hungary, August 2 - 10, 2019;

2. the sign when the OIAA - J was to take place - Edition I, March 28 - April 3, 2020, Romania (Edition I, proposal);

3. the sign of the OIAA - J - Edition I, November, 2022.

b) Each of the 3 specified Olympic events can be considered to have taken place on the day when the Sun, viewed from Earth, was in the middle of the angular range corresponding to each of the 3 signs.

1. *Determine* the time interval between any two of these three events:

$$\Delta t_{\mathrm{H-R,1}};\ \Delta t_{\mathrm{R,1-R,2}};\ \Delta t_{\mathrm{H-R,2}}.$$

2. *Estimate*, by direct measurements on the drawing, by means of a protractor, the value of the angle between the position vector of the Earth, in relation to the Sun, corresponding to each of the three specified moments, ie the directions of the vectors $\vec{r}_{\mathrm{H}}$, $\vec{r}_{\mathrm{R,1}}$ and respectively $\vec{r}_{\mathrm{R,2}}$, and the direction of the apse line, Aph – Ph, ie the angles α_{H}, $\alpha_{\mathrm{R,1}}$ and respectively $\alpha_{\mathrm{R,2}}$.

c) *Determine* the area of the surface described by the position vector of the center of the Earth, $\vec{r}$, in relation to the center of the Sun:

1. from position $\vec{r}_{\mathrm{Hungry}}$ to position $\vec{r}_{\mathrm{Romania,1}}$;

2. from position $\vec{r}_{\mathrm{Romania,1}}$ to position $\vec{r}_{\mathrm{Romania,2}}$;

3. from position $\vec{r}_{\mathrm{Hungry}}$ to position $\vec{r}_{\mathrm{Romania,2}}$.

d) *Determine*:

1. the distance between the Earth and the Sun, on the day when in Suceava, in 2020, the IOAA for Juniors, Edition I should have taken place, $r_{\mathrm{R,1}}$, if, for the ellipse representing the Earth's orbit around the Sun, the following are known: large semiaxis, $a = 149\,597\,500\ \mathrm{km}$ and small semiaxis, $b = 149\,580\,670\ \mathrm{km}$;

2. the acceleration of the center of the Earth, on the day when in Suceava, in 2020, the IOAA for Juniors should have taken place, $a_{\mathrm{E,1}}$, and compared with the gravitational acceleration in the gravitational field of the Sun, corresponding to the distance $r_{\mathrm{R,1}}$ from the center of the Sun, $g_{\mathrm{S,1}}$;

3. the components of the speed of the center of the Earth, $\vec{\mathrm{v}}_{\mathrm{R}}$, parallel to the major axis of the ellipse and respectively perpendicular to the major axis of the ellipse, in the days of the International Olympics in Romania, $\mathrm{v}_{//}$ and respectively $\mathrm{v}_{\perp}$.

It is known that the kinetic moment of the Earth in relation to the center of the Sun and the total mechanical energy of the Earth-Sun system are given by the expressions:

$$L = M_{\mathrm{E}} b \cdot \sqrt{\frac{K M_{\mathrm{S}}}{a}};\ E = -K\,\frac{M_{\mathrm{E}} M_{\mathrm{S}}}{2a}.$$

Given: gravitational attraction constant, $K = 6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}$; mass of the Sun, $M_{\mathrm{S}} = 1.989 \cdot 10^{30}$ kg.

