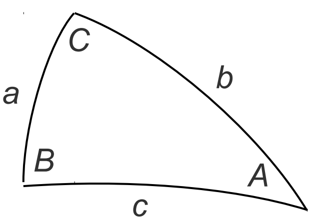
Astronomical and physical constants

Astronomical unit (AU)	1.4960×10^{11} m
Light year (ly)	9.4605×10^{15} m = 63 240 AU
Parsec (pc)	3.0860×10^{16} m = 206 265 AU
1 Sidereal year	365.2564 solar days
1 Tropical year	365.2422 solar days
1 Calendar year	365.2425 solar days
1 Sidereal day	$23^{h}\ 56^{m}\ 04^{s}.091$
1 Solar day	$24^{h}\ 03^{m}\ 56^{s}.555$ units of sidereal time
Mass of Earth	5.9736×10^{24} kg
Mean radius of Earth	6.371×10^{6} m
Equatorial radius of Earth	6.378×10^{6} m
Mean velocity of Earth on its orbit	29.783 km s^{-1}
Mass of Moon	7.3490×10^{22} kg
Radius of Moon	1.737×10^{6} m
Mean Earth – Moon distance	3.844×10^{8} m
Mass of Sun	1.98892×10^{30} kg
Radius of Sun	6.96×10^{8} m
Effective temperature of the Sun	5 780 K
Luminosity of the Sun	3.96×10^{26} J s^{-1}
Solar constant	1366 W m^{-2}
Brightness of the Sun in V-band	−26.8 mag.
Absolute brightness of the Sun in V-band	4.75 mag.
Absolute bolometric brightness of Sun	4.72 mag.
Angular diameter of the Sun	30′
Speed of light in vacuum (c)	2.9979×10^{8} m s^{-1}
Gravitational constant (G)	6.6738×10^{-11} N m^{2} kg^{-2}
Boltzmann constant (k)	1.381×10^{-23} m kg s^{-2} K^{-1}
Stefan−Boltzmann constant (σ)	5.6704×10^{-8} kg s^{-3} K^{-4}
Planck constant (h)	6.6261×10^{-34} J s
Wien's constant (b)	2.8978×10^{-3} m K
Hubble constant (H_0)	70 km s^{-1} Mpc^{-1}
electron charge (e)	1.602×10^{-19} C
Current inclination of the ecliptic (ε)	23º 26.3′
Coordinates of the northern ecliptic pole for epoch 2000.0 (α_E , δ_E)	$18^{h}\ 00^{m}\ 00^{s}$, + 66º 33.6′
Coordinates of the northern galactic pole for epoch 2000.0 (α_G , δ_G)	$12^{h}\ 51^{m}$, + 27º 08′

You can try to solve an equation $x = f(x)$ using iteration: $x_{n+1} = f(x_n)$.



Basic equations of spherical trigonometry

$$\sin a \sin B = \sin b \sin A \ ,$$
$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \ ,$$
$$\cos a = \cos b \cos c + \sin b \sin c \cos A \ .$$

5th IOAA SILESIA POLAND 2011

Short theoretical questions

Each question max 10 points

1. Most single-appearance comets enter the inner Solar System directly from the Oort Cloud. Estimate how long it takes a comet to make this journey. Assume that in the Oort Cloud, 35 000 AU from the Sun, the comet was at aphelion.

2. Estimate the number of stars in a globular cluster of diameter 40 pc, if the escape velocity at the edge of the cluster is 6 km s^{-1} and most of the stars are similar to the Sun.

3. On 9 March 2011 the Voyager probe was 116. 406 AU from the Sun and moving at 17.062 km s^{-1}. Determine the type of orbit the probe is on: (a) elliptical, (b) parabolic, or (c) hyperbolic. What is the apparent magnitude of the Sun as seen from Voyager?

