SHORT PROBLEMS

1. Several exoplanets have been observed in the Gliese 876 system $(\mathrm{M_G} = 0.33 \pm 0.03\mathrm{M_\odot})$ as given in the following table,

Gliese System	Mass	Semi Major Axis (AU)
Gliese 876 b	2.276 M_J	0.2083
Gliese 876 c	0.714 M_J	0.1296
Gliese 876 d	6.8 $M_\oplus$	0.0208
Gliese 876 e	15 $M_\oplus$	0.334

where $\mathrm{M_\odot}$ is mass of Sun, $\mathrm{M_J}$ is mass of Jupiter ($M_J = 1.89813 \times 10^{27}\ kg$), and $\mathrm{M_\oplus}$ is mass of Earth. Assume that all these planets revolve around Gliese 876 in the same direction. Two planets are said to be in resonant orbits if the synodic period of one planet with respect to the other planet is an integer multiple of the orbital period of the second planet.
Find if any of the exoplanets of Gliese 876 system may have resonant orbits.

2. One satellite of a planet has an orbital period of 7 days, 3 hours, 43 minutes, and the semi major axis is 15.3 times the mean radius of the planet. The Moon has an orbital period of 27 days, 7 hours, 43 minutes and the semi major axis is 60.3 times the Earth's mean radius. Assume that the mass of the moon and the satellite is negligible compared to the mass of the planet. Calculate the ratio of the planet's mean density to that of the Earth.

3. On 27 May 2015 at 02:18:49, the occultation of the star HIP 89931 (δ −Sgr) by the asteroid 1285 Julietta was observed from Borobudur temple, which was located at the center of the asteroid shadow path. It lasted for only 6.201 s. Assume that Earth's orbit is circular and the orbit of Julietta is on the ecliptic plane and revolves in the same direction as Earth. At the occultation, Julietta is near its aphelion. At the time of the occultation the distances of Julietta from the Sun and the Earth are 3.076 AU and 2.156 AU respectively. Find the approximate diameter of asteroid Julietta, if the semi major axis of Julietta is $a = 2.9914$ AU.

4. Let us assume that an observer using a hypothetical, far-infrared, Earth-sized telescope (wavelength range 20 to 640 μm) found a static and neutral supermassive black hole with a mass of 2.1 x 10^{10} $M_\odot$. Determine the maximum distance at which this black hole can be resolved by the observer.

5. An observer is trying to determine an approximate value of the orbital eccentricity of a man-made satellite. When the satellite was at apogee, it was observed to have moved by $\Delta\theta_1 = 2'44''$ in a short time. When the radius vector connecting Earth and the satellite is perpendicular to the major axis (true anomaly is equal to 90°), within the same duration of time, it was observed to have moved by $\Delta\theta_2 = 21'17''$. Assume that the observer is located at the center of the Earth. Find an approximate value of the eccentricity of the satellite's orbit.

6. At the start of every observation, a radio telescope is pointed at a point-source calibrator that has a known flux density of 21.86 Jy outside the Earth's atmosphere. However, on a certain date, the measured flux density of the calibrator source was 14.27 Jy. If the calibrator source was at an altitude of 35 degrees, estimate the zenith atmospheric optical depth, τ_z.

7. A galaxy at the boundary of a galaxy cluster of radius 10 Mpc is expected to escape from the cluster if it has an initial velocity of at least 700 km/s relative to the center of the cluster. Calculate the density of the cluster.

8. A strong continuum radio signal from a celestial body has been observed as a burst with a very short duration of 700 μs. The observed flux density at a frequency of 1660 MHz is measured to be 0.35 kJy. If the distance from the source is known to be 2.3 kpc, estimate the brightness temperature of this source.

9. Assume that the Sun is a perfect blackbody. Venus is also assumed to be a blackbody, with temperature T_V, and it is in thermal equillibrium (i.e. it is radiating about as much energy as it receives from the Sun) at its orbital distance of 0.72 AU. Suppose that at closest approach to Earth, Venus has an angular diameter of about 66 arcsec. What is the flux density of Venus at the closest approach to Earth as observed by a radio telescope at an observing frequency of 5 GHz?

10. A molecular hydrogen cloud is known to have a temperature T = 115 K. The hydrogen atoms (assumed spherical) have (covalent) radius $r_H = 0.37\ x\ 10^{-10} m$ and the separation centre-to-centre distance between the two atoms is $d_{H_2} = 0.74\ x\ 10^{-10} m$. Assume that the molecules are in thermal equilibrium. Estimate the frequency at which they will radiate due to molecular rotational excitation.

