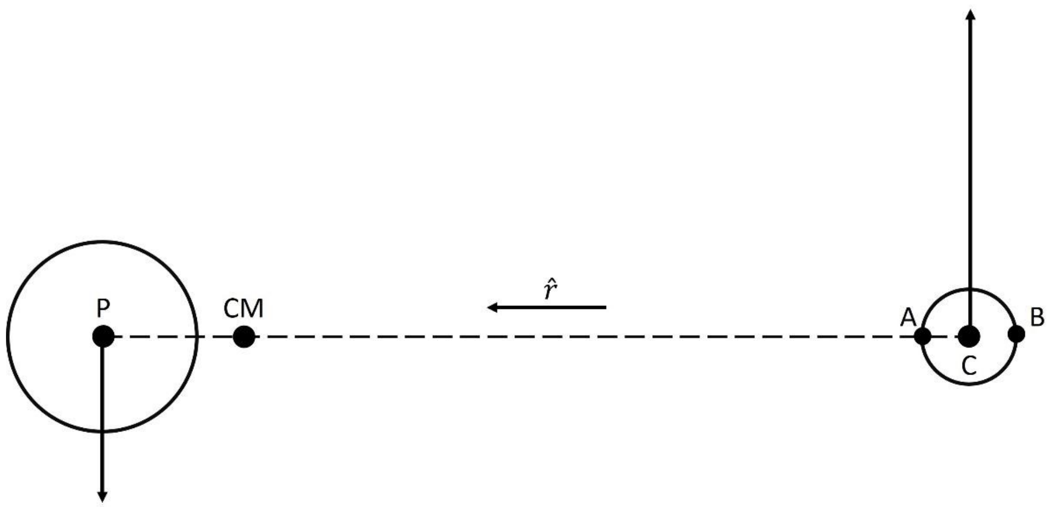
SOLUTION

TQ9 [15 points]



Let P and C be the centers of Pluto and Charon respectively, and CM their center of mass. Clearly $\overline{CM - P} = \frac{1}{9}R$ and $\overline{CM - C} = \frac{8}{9}R$. The effective gravitational field at points A and B are given by:

[By recognizing the vectorial nature of gravities/forces/fields] [4 points]

$$g_A = \frac{GM_P}{(R-r)^2}\hat{r} - \frac{GM_C}{r^2}\hat{r} - \omega^2\left(\tfrac{8}{9}R - r\right)\hat{r}$$ [1 point]

$$g_B = \frac{GM_P}{(R+r)^2}\hat{r} + \frac{GM_C}{r^2}\hat{r} - \omega^2\left(\tfrac{8}{9}R + r\right)\hat{r}$$ [1 point]

where $\omega^2 = 9\frac{GM_C}{R^3}$ is the angular speed common to both Pluto and Charon. Thus, the gravitational accelerations at these points are:

[by finding angular speed] [2 points]

$$g_A = \frac{GM_P}{(R-r)^2} - \frac{GM_C}{r^2} - \frac{GM_C}{R^2}\left(8 - 9\tfrac{r}{R}\right)$$

$$g_B = \frac{GM_P}{(R+r)^2} + \frac{GM_C}{r^2} - \frac{GM_C}{R^2}\left(8 + 9\tfrac{r}{R}\right)$$

Factorizing the expression to obtain formulas proportional to Charon’s mass [1 point]:

$$g_A = -\frac{GM_C}{r^2}\left[1 - \frac{M_P}{M_C}\left(\frac{r}{R}\right)^2 \frac{1}{\left(1-\frac{r}{R}\right)^2} + \left(\frac{r}{R}\right)^2\left(8 - 9\frac{r}{R}\right)\right]$$

$$g_B = \frac{GM_C}{r^2}\left[1 + \frac{M_P}{M_C}\left(\frac{r}{R}\right)^2 \frac{1}{\left(1+\frac{r}{R}\right)^2} - \left(\frac{r}{R}\right)^2\left(8 + 9\frac{r}{R}\right)\right]$$

Replacing the numerical value of the ratio of the two masses:

$$g_A = -\frac{GM_C}{r^2}\left[1 - \left(\frac{r}{R}\right)^2 \frac{8}{\left(1-\frac{r}{R}\right)^2} + \left(\frac{r}{R}\right)^2\left(8 - 9\frac{r}{R}\right)\right]$$

$$g_B = \frac{GM_C}{r^2}\left[1 + \left(\frac{r}{R}\right)^2 \frac{8}{\left(1+\frac{r}{R}\right)^2} - \left(\frac{r}{R}\right)^2\left(8 + 9\frac{r}{R}\right)\right]$$

Noting that the expressions in square brackets are dimensionless, it is helpful to replace the given values of r and R, resulting [2 points by expressions of g_A and g_B]:

$$g_A = -\frac{GM_C}{r^2}\,(0,99929)$$

$$g_B = \frac{GM_C}{r^2}\,(0,99933)$$

The percentage difference with respect to Charon’s normal gravity $g_0 = \frac{GM_C}{r^2}$ is

$$\frac{\left||g_A| - |g_B|\right|}{g_0} \cdot 100\% = 0,004\% = 4\times10^{-3}\%$$ [1 point]