4. Assuming that Phobos moves around Mars on a perfectly circular orbit in the equatorial plane of the planet, give the length of time Phobos is above the horizon for a point on the Martian equator. Use the following data:
   Radius of Mars R_{Mars} = 3 393 km Rotational period of Mars T_{Mars} = 24.623 h. Mass of Mars $M_{Mars} = 6.421 \times 10^{23}$kg Orbital radius of Phobos R_P = 9 380 km .

5. What would be the diameter of a radiotelescope working at a wavelngth of λ = 1 cm with the same resolution as an optical telescope of diameter D = 10 cm?

6. Tidal forces result in a torque on the Earth. Assuming that, during the last several hundred million years, both this torque and the length of the sidereal year were constant and had values of 6.0×10^{16} N m and 3.15×10^{7} s respectively, calculate how many days there were in a year 6.0 x 10^8 years ago. Moment of inertia of a homogeneous filled sphere of radius R and mass m is $I = \frac{2}{5} m\, R^2$

7. A satellite orbits the Earth on a circular orbit. The initial momentum of the satellite is given by the vector **p**. At a certain time, an explosive charge is set off which gives the satellite an additional impulse Δ**p**, equal in magnitude to |**p**|. Let α be the angle between the vectors **p** and Δ**p**, and β between the radius vector of the satellite and the vector Δ**p**. By thinking about the direction of the additional impulse Δ**p**, consider if it is possible to change the orbit to each of the cases given below. If it is possible mark YES on the answer sheet and give values of α and β for which it is possible. If the orbit is not possible, mark NO.

   (a) a hyperbola with perigee at the location of the explosion.

(b) a parabola with perigee at the location of the explosion.

(c) an ellipse with perigee at the location of the explosion.

(d) a circle.

(e) an ellipse with apogee at the location of the explosion.

Note that for $\alpha = 180^\circ$ and $\beta = 90^\circ$ the new orbit will be a line along which the satellite will free fall vertically towards the centre of the Earth.

8. Assuming that dust grains are black bodies, determine the diameter of a spherical dust grain which can remain at 1 AU from the Sun in equilibrium between the radiation pressure and gravitational attraction of the Sun. Take the density of the dust grain to be $\varrho = 10^3$ kg m^{-3}.

9. Interstellar distances are large compared to the sizes of stars. Thus, stellar clusters and galaxies which do not contain diffuse matter essentially do not obscure objects behind them. Estimate what proportion of the sky is obscured by stars when we look in the direction of a galaxy of surface brightness $\mu = 18.0$ mag arcsec^{-2}. Assume that the galaxy consists of stars similar to the Sun.

10. Estimate the minimum energy a proton would need to penetrate the Earth's magnetosphere. Assume that the initial penetration is perpendicular to a belt of constant magnetic field 30 μT and thickness 1.0 x 10^4 km. Prepare the sketch of the particle trajectory. (Note that at such high energies the momentum can be replaced by the expression E/c. Ignore any radiative effects).

11. Based on the spectrum of a galaxy with redshift $z = 6.03$ it was determined that the age of the stars in the galaxy is from 560 to 600 million years. At what z did the epoch of star formation occur in this galaxy? Assume that the current age of the Universe is $t_0 = 13.7 \times 10^9$ years and that the rate of expansion of the Universe is given by a flat cosmological model with cosmological constant $\Lambda = 0$. (In such a model the scale factor $R \propto t^{2/3}$, where t is the time since the Big Bang.)

12. Due to the precession of the Earth's axis, the region of sky visible from a location with fixed geographical coordinates changes with time. Is it possible that, at some point in time, Sirius will not rise as seen from Krakow, while Canopus will rise and set? Assume that the Earth's axis traces out a cone of angle 47°. Krakow is at latitude 50.1° N; the current equatorial coordinates (right ascension and declination) of these stars are:
Sirius (α CMa) : $6^h\,45^m$, $-16^\circ\,43'$
Canopus (α Car) : $6^h\,24^m$, $-52^\circ\,42'$

13. The equation of the ecliptic in equatorial coordinates (α, δ) has the form:
$\delta = \arctan(\sin\alpha \tan\varepsilon)$,
where ε is the angle of the celestial equator to the ecliptic plane. Find an analogous relation $h = f(A)$ for the galactic equator in horizontal coordinates (A, h) for an observer at latitude $\varphi = 49^\circ\,34'$ at local sidereal time $\theta = 0^h\,51^m$.