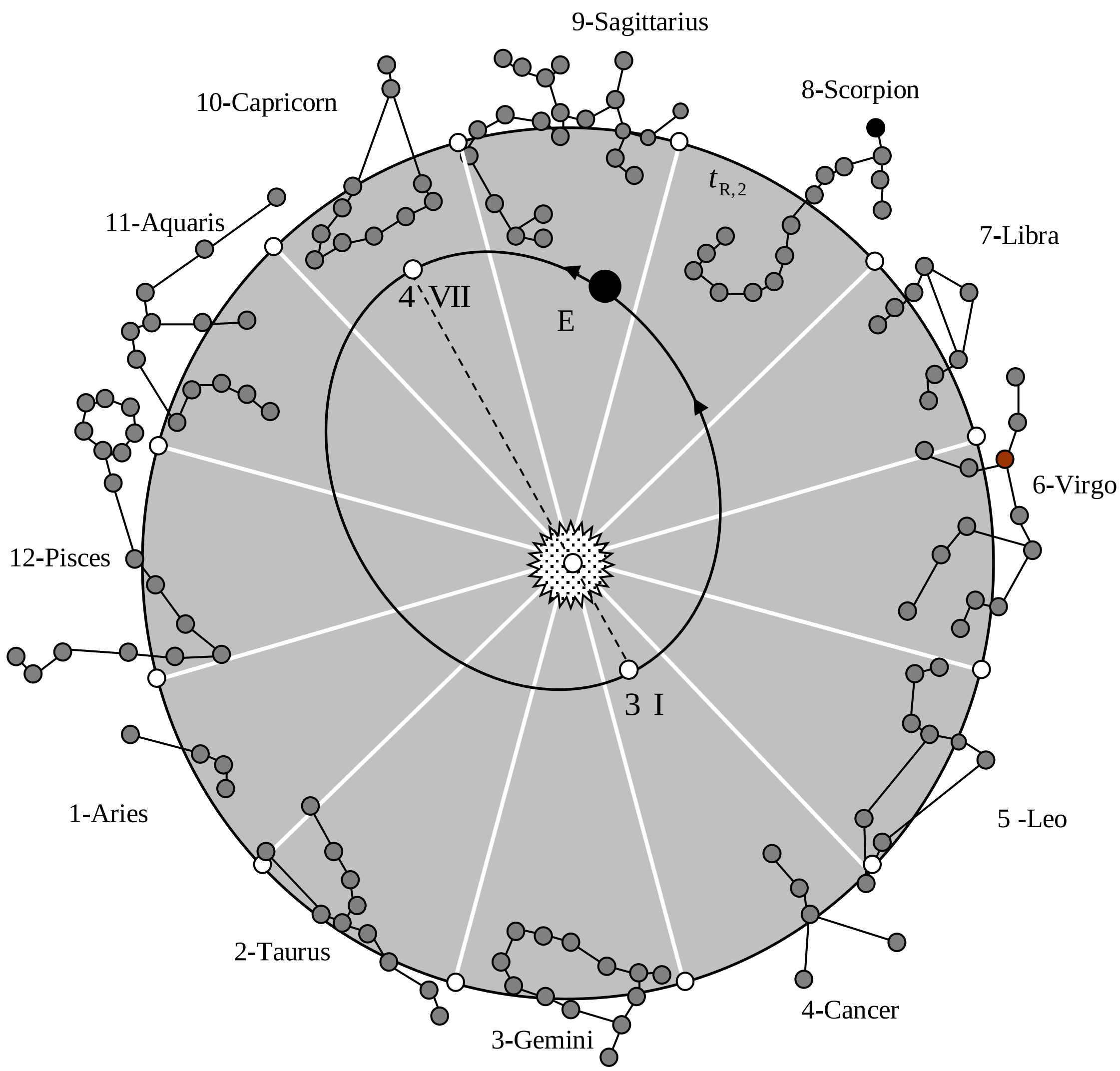

Figure 1, reproduced in enlarged format!
This image will be worked on and will be handed in with the competition sheets!

