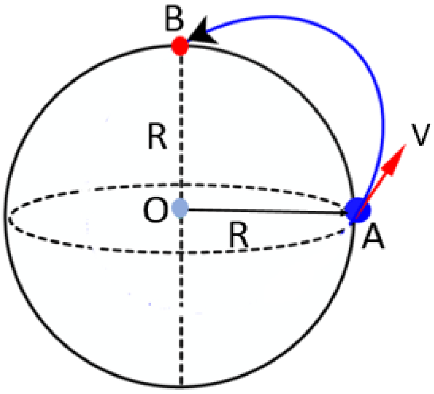
SOLUTION

TQ11 [15 points]

11.1



[1 point]

$$EM = -\frac{GMm}{R} + \frac{1}{2}mv^2$$ [1 point]

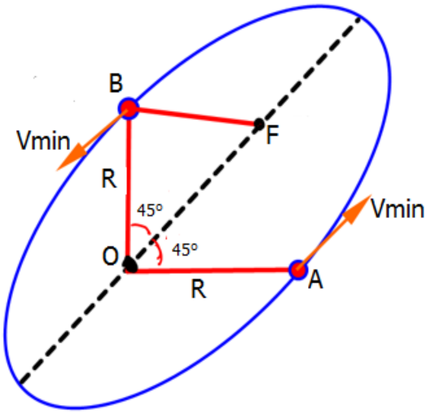
At points A and B, the v is the same; so, if v is minimum, EM is minimum:

We know that the mechanical energy for an elliptical orbit is given by:

$$EM = -\frac{GMm}{2a}$$ [1 point]

where 2a is the length of the major axis.

Since EM must be minimal, then 2a must be minimal [1 point]



[1 point]

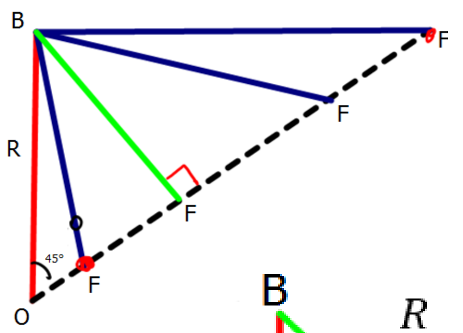
O: Center of the Earth and one of the focus of the ellipse.

F: the other focus of the ellipse

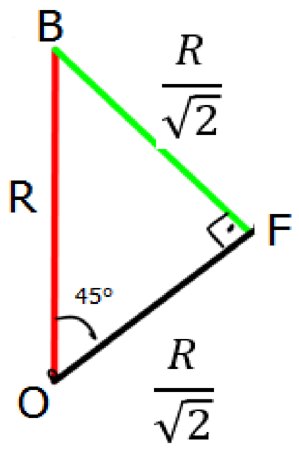
$$OB + BF = 2a$$ [1 point]

$OB + BF = 2a$, must be minimal.

F takes an arbitrary position, so to minimize, BF must be perpendicular to OF:



[2 points]



[1 point]

$$OB + BF = R + \frac{R}{\sqrt{2}}$$ [1 point]

$$2a = R + \frac{R}{\sqrt{2}}$$

This is the minimum value it can take

Correspondingly, the expression for the minimum velocity is

$$v = \sqrt{\frac{2GM}{(1+\sqrt{2})R}}$$ [1 point]

Using the M and R values for the Earth, the minimum velocity is

$$v = 7195\ \frac{m}{s}$$ [1 point]

11.2 Furthermore, $OF = 2\, e \cdot a$, where e is the eccentricity

$$\frac{R}{\sqrt{2}} = 2\, e\, \frac{R}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$$ [2 points]

$$e = \frac{1}{1+\sqrt{2}} = 0.414$$ [1 point]