SOLUTION

TQ4 [10 points]

4. 1 To get the number of photons per second we have to multiply the Flux by the area of the dish, to know the incoming energy per second, and divide this by the energy of a photon.

For $\lambda_1 = 0.32\ mm = 3.2\times10^{-4} m$:

Energy of a photon: $E = \frac{hc}{\lambda} = 6.2076 \times 10^{-22} Jules$ [1 point]

Area of the disk: $A = \pi R^2 = 113\ m^2$

Number of photons per second:

$$n_1 = \frac{flux\times A}{E} \approx 1820\ photon/s$$ [1 point]

4.2 Same calculation, just changing the energy of the individual photons:

For $\lambda_2 = 8.6\ mm = 8.6 \times 10^{-3} m$

Energy of a photon: $E = \frac{hc}{\lambda} = 2.31 \times 10^{-23} Jules$ [1 point]

Number of photons per second:

$$n_2 = \frac{flux\times A}{E} \approx 48900\ photon/s$$ [1 point]

4.3 Spatial resolution of a single telescope is given by:

$$\theta = 1.22\frac{\lambda}{D}$$

where D represents the diameter of the dish. This value will be given in radians, so it must be converted to arcsec afterwards.

For a frequency of 74.9 GHz, the corresponding wavelength is:

$$\lambda = \frac{c}{f} = 4\ mm$$ [1 point]

And the spatial resolution:

$$\theta = 1.22\frac{4mm}{12\ m} \approx 83.9\ arcsec$$ [1 point]

4.4 For an array the correct expression is:

$$\theta = \frac{\lambda}{B} \quad \text{or} \quad \theta = \frac{\lambda}{2B}$$ [1 point]

being B the longest baseline in the array.

So in this case:

$$\theta = \frac{4\,mm}{16\,km} \approx 0.0516\ arcsec \quad \text{or} \quad 0.0258\ arcsec$$ [1 point]

4.5 The SEFD is a characteristic flux of a system, found by dividing the characteristic energy associated to the so-called temperature of the antenna by its effective area:

$$SEFD = \frac{2kT_{sys}}{A_e}$$ [1 point]

As no additional information is given about the effective area, the actual physical area of the array should be used:

$$A = 54\left(\pi\times6^2\right) + 12\left(\pi\times3.5^2\right) \approx 6569\ m^2$$ [0.5 point]

Substituting the Boltzmann constant, the given temperatura of the antenna, and converting the answer to Jansky we get:

$$SEFD \approx 290.5\ Jy$$ [0.5 point]

NOTES:

Question	We indicate the answer must be:	Tolerance
4.a	Approximated to the nearest integer	+/- 10 photons
4.b	Approximated to the nearest integer	+/- 10 photons
4.c	2 digit of precision	[83.0 , 85.0]
4.d	2 digits of precision	0.05 exact
4.e	2 digit of precision	[289.0 , 291.0]