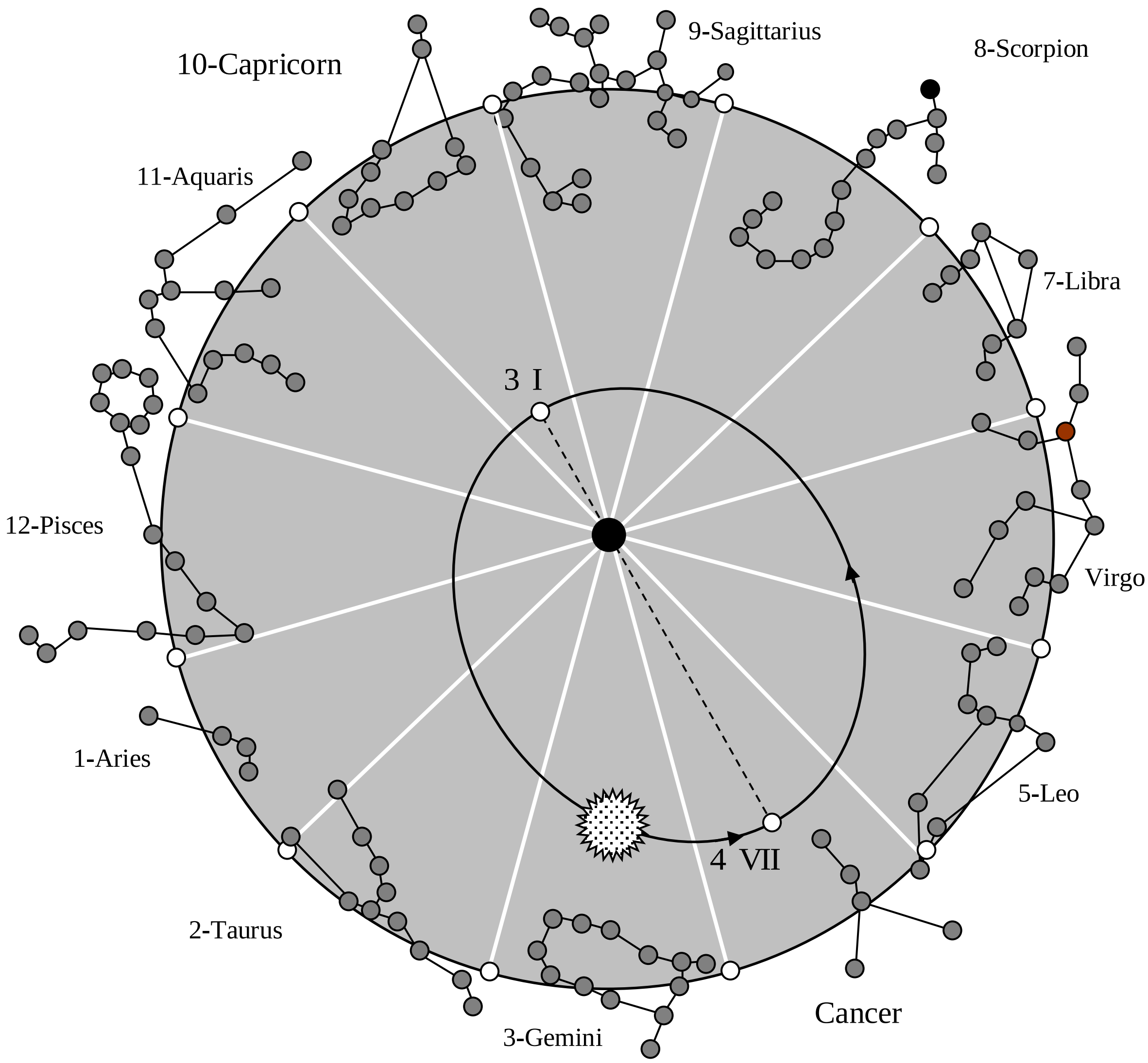

Figure 2, reproduced in enlarged format!
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Solving

a)

1) In the drawing in figure 3 are represented the positions of the Earth on its elliptical orbit around the Sun, as well as the positions of the projections of the Sun in the zodiacal belt, corresponding to the moments of the three events, specified in the statement of the problem.

