Problem Booklet
2019/20 & 2020/21

Czech Astronomy Olympiad

Board of Organizers of the Czech Astronomy Olympiad

Prague, 2022

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Contents

Introduction **4**
Foreword 4
Legend and acknowledgements 5

Theoretical Problems **6**
Geometry, time and instrumentation 6
Solar system 10
Stellar astronomy 21
Binary stars and exoplanets 25
Cosmology and relativity 30

Practical Problems **34**

Introduction

Foreword

Dear friends,

after a two-year break, you can now get your hands on a new edition of the Czech Astronomy Olympiad problem book. Czech Astronomical Society, the main organizer of the Olympiad, is a respected and amazing association with more than 100 years of tradition, where amateur and professional astronomers get together in a single organization. Its 600 members work in 17 branches, covering the most important areas ranging from solar astronomy over variable stars to cosmology. Aside from the research topics, the society concentrates immense effort into working with young astronomers. In addition to the Olympiad (which itself attracts about 10 000 participants annually), it organizes summer schools and camps for children and youth together with astronomical courses, clubs and more.

The Olympiad is divided into four age groups (called – from the oldest to the youngest – AB, CD, EF and GH), each having three stages. The first round takes place at school with its main objective being to attract pupils to astronomy and motivate them to further work. In the second (regional) round, participants have to solve more complex problems as well as to perform observational tasks. The best participants go forward to the national rounds held in Opava and Prague in March and May, respectively, the winners of which then meet at the qualification camps for the IAO and IOAA.

The Covid-19 pandemic has also affected the student competitions. In March 2020 it was necessary to switch the competitions to online mode from one day to the next. The national rounds, which are a popular platform for meetings and exchanges, had to be held remotely. Up to twelve invigilators supervised the proceedings of the final rounds, the contestants had to be in the field of view of their cameras all the time and had to have their microphones on. Although we may have unwittingly got a glimpse into the room of a future author of a major discovery or maybe even a Nobel Prize winner, we hope that this unusual online experiment will not be continued in the coming years.

Even in this challenging period, the authors have prepared interesting tasks and you can now immerse yourself in solving them.

We wish you good health, dark skies and pleasant reading!

On behalf of the Organizers of the Czech AO

Jan Kožuško

Legend and acknowledgements

Each problem presented in this booklet comes with its name and ID code containing information about the place of its original use in the Olympiad. For instance, "CD/R/2-20" denotes the second problem in the regional round of the CD category in the year 2019/20. Most problems have their answers shown in small print.

Majority of the competition problems are original work of its organizers. Credits for the problems presented in this volume are as follows:

Martin Blaschke: CD/N/1-21; *Jindřich Jelínek:* AB/R/5-20, AB/R/1-21, CD/N/6-21; *Pavel Kůs:* AB/R/4-20, AB/N/3-21; *Jaromír Mielec:* AB/N/1-21, AB/N/7-21; *Václav Pavlík:* EF/R/1-20, EF/R/2-20, EF/R/1-21, EF/R/2-21, EF/R/3-21, EF/N/1-21, EF/N/2-21, EF/N/3-21, EF/N/4-21; *Jiří Vala:* AB/N/2-21, CD/R/1-20; *Ondřej Theiner:* AB/R/2-21, AB/N/6-21; *Jakub Vošmera:* AB/R/1-20, AB/R/2-20, AB/R/3-20, AB/R/3-21, AB/N/4-21, AB/N/5-21, CD/R/2-20, CD/R/3-20, CD/R/1-21, CD/R/2-21, CD/N/2-21, CD/N/3-21, CD/N/4-21, CD/N/5-21

We also acknowledge that the problems AB/N/5-21, CD/R/2-21, AB/N/4-21, CD/R/1-21 and CD/N/5-21 were inspired by the *Russian AO*.

The reader certainly would not be able to enjoy the problems in their present form were it not for the work of *Petr Kulhánek*, *Ota Kéhar* and *Michal Švanda* who carefully reviewed all questions.

Finally, we want to express our immense gratitude to the director of the Prague Observatory and Planetarium, *Jakub Rozehnal*, and the vice-dean of Faculty of Philosophy and Science in Opava, *Tomáš Gráf*, for kindly providing the venue and related services for the national rounds. We also thank *Jan Kožuško*, *Lenka Soumarová* for helping make the Czech Astronomy Olympiad happen by diligently providing their support.

