SOLUTION

TQ12 [15 points]

12.1 For a satellite that describes a uniform circular motion of radius r around the star and mass M, we have:

$$v = \left[R_{Hodograph}\right] = \sqrt{\frac{GM}{r}} \quad \text{[1 point]}$$

12.2

$$\vec{a} = -\frac{Gm}{r^2}\hat{r} \quad \text{[1 point]}$$

$$L = mr^2\omega \quad \text{[1 point]}$$

$$\rightarrow \vec{a} = -\frac{GMm\omega}{L}\hat{r} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\frac{GMm}{L}\frac{\Delta\theta}{\Delta t} = \left|\frac{\Delta v}{\Delta t}\right|$$

$$\Delta v = \pm\frac{GMm}{L}\Delta\theta$$

$$k = \frac{GMm}{L} \quad \text{[2 points]}$$

12.3

$$L = mv_c R \quad \text{[1 point]}$$

$$k_c = \frac{GMm}{L} = v_c = \sqrt{\frac{GM}{R}} \quad \text{[1 point]}$$

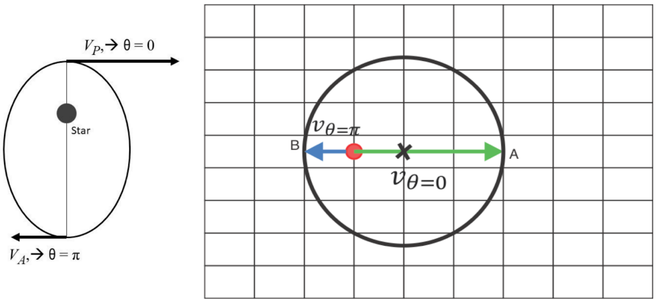
12.4 In the following scheme, the hodographic circumference has been constructed as follows:

- For $\theta = 0$, the velocity vector is drawn in arbitrary units (3, for instance) corresponding to the velocity in the periastron, v_P. Its head determines the point A.
- For $\theta = \pi$, the velocity vector is drawn diametrically opposite in the apastron, v_A .also in arbitrary units. Its head determines point B. The hodograph must pass through points A and B. Therefore, the radius of the hodograph must be equal to:

  $$R_{Hodograph} = \frac{v_P + v_A}{2} \quad \text{[1 point]}$$

  And the "distance" *d* from the star to the center of the circumference will be:

  $$d = \frac{v_P - v_A}{2} \quad \text{[1 point]}$$



[2 points]

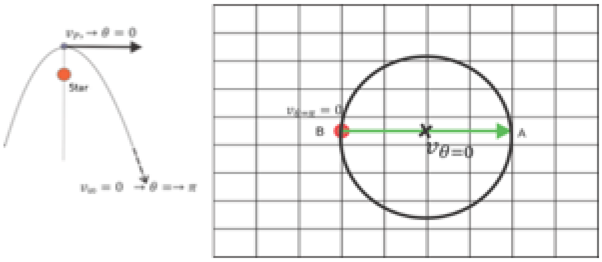
12.5 In the following scheme, the hodographic circumference has been constructed as follows:

- For $\theta = 0$, the velocity vector is drawn in arbitrary units (4, for instance) corresponding to the velocity in the periastron, that is v_P. Its head determines the point A. Then the velocity vector is drawn diametrically opposite in the apastron, that is, $v_A = 0$. Its head determines point B that coincides with the star. The hodograph must pass through points A and B. Therefore, the radius of the hodograph must be equal to:

$$R_{Hodograph} = \frac{v_P}{2}$$ [1 point]

And the distance d from the star to the center of the hodographic circumference

$$d = \frac{v_P}{2}$$ [1 point]



[2 points]