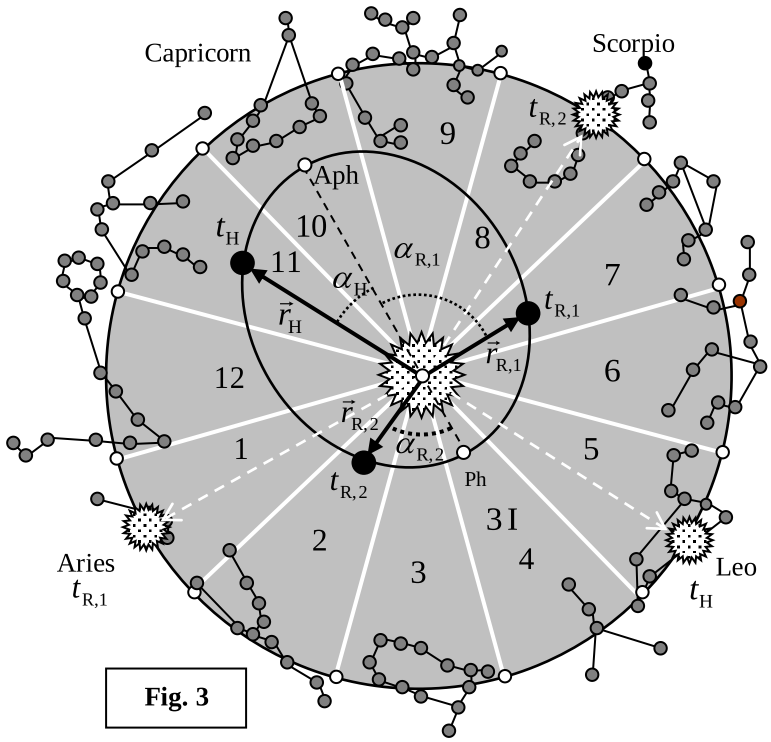

Fig. 3

In the drawing in figure 4, the positions of the Sun on its apparent elliptical orbit around the Earth are represented, as well as the positions of the projections of the Sun in the zodiac belt, corresponding to the moments of the three events, specified in the statement of the problem.

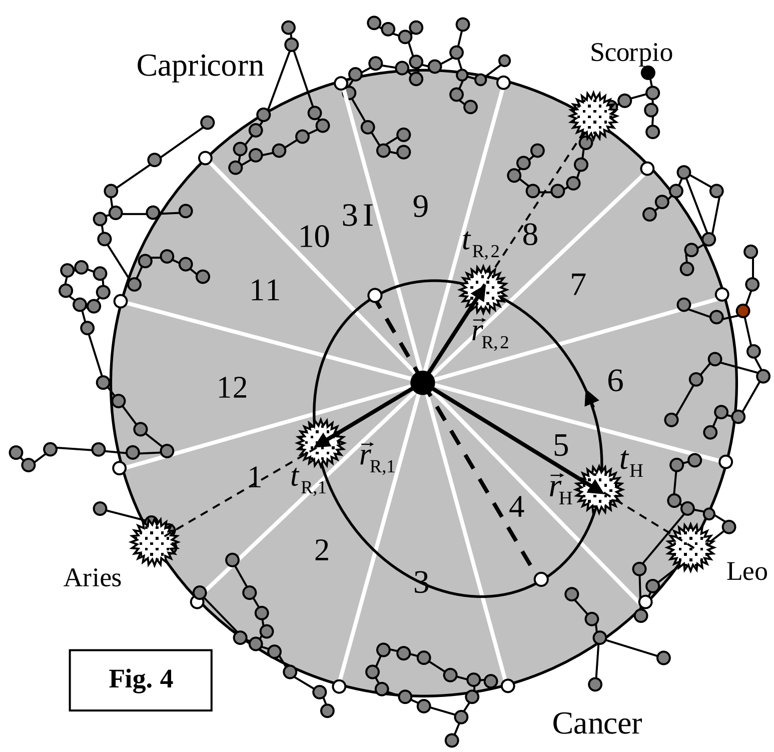

Fig. 4

In these conditions, we identify:

1. the announcement from Hungary was made in the sign of LEO, in the middle of it, on August 6, 2019;

2. the event proposed to take place in Romania was to take place in the sign of ARIES, in the middle of it, on April 4, 2020;

3. the International Olympics of Astronomy and Astrophysics, for Juniors, Edition I, takes place in Romania, in November, 2022, in the sign of SCORPIO.

b) The position vectors of the Earth, in relation to the Sun, as well as the moments corresponding to the three events, are:

1. for IOAA – 2019, Hungary: $\vec{r}_{\mathrm{H}}$; $t_{\mathrm{H}} =$ 6 August, 2019;

2. for IOAA – J – 2020, Romania - proposal: $\vec{r}_{\mathrm{R},1}$; $t_{\mathrm{R},1} =$ 4 April, 2020,

3. for IOAA – J – 2022, Romania: $\vec{r}_{\mathrm{R},2}$; $t_{\mathrm{R},2} =$

so that the time intervals between any two of these three events are:

$$\Delta t_{\mathrm{H-R,1}} = t_{\mathrm{R},1} - t_{\mathrm{H}} = 223 \text{ days};$$

$$\Delta t_{\mathrm{R,1-R,2}} = t_{\mathrm{R},2} - t_{\mathrm{R},1} = \text{ days};$$

$$\Delta t_{\mathrm{H-R,2}} = t_{\mathrm{R},2} - t_{\mathrm{H}} = \Delta t_{\mathrm{H-R,1}} + \Delta t_{\mathrm{R,1-R,2}} = 223 \text{ days} + 886 \text{ days} = 1109 \text{ days}.$$