Theoretical problems

Geometry, time and instrumentation

Astronomical twilight EF/N/2-21

The motion of the real Sun in the sky (the one that we observe) is uneven. The length of the apparent solar day varies throughout the year so that people have established the mean solar day that is exactly 24 hours long.

a) Calculate the angular distance that the Sun traverses in the sky in one mean solar hour. Round the result to whole degrees.
b) Astronomical twilight ends when the centre of the sun's disc falls 18 degrees below the horizon. Draw a diagram of sunset at a latitude of 50°. Mark the horizon and the trajectory of the setting Sun above and below the horizon (represent it by a straight line for simplicity) and label the part that the Sun travels from sunset to the end of astronomical twilight by x. Additionally, mark the position of the Sun at the end of astronomical twilight (using a full circle (●)).
c) At what time after sunset does astronomical twilight end at a latitude of 50°? For the sake simplicity, consider the sunset to be the moment when the center of the solar disk crosses the horizon, and also neglect atmospheric refraction. Give the answer in mean solar hours.

Hint: Knowledge of planar geometry is sufficient for solving the problem.

[a) 15°, c) 1.9 h]

Unknown location CD/N/3-21

An observer at an unknown location on Earth spotted a star located exactly at zenith at 23:00 local time. The next day, again at 23:00 local time, he noticed that the star's zenith distance had changed to $z = 0.8°$. Determine the latitude φ of the observer.

[35.5°]

Field of view CD/N/4-21

Determine the diameter d of the telescope's field of view (in arcseconds) if we know that for Polaris (α UMi, right ascension $\alpha = 2^{\mathrm{h}}\,31^{\mathrm{m}}\,49^{\mathrm{s}}$, declination $\delta = 89^\circ\,15'\,51''$) it took exactly $t = 6\,\mathrm{h}$ to get from the centre of the field of view to its edge. The telescope is placed on a non-motorized mount.

[2.1°]

Amazing precession AB/R/1-20

The goal of this problem is to determine how the visible sky changes for individual observers on Earth during a single precession cycle. Assume that the atmospheric refraction at the horizon is $\rho = 35'$ and that the inclination of the ecliptic with respect to the celestial equator is equal to $\varepsilon = 23.4^\circ$.

a) What percentage P_{N} of the sky can we see from the North Pole in one year?

b) What percentage P_{eq} of the sky can we see over the course of a year from the equator?

c) Express the fraction $P(\phi)$ of the sky (in percent) visible over the course of a year as a function of the latitude ϕ of the observer.

d) What percentage $\tilde{P}_{\mathrm{N}}$ of the sky can we see from the North Pole over the course of 50 000 years?

e) At what places on Earth can all the directions in the sky be seen over the course of 50 000 years?

f) Express the fraction $\tilde{P}(\phi)$ of the sky that can be seen over 50 000 years as a function of the latitude of the observer ϕ.

g) Express the ecliptic coordinates (J2000.0) of the North and South Celestial Poles as a function of time.

h) Express the equatorial coordinates (J2000.0) of the North and South Celestial Poles as a function of time.

i) Using a sky map or a suitable computer software, explore the area near the trajectories of the North and South Celestial Poles. List all bright stars that may approximately serve as the North or South "Polaris" in the future (for each star, give the minimum angular distance to the given celestial pole and the corresponding year).

[a) $P_{\mathrm{N}} = 50.5\,\%$, b) $P_{\mathrm{eq}} = 100\,\%$, c) $P(\phi) = [1 + \cos(|\phi| - \rho)]/2$ when we have $\rho \leqslant |\phi| \leqslant 90^\circ$ and $P(\phi) = 1$ when we have $|\phi| \leqslant \rho$, d) $\tilde{P}_{\mathrm{N}} = 70.3\,\%$, e) $|\phi| \leqslant \rho + \varepsilon$, f) $\tilde{P}(\phi) = [1 + \cos(|\phi| - \rho - \varepsilon)]/2$ when we have $|\phi| > \rho + \varepsilon$ and $\tilde{P}(\phi) = 1$ when we have $|\phi| \leqslant \rho + \varepsilon$, g) $\beta_{\pm}(t) = \pm 66.6^\circ$, $\lambda_{\pm}(t) = \pm 90^\circ - (t/T_{\mathrm{p}}) \times 360^\circ$, h) $\cos\delta_{\pm}(t) = \sqrt{\sin^2\varepsilon\sin^2\omega t + \sin^2\varepsilon\cos^2\varepsilon(1 - \cos\omega t)^2}$, $\cos\alpha_{\pm}(t) = \mp\sin\varepsilon\sin\omega t / \sqrt{\sin^2\varepsilon\sin^2\omega t + \sin^2\varepsilon\cos^2\varepsilon(1 - \cos\omega t)^2}$, $\sin\alpha_{\pm}(t) = \mp\sin\varepsilon\cos\varepsilon(1 - \cos\omega t) / \sqrt{\sin^2\varepsilon\sin^2\omega t + \sin^2\varepsilon\cos^2\varepsilon(1 - \cos\omega t)^2}$, i) Northern