14. Estimate the number of solar neutrinos which should pass through a 1 m^2 area of the Earth's surface perpendicular to the Sun every second. Use the fact that each fusion reaction in the Sun produces 26.8 MeV of energy and 2 neutrinos.

15. Given that the cosmic background radiation has the spectrum of a black body throughout the evolution of the Universe, determine how its temperature changes with redshift z. In particular, give the temperature of the background radiation at the epoch $z \approx 10$ (that of the farthest currently observed objects). The current temperature of the cosmic background radiation is 2.73 K .

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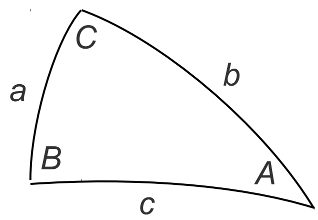

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Long theoretical questions

Instructions

1. You will receive in your envelope an English and native language version of the questions.

2. You have 5 hours to solve 15 short (tasks 1-15) and 3 long tasks.

3. You can use only the pen given on the desk.

4. The solutions of each task should be written on the answer sheets, starting each question on a new page. Only the answer sheets will be assessed.

5. You may use the blank sheets for additional working. These work sheets will not be assessed

6. At the top of each page you should put down your code and task number.

7. If solution exceeds one page, please number the pages for each task.

8. Draw a box around your final answer.

9. Numerical results should be given with appropriate number of significant digits with units.

10. You should use SI or units commonly used in astronomy. Points will be deducted if there is a lack of units or inappropriate number of significant digits.

11. At the end of test, all sheets of papers should be put into the envelope and left on the desk.

12. In your solution please write down each step and partial result.

Long theoretical questions (max 30 points each)

1. A transit of duration 180 minutes was observed for a planet which orbits the star HD209458 with a period of 84 hours. The Doppler shift of absorption lines arising in the planet's atmosphere was also measured, corresponding to a difference in radial velocity of 30 km/s (with respect to observer) between the beginning and the end of the transit. Assuming a circular orbit exactly edge-on to the observer, find the approximate radius and mass of the star and the radius of the orbit of the planet.

2. Within the field of a galaxy cluster at a redshift of $z = 0.500$, a galaxy which looks like a normal elliptical is observed, with an apparent magnitude in the B filter $m_B = 20.40$ mag.

   The luminosity distance corresponding to a redshift of $z = 0.500$ is $d_L = 2754$ Mpc.

   The spectral energy distribution (SED) of elliptical galaxies in the wavelength range 250 nm to 500 nm is adequately approximated by the formula:

$$L_\lambda(\lambda) \propto \lambda^4$$

   (i.e., the spectral density of the object's luminosity, known also as the monochromatic luminosity, is proportional to λ^4.)

   a) What is the absolute magnitude of this galaxy in the B filter ?

   b) Can it be a member of this cluster? (write YES or NO alongside your final calculation)

   <u>Hints:</u> Try to establish a relation that describe the dependence of the spectral density of flux on distance for small wavelength interval. Normal elliptical galaxies have maximum absolute magnitude equal to -22 mag.

3. The planetarium program 'Guide' gives the following data for two solar mass stars:

Star	1	2
Right Ascension	$14^h 29^m 44.95^s$	$14^h 39^m 39.39^s$
Declination	–62º 40′ 46.14″	–60 50′ 22.10″
Distance	1.2953 pc	1.3475 pc
Proper motion in R.A.	–3.776 arcsec / year	–3.600 arcsec / year
Proper motion in Dec.	0.95 arcsec / year	0.77 arcsec / year

   Based on these data, determine whether these stars form a gravitationally bound system. Assume the stars are on the main sequence. Write YES of bound or NO if not bound alongside your final calculation.

   Note: the proper motion in R.A. has been corrected for the declination of the stars.