From direct measurements, performed on the drawing in Figure 3, it results:

$$\alpha_{\mathrm{H}} \approx 30^0;$$

$$\alpha_{\mathrm{R},1} \approx 90^0;\ \vec{r}_{\mathrm{R},1} \perp (\mathrm{Aph-Ph});$$

$$\alpha_{\mathrm{R},2} \approx 60^0.$$

c)

1. It is known that in the evolution of the Earth in the elliptical orbit around the Sun, according to Keppler's third law, the areolar velocity of the Earth is constant, so that:

$$\Omega = \frac{\Delta A}{\Delta t} = \text{constant};$$

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\mathrm{H-R,1}}}{\Delta t_{\mathrm{H-R,1}}};$$

$$\Delta A_{\mathrm{H-R,1}} = \frac{\Delta t_{\mathrm{H-R,1}}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\mathrm{H-R,1}}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta t_{\mathrm{H-R,1}} = 223 \text{ days};\ T = 365.256 \text{ days};$$

$$a = 149\,597\,500 \text{ km};\ b = 149\,580\,670 \text{ km};$$

$$\Delta A_{\mathrm{H-R,1}} = \frac{223 \text{ days}}{365.256 \text{ days}} \cdot 3.14 \cdot 149\,597\,500 \text{ km} \cdot 149\,580\,670 \text{ km};$$

$$\Delta A_{\mathrm{H-R,1}} = \frac{223}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670 \text{ km}^2;$$

$$\Delta A_{\mathrm{H-R,1}} = 4.2897 \cdot 10^{16} \text{ km}^2.$$

2.

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{R},1-\text{R},2}}{\Delta t_{\text{R},1-\text{R},2}};$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{\Delta t_{\text{R},1-\text{R},2}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\text{R},1-\text{R},2}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{886\ \text{days}}{365.256\ \text{days}} \cdot 3.14 \cdot 149\,597\,500\ \text{km} \cdot 149\,580\,670\ \text{km};$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{886}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670\ \text{km}^2;$$

$$\Delta A_{\text{R},1-\text{R},2} = 1.704 \cdot 10^{17}\ \text{km}^2;$$

3.

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{H}-\text{R},2}}{\Delta t_{\text{H}-\text{R},2}}; \quad \frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{H}-\text{R},1}}{\Delta t_{\text{H}-\text{R},1}};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{\Delta t_{\text{H}-\text{R},2}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\text{H}-\text{R},2}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta t_{\text{H}-\text{R},2} = 1109\ \text{days}; \ T = 365.256\ \text{days};$$

$$a = 149\,597\,500\ \text{km}; \quad b = 149\,580\,670\ \text{km};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{1109\ \text{days}}{365.256\ \text{days}} \cdot 3.14 \cdot 149\,597\,500\ \text{km} \cdot 149\,580\,670\ \text{km};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{1109}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670\ \text{km}^2;$$

$$\Delta A_{\text{H}-\text{R},2} = 2.133 \cdot 10^{17}\ \text{km}^2.$$

d)

1. According to the notations in figure 5, representing the position of the Earth, in relation to the Sun, on the day of, when, as we have shown, $\vec{r}_{\text{R},1} \perp (\text{Aph} - \text{Ph})$, it results:

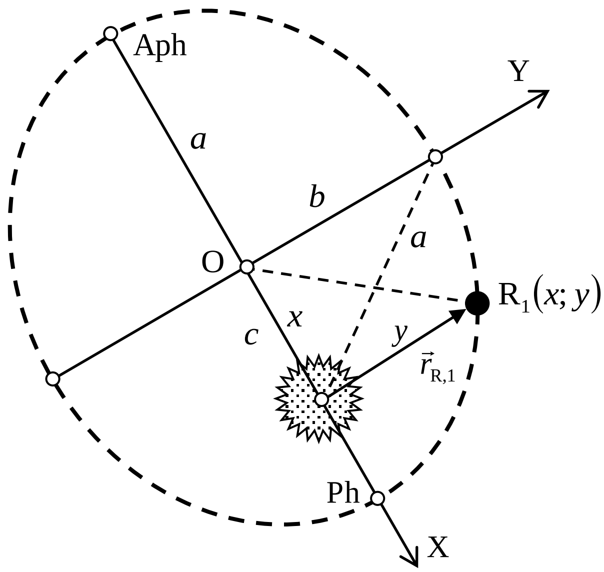

Fig. 5

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1; \ \ x=c; \ y=r_{R,1};$$

$$c^2=a^2-b^2;$$

$$\frac{c^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1; \ \frac{a^2-b^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1;$$

$$1-\frac{b^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1; \ \frac{r_{R,1}^2}{b^2}=\frac{b^2}{a^2};$$

$$r_{R,1}^2=\frac{b^4}{a^2};$$

$$r_{R,1}=\frac{b^2}{a},$$

representing the distance between the center of the Earth and the center of the Sun on the day when the International Olympiad in Astronomy and Astrophysics should have taken place, for Juniorim Edition I, on March 28 - April 3, 2020, in Romania;

$$a=149\,597\,500 \ \text{km}; \ \ b=149\,580\,670 \ \text{km};$$

$$r_{R,1}=149\,563\,841.9 \ \text{km}.$$

2.

$$K\frac{M_E M_S}{r_{R,1}^2}=M_E a_{E,1}; \ a_{E,1}=K\frac{M_S}{r_{R,1}^2};$$

$$K=6.67\cdot10^{-11}\ \text{Nm}^2\text{kg}^{-2}; \ M_S=1.989\cdot10^{30}\ \text{kg};$$

$$a_{E,1}=\frac{6.67\cdot10^{-11}\cdot1.989\cdot10^{30}}{(149\,563\,841.9)^2\cdot10^6}\ \frac{\text{m}}{\text{s}^2};$$

$$a_{E,1}\approx 0.006\,\frac{\text{m}}{\text{s}^2}.$$

$$g_{S,1} = K \cdot \frac{M_S}{r_{R,1}^2} = a_{E,1}.$$

3. Corresponding to the position R_1 of the Earth, represented in the drawing in Figure 6, when its kinetic moment is:

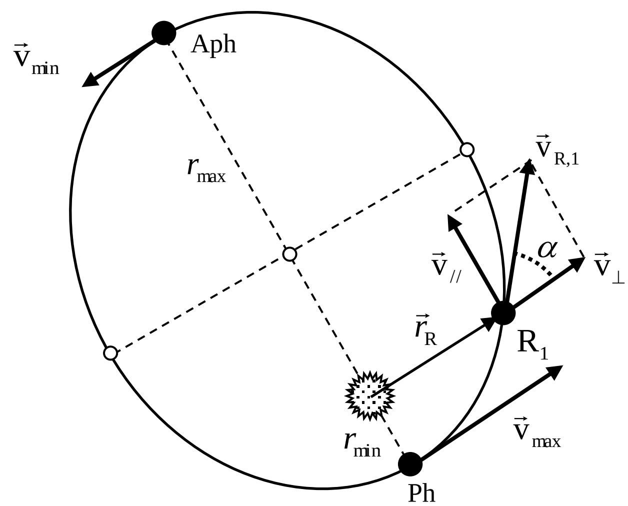

Fig. 6

$$\vec{L}_{R,1} = \vec{r}_{R,1} \text{ x } M_E\, \vec{v}_{R,1};\; L_{R,1} = r_{R,1} M_E\, v_{R,1} \cdot \sin\left(\vec{r}_{R,1};\vec{v}_{R,1}\right) = r_{R,1} M_E\, v_{R,1} \cdot \sin\alpha,$$

according to the law of conservation of kinetic momentum, it follows:

$$v_{R,1} \cdot \sin\alpha = v_{//};$$

$$L_{R,1} = r_{R,1} M_E\, v_{//};\; L = M_E b \cdot \sqrt{\frac{KM_S}{a}};$$

$$M_E b \cdot \sqrt{\frac{KM_S}{a}} = r_{R,1} M_E\, v_{//};\; r_R = \frac{b^2}{a};$$

$$v_{//} = \frac{a}{b} \cdot \sqrt{\frac{KM_S}{a}};$$

$$K = 6.67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};\; M_S = 1.989 \cdot 10^{30}\ \text{kg};$$

$$a = 149\,597\,500\ \text{km};\;\; b = 149\,580\,670\ \text{km};$$

$$v_{//} = \frac{149\,597\,500\ \text{km}}{149\,580\,670\ \text{km}} \cdot \sqrt{\frac{6.67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2} \cdot 1.989 \cdot 10^{30}\ \text{kg}}{149\,597\,500 \cdot 10^3\ \text{m}}};$$

$$v_{//} = \frac{149\,597\,500}{149\,580\,670} \cdot \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 1.989 \cdot 10^{30}}{149\,597\,500 \cdot 10^3}}\ \frac{\text{m}}{\text{s}};$$

$$v_{//} = \frac{149\,597\,500}{149\,580\,670} \cdot \sqrt{\frac{6.67 \cdot 1.989}{149\,597\,500}} \cdot 10^8\ \frac{\text{m}}{\text{s}};$$

$$v_{//} = \frac{\sqrt{149\,597\,500 \cdot 6.67 \cdot 1.989}}{149\,580\,670} \cdot 10^8\ \frac{\text{m}}{\text{s}};$$

$$\mathrm{v}_{//} = 29\,782.9\,\frac{\mathrm{m}}{\mathrm{s}};\ \mathrm{v}_{//} = 29\,782.9\cdot 10^{-3}\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{//} = 29.7829\,\frac{\mathrm{km}}{\mathrm{s}}.$$

According to the law of conservation of mechanical energy it results:

$$E = -K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{2a};$$

$$E_{\mathrm{R,1}} = \frac{M_{\mathrm{E}}\,\mathrm{v}_{\mathrm{R}}^2}{2} - K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{r_{\mathrm{R,1}}};$$

$$\frac{M_{\mathrm{E}}\,\mathrm{v}_{\mathrm{R,1}}^2}{2} - K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{r_{\mathrm{R,1}}} = -K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{2a};$$

$$\frac{\mathrm{v}_{\mathrm{R,1}}^2}{2} - K\frac{M_{\mathrm{S}}}{r_{\mathrm{R,1}}} = -K\frac{M_{\mathrm{S}}}{2a};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{KM_{\mathrm{S}}\left(\frac{2}{r_{\mathrm{R,1}}} - \frac{1}{a}\right)} = \sqrt{KM_{\mathrm{S}}\cdot\frac{2a - r_{\mathrm{R,1}}}{r_{\mathrm{R,1}}a}};$$

$$K = 6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}};\ M_{\mathrm{S}} = 1.989\cdot 10^{30}\ \mathrm{kg};\ r_{\mathrm{R,1}} = 149\,563\,841{,}9\ \mathrm{km};\ a = 149\,597\,500\ \mathrm{km};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}\cdot 1.989\cdot 10^{30}\ \mathrm{kg}\cdot\frac{2\cdot 149\,597\,500\ \mathrm{km} - 149\,563\,841.9\ \mathrm{km}}{149\,563\,841.9\ \mathrm{km}\cdot 149\,597\,500\ \mathrm{km}}};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}\cdot 1.989\cdot 10^{30}\ \mathrm{kg}\cdot\frac{2\cdot 149\,597\,500 - 149\,563\,841.9}{149\,563\,841.9\cdot 149\,597\,500\cdot 10^3\ \mathrm{m}}};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\cdot 1.989\cdot 10^{30}\cdot\frac{2\cdot 149\,597\,500 - 149\,563\,841.9}{149\,563\,841.9\cdot 149\,597\,500\cdot 10^3}\cdot 10^8}\ \frac{\mathrm{m}}{\mathrm{s}};$$

$$\mathrm{v}_{\mathrm{R,1}} = 29\,786.25\,\frac{\mathrm{m}}{\mathrm{s}} = 29\,786.25\cdot 10^{-3}\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{\mathrm{R,1}} = 29.7862\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{//} = 29.7829\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{\perp} = \sqrt{\mathrm{v}_{\mathrm{R,1}}^2 - \mathrm{v}_{//}^2} = 0.4433\,\frac{\mathrm{km}}{\mathrm{s}}.$$