Celestial Pole: α Lyr (4° 50′, year 13 790), Southern Celestial Pole: ι Car (30′, year 8060), δ Vel (35′, 9230), γ Vel (2° 10′, year 10 780), β Hyi (1°40′, year 25 520)]

Christmas surprise AB/R/4-20

Lukáš is a young astronomer and as a Christmas present, he received a Kepler-type telescope with a diameter $D = 6\,\text{cm}$ and a focal length $f = 80\,\text{cm}$. The Moon was currently visible in the sky and Lukáš, wanting to test the telescope immediately, went out into the yard of his house in Ostrava (49°50′8′ N, 18°17′34″ E and pointed it at the Moon.

a) By what factor is the number of photons coming from the Moon to Lukáš's eye greater when he uses the telescope than in the situation when he looks at the Moon directly? Lukáš's eyes are well adapted to darkness, so that his pupils have a diameter of $D_{\text{pup}} = 8\,\text{mm}$.
b) What is the factor by which the Moon will be magnified in the eyepiece of focal length $f' = 10\,\text{mm}$?
c) What is the smallest angular distance (in angular seconds) of two objects for the telescope to distinguish them from each other?
d) If these objects were to be two opposite sections of a crater rim on the Moon, what would be the diameter l of that crater in kilometers? Determine the mean distance of the Moon from the Earth by using its period $T = 27.3\,\text{d}$ and the mass of the Earth $M = 5.97\times10^{24}\,\text{kg}$. Neglect the dimensions of the Earth and the Moon.
e) If he points the telescope so that the Moon crosses the diameter of the field of view, what is the time Δt of the crossing in minutes? Assume that at the start of the observation, the entire disk was visible and was touching the edge of the field of view, and that at the end the entire disk disappeared the field of view. The field of view (FOV) of the telescope is FOV $= 0.8°$. The mount of Lukáš's telescope is not equipped with a clock drive.

During his observations, he noticed that an object had emerged at the edge of the lunar disk and continued to move in a straight line across the disk. Because Lukáš is still young and believes in Santa Claus, he refocused on the flying object out of curiosity.

f) Assuming that he has to move the eyepiece by a distance $d = 1\,\text{mm}$, at what altitude H (in meters) above the surface of the Earth is the object flying? The (angular) altitude of the Moon above the horizon at the time of the observation was $h = 30°$.

[a) 500, b) 80, c) 2.3″, d) 4.2 km, e) 5 min, f) 320 m]

Spy satellite **AB/N/5-21**

Consider an artificial satellite orbiting the Earth along a circular orbit lying in the equatorial plane, moving in the direction of the Earth's rotation. The satellite orbits at altitude $H = 500\,\mathrm{km}$ above the Earth's surface. For the purposes of this problem, assume that the Earth has the shape of an ideal sphere with radius $R_\mathrm{E} = 6\,371\,\mathrm{km}$ and mass $M_\mathrm{E} = 5.972 \times 10^{24}\,\mathrm{kg}$. Let us also denote the sidereal period of the Earth's rotation by $T_0 = 23^\mathrm{h}\,56^\mathrm{m}\,4^\mathrm{s}$.

a) Calculate the geocentric velocity u of the motion of an observer located on the equator on the Earth's surface. Assume that the observer is not moving with respect to the surface. Express the result in terms of R_E and T_0, as well as numerically in $\mathrm{km\,s^{-1}}$.
b) Calculate the orbital velocity v and the orbital period T of the satellite. Express the results in terms of G, M_E, R_E, H (where G is Newton's gravitational constant), as well as numerically (in $\mathrm{km\,s^{-1}}$ and in hours, respectively)

Consider now that the purpose of the satellite is to spy on various targets on the surface of the Earth.

c) Determine the maximum and minimum latitude $\pm|\phi_\mathrm{max}|$ of the areas that the satellite can monitor. Give the numerical value in degrees. Will the refraction theoretically increase or decrease this area?

