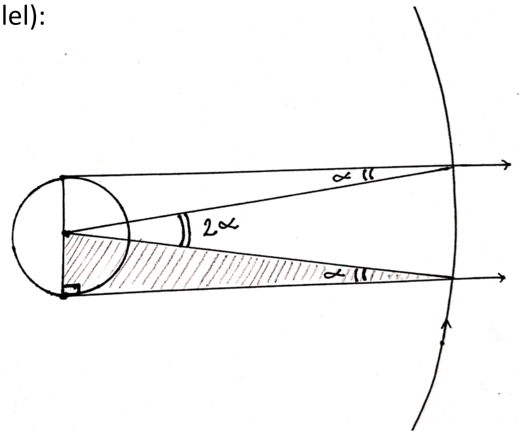
SOLUTION

TQ 10 [15 points]

10.1 The maximum duration occurs if Earth passes exactly along the diameter of the Sun. Now consider the following figure, where Sun rays are depicted as traveling to the right towards the hypothetical distant observer (reason why they can be considered parallel):



The arc along the circumference associated to the transit, 2α, can be easily found looking at the shaded triangle:

$$\alpha = sin^{-1}(\frac{R_{\odot}}{R_{orb}})$$ [2 points]

being R_{orb} the orbital radius of the Earth. Now the time of the transit can be found by considering the angular velocity of the Earth, for instance by means of the following proportionality relations:

$$\frac{t_{tr}}{T_{orb}} = \frac{2\alpha}{2\pi} = \frac{sin^{-1}(\frac{R_{\odot}}{R_{orb}})}{\pi}$$ [2 points]

Substituting with the known values for these quantities we get, i.e.,

$$t_{tr} \sim 12.97\,h = 12h\,58m$$ [1 point]

10.2 First of all it must be said that the minimum orbital period is obtained if we assume that the transit occurs along the diameter of the star. In other cases, the planet would be crossing a shorter path in front of the star during the same time, meaning that it would have a smaller angular velocity and therefore a longer period.

That said, it means that we can resort to the same expression of 10.1, yet this time we need 2 additional elements:

- Using the small angle approximation:

$$sin(\alpha) \sim \alpha$$ [1 point]

- Invoking the full expression for the orbital period:

$$T_{orb} = \frac{2\pi}{\sqrt{GM_*}} R_{orb}^{3/2}$$ [1 point]

Combining these results, we get:

$$\frac{t_{tr}}{T_{orb}} = \frac{t_{tr}}{\frac{2\pi}{\sqrt{GM_*}} R_{orb}^{3/2}} = \frac{\frac{R_*}{R_{orb}}}{\pi}$$ [2 points]

And solving for R_{orb}:

$$R_{orb} = \frac{GM_* t_{tr}^2}{2^2 R_*^2}$$ [2 points]

Finally putting this last result back into the expression for the orbital period:

$$T = \frac{\pi G M_*}{4R^3} t_{tr}^3$$ [2 points]

At this point, just a numerical evaluation is needed, though a more elegant solution can be achieved by means of scaling relations by noting that this very same expression must be true in the case of the Earth-Sun system as seen from far away. Having that 31 minutes is the 4% of 12.94h:

$$T = 365.25 \cdot \frac{0.1}{0.1^3} \cdot 0.04^3 \sim 2.3\ días \sim 2d\ 7h$$ [2 points]

These values were inspired by the planetary system Trappist 1.