11. The mass density of an object is inversely proportional to the radial distance from the center of the object with a factor of proportionality $\alpha = 5.0 \times 10^{13}\ \ kg/m^2$. If the escape velocity at the surface of the object is $v_0 = 1.5 \times 10^4\ m/s$, calculate the total mass of the object.

12. A proton with a kinetic energy of $1 GeV$ propagates out from the surface of the Sun towards the Earth. Neglecting the magnetic field of the Sun, calculate the travel time of the proton as seen from the Earth.

13. Volcanic activity on Io, whose rotation period synchronizes with its orbital period, was proposed to be the result of tidal heating mainly from Jupiter. The resultant tidal force on a body is the difference in gravitational force experienced by the near and far sides of that body due to another body. Measurements of the surface distortion of Io via satellite radar altimeter mapping indicate that the surface rises and falls by up to 100m during one-half orbit. Only the surface layers will move by this amount. Interior layers within Io will move by a smaller amount, and thus we assume that on average the entire mass of Io is moved through 50m. Assume that Io is considered as two hemispheres each treated as a point mass. Calculate the *average power* of the tidal heating on Io.

Hint: you can use the following approximation $(1+x)^n \approx 1+nx$ for small *x*.

The mass of Io is $m_{Io} = 8.931938 \times 10^{22}\ kg$

The perijove distance is r_{peri} = 420000 km

The apojove distance is r_{apo} = 423400 km

The orbital period of Io is 152853 s

The radius of Io is R_{Io} = 1821.6 km.

14. Suppose we live in a static and infinitely large universe where the average density of stars is $n = 10^9\ \mathrm{Mpc}^{-3}$ and the average stellar radius is equal to the solar radius. Assume that standard Euclidean geometry holds true in this universe. How far, on average, could you see in any direction before your line of sight strikes a star? Please write your answer in Mpc.

15. An airplane was flying from Lima, capital of Peru ($12^{o}2'S$ and $77°1'W$) to Yogyakarta ($7°47'S$ and $110^{o}26'E$), near the venue of the 9th IOAA. The airplane chooses the shortest flight path from Lima to Yogyakarta. Find the latitude of the southernmost point of the flight path.

LONG PROBLEMS

1. A moon is orbiting a planet such that the plane of its orbit is perpendicular to the surface of the planet where an observer is standing. After some necessary scaling, suppose the orbit satisfies the following equation:

$$9\left(\frac{x}{2}+\frac{\sqrt{3}y}{2}-4\right)^2+25\left(-\frac{\sqrt{3}x}{2}+\frac{y}{2}\right)^2=225$$

Consider Cartesian coordinates where x is on the horizontal plane and y is on the zenith of the observer. Let r be the radius of the moon. Assume that the period of rotation of the planet is much larger than the orbital period of the moon. Ignore the atmospheric refraction.

a. Calculate the semimajor and semiminor axis of the ellipse.

b. Calculate the zenith angle of perigee.

c. Determine $\tan\frac{\theta}{2}$ where θ is the elevation angle (altitude of the upper tangent of the moon) when the moon looks largest to the observer.

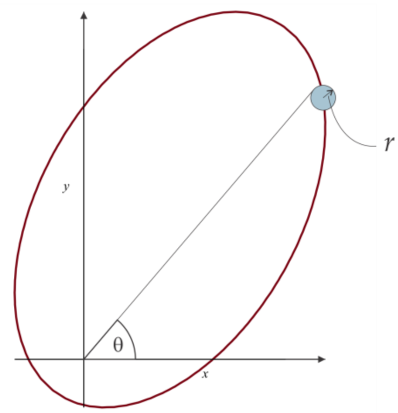

Figure 1

2. Two massive stars A and B with masses m_A and m_B are separated by a distance d. Both stars orbit around their center of mass under gravitational force. Assume their orbits are circular and lie on the *X*-*Y* plane whose origin is at the stars' center of mass (see Figure 2)

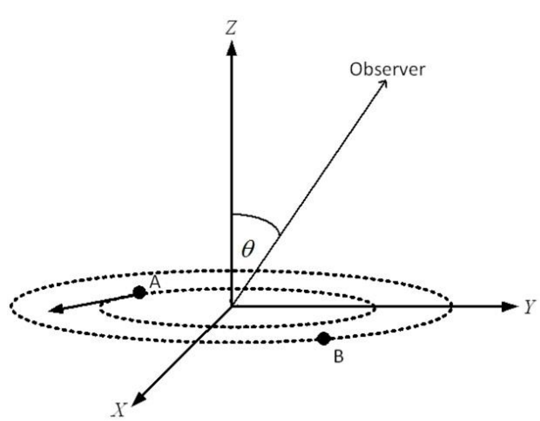

Figure 2

a. Find the expressions for the tangential and angular speeds of star A.

An observer standing on the *Y-Z* plane (see Figure 2) sees the stars from a large distance with an angle θ relatively to the *Z*-axis. He measures that the velocity component of star A to his line of sight has the form $K\ cos(\omega t + \varepsilon)$, where *K* and ε are positive.

b. Express $K^3/\omega G$ in terms of m_A, m_B , and θ where *G* is the universal gravitational constant.

Assume that the observer then identifies that star A has mass equal to $30M_S$ where M_S is the Sun's mass. In addition, he observes that star B produces X-rays and then realizes that it could be a neutron star or a black hole. This conclusion would depend on m_B, i.e.: i) If $m_B < 2M_S$, then B is a neutron star; ii) If $m_B > 2M_S$, then B is a black hole.

c. A measurement by the observer shows that $\frac{K^3}{\omega G} = \frac{1}{250} M_S$. In practice, the value of θ is usually not known. What is the condition on θ for star B to be a black hole?

3. Suppose a static spherical star consists of *N* neutral particles with radius *R* (see Figure 3).

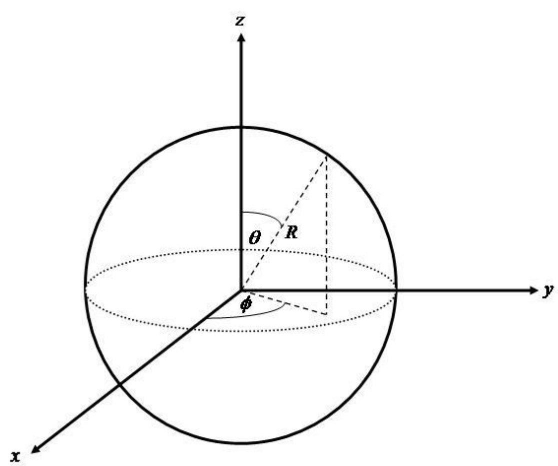

Figure 3

with $\mathbf{0 \le \theta \le \pi}$, $\mathbf{0 \le \phi \le 2\pi}$, satisfying the following equation of states

$$P\,V = N\,k\,\frac{T_R - T_0}{\ln(T_R/T_0)} \tag{1}$$

where P and V are the pressure inside the star and the volume of the star respectively, k is the Boltzmann constant. T_R and T_0 are the temperatures at the surface $r = R$ and the temperature at the center $r = 0$ respectively. Assume that $T_R \leq T_0$.

a. Simplify the stellar equation of state (1) if $\Delta T = T_R - T_0 \approx 0$ (this is called ideal star) (Hint: Use the approximation $\ln(1 + x) \approx x$ for small x)

Suppose the star undergoes a quasi-static process, in which it may slightly contract or expand, such that the above stellar equation of state (1) still holds.

The star satisfies first law of thermodynamics

$$Q = \Delta M c^2 + W \tag{2}$$

where Q, M, and W are heat, mass of the star, and work respectively, while c is the light speed in the vacuum and $\Delta M \equiv M_{\text{final}} - M_{\text{initial}}$.

In the following we assume T_0 to be constant, while $T_R \equiv T$ varies.

b. Find the heat capacity of the star at constant volume C_v in term of M and at constant pressure C_p expressed in C_v and T (Hint: Use the approximation $(1 + x)^n \approx 1 + nx$ for small x)

Assuming that C_v is constant and the gas undergo the isobaric process so the star produces the heat and radiates it outside to the space.

c. Find the heat produced by the isobaric process if the initial temperature and the final temperature are T_i and T_f, respectively.

d. Suppose there is an observer far away from the star. Using information from part c., estimate the distance of the observer from the star.

For the next parts, assume the star is the Sun.

e. If the sunlight is monochromatic with frequency 5×10^{14} Hz, estimate the number of photons radiated by the Sun per second.

f. Calculate the heat capacity C_v of the Sun assuming its surface temperature varies from 5500 K to 6000 K in one second.