For the rest of the problem, neglect atmospheric refraction.

d) What part p of the Earth's surface can the satellite observe at one moment? What part p' of the Earth's surface can the satellite observe in the long term? Express the results in terms of R_E and H, as well as numerically as a percentage.

Now let us assume that the monitored target is located on the equator.

e) Determine the period T_s, with which the satellite passes directly above the target. Give the result in hours.
f) Calculate the time ΔT for which the satellite can monitor its target within each period T_s. Give the answer numerically in minutes. What fraction τ of total represents the duration when the satellite is able to track the target? Consider the weather over the observed target to be ideal.
g) Express the angle $\alpha(t)$ between the position vectors of the satellite and the target (with respect to Earth's centre) in terms of t, T_s, t_i and ΔT.
h) Express the angular velocity Ω with which the camera needs to rotate in order to follow the target (with respect to the body of the satellite) depending on the time $t \in \langle t_\mathrm{i}, t_\mathrm{f}\rangle$, where t_i, respectively t_f are the times of the beginning, respectively of the end of the satellite's visibility within each

observation window. First perform the calculation assuming zero sidereal period of the satellite rotation: show that in that case it is possible to write

$$\Omega(t) = \frac{2\pi}{T_\mathrm{s}} \frac{1 - \cos|\phi_\mathrm{max}| \cos\alpha(t)}{1 + \cos^2|\phi_\mathrm{max}| - 2\cos|\phi_\mathrm{max}|\cos\alpha(t)} \,. \tag{1}$$

i) Express the angular velocities of the camera Ω_i, Ω_f and Ω_c at times $t = t_\mathrm{i}$, $t = t_\mathrm{f}$ and $t = t_\mathrm{c} \equiv t_\mathrm{i} + \Delta T/2$ using T_s and $\cos|\phi_\mathrm{max}|$. Calculate their numerical values in degrees per second.

j) How would the values of $\Omega(t)$, Ω_i, Ω_f and Ω_c change assuming captured rotation of the satellite (sidereal period of satellite rotation is equal to T_0 in the direction of the satellite orbit)?

Note: tasks i) and j) can be solved using the relation (1) for $\Omega(t)$ without point penalty, even if you were not able to derive it in part h).

[a) 0.465 km s^{-1}, b) $v = \sqrt{GM_\mathrm{E}/(R_\mathrm{E}+H)} = 7.613\,\mathrm{km\,s^{-1}}$, $T = 2\pi\sqrt{(R_\mathrm{E}+H)^3/GM} = 1.577\,\mathrm{h}$, c) $\pm 22°$, d) $(1/2)H/(R_\mathrm{E}+H) = 3.6\,\%$, $\sqrt{2R_\mathrm{E}H + H^2}/(R_\mathrm{E}+H) = 37\,\%$, e) 1.688 h, f) 12.4 min, 0.12, g) $(2\pi/T_\mathrm{s})(t - t_\mathrm{i} - \Delta T/2)$, i) $\Omega(t_\mathrm{i}) = \Omega(t_\mathrm{f}) = 2\pi/T_\mathrm{s} = 0.059°\,\mathrm{s^{-1}}$, $\Omega(t_\mathrm{c}) = (2\pi/T_\mathrm{s})[1/(1-\cos|\phi_\mathrm{max}|)] = 0.82°\,\mathrm{s^{-1}}$, j) just add $2\pi/T_0$]

Solar system

The transit of Mercury EF/R/1-20

On 11–12 November 2019 (depending on the time zone) we had the chance to observe a rare astronomical event – the transit of the planet Mercury across the solar disk. In this problem, we will take a closer look at it.

a) What planets can we see transiting the Sun from the Earth?

b) What must be the positions of the Earth and the Sun relative to Mercury for a transit to occur?

c) Consider that during this daytime observation, the eye had a pupil diameter of 1.4 mm. What is the angular size of Mercury if it became visible as a disk only at ten times the magnification of the telescope? Round the result to whole arc seconds.

Hint: The human eye is most sensitive to light at a wavelength of 550 nm.

d) How far was Mercury from the Earth at the time of the transit? Give the result in astronomical units (au) rounded to the nearest hundredth.

Hint: For small angles, $\tan\alpha \approx \alpha$ if α is expressed in radians.

[a) Mercury, Venus, b) Mercury passing through the plane of ecliptic, Earth in opposition w.r.t. Mercury, c) 10″, d) 0.68 au]

Earth observations EF/R/2-21

Three "alien" scientists are simultaneously looking at the Earth. The Venusian has taken up residence in a cloud base on the planet Venus, the Martian on the surface of the planet Mars, and the Lunarian on the near side of our Moon. They record their measurements in a table. First, they find the synodic orbital period of their respective bodies with respect to Earth – i.e., the time it takes for their body to re-enter opposition with the Sun relative to Earth (we neglect the inclination of their orbits relative to the ecliptic). By observing the Sun's motion between the stars, the Venusian and the Martian found their sidereal orbital period around the Sun, while the Lunarian estimated the Earth's sidereal orbital period.

period	**Venus**	**Mars**	**Moon**	**Earth**
synodic (T_{syn})	583.9 d	780.0 d	29.5 d	—
sidereal (T_{sid})	224.7 d	687.0 d	—	365.26 d

a) For each observer, determine whether he can see all phases of the Earth – from new Earth through full Earth to new Earth. Write "Yes" or "No" and explain in ONE sentence.
b) For those who can see all phases, determine how long it takes for the phases to repeat themselves.
c) The Venusian and the Martian can only communicate with each other when the distance between their planets reaches its minimum. With what period does this occur if we neglect the eccentricities of both orbits?

Hint: It is convenient to consider the angular velocities of the planets.

d) Using the values given above, calculate how many times farther away from the Sun the planet Mars orbits than the Earth. Neglect the eccentricities of both orbits.
e) The Lunarian has better tracking equipment than the Venusian, and is also closer to Mars, so he can pick up the Martian's signal all the way from the quadrature, through the opposition, to the next Earth–Mars quadrature. By a quadrature we mean such relative position of the outer planets so that the planet–Earth–Sun angle is equal to 90°. Calculate how long this period lasts. Neglect the eccentricities and inclinations of the planetary orbits and the Earth–Moon distance (i.e., proceed as if the Lunarian was based on Earth).

[c) 333.9 d, d) 1.52 au, e) 211.7 d]

Mars Orbital Station EF/N/4-21

Imagin that we have time-traveled to the near future, where humans regularly travel to Mars and they built the first orbital station (similar to the ISS) orbiting around Mars and called it the *Mars Space Station* (MSS). The MSS orbits in the equatorial plane in the same direction as Mars rotates around its axis. Moreover, living or working on terraformed Mars is not so different from Earth, because one sidereal day on Mars lasts 24.62 hours. Further parameters of Mars can be found in the table.

radius	mass	semi-major axis	orbital period
3396 km	6.42×10^{23} kg	1.52 au	1.88 yr

a) The MSS orbits along a circular orbit at an altitude of 300 km above the surface of Mars. Find the speed neccessary for it to stay on this circular orbit. Give the answer in m/s.
b) What is the orbital period of MSS? Give the answer in minutes.
c) MSS communicates with a radiotelescope located at the equator, whose antenna can detect signal from anything above the horizon. What part of the orbit of MSS is covered by the range of the radiotelescope? (Atmospheric effects are to be neglected.) Give the answer in km.
d) How long can the MSS communicate with the radiotelescope during one orbit if it orbits in the same direction as Mars rotates around its axis? Give the result in minutes.
e) A supply ship of negligible mass left the MSS in the opposite direction than the MSS orbits Mars. However, the magnitude of its initial velocity was equal to the MSS orbital velocity. Immediately after takeoff, the pilot realized that he had forgotten to unload one of the supply boxes. With his current speed, he would catch MSS in half of the period. However, he did not want to wait that long and had plenty of fuel, so he ignited the auxiliary jets and increased his speed 1.5 times. How long will it take to catch up with the MSS at this velocity? Neglect all connection manoeuvres, we are purely interested in the time until the moment of encounter.

[a) 3 404 m s^{-1}, b) 114 min, c) 2 998 km, d) 16.0 min, e) he will not catch up with MSS]

Martian CD/R/1-20

In 2015, the science fiction film The Martian about an astronaut Mark Watney was made, who, along with other astronauts, was sent on the Ares 3 mission to the planet Mars. There, he almost died in a dust storm. When he woke up, he found that the rest of the crew had left the planet and he was left on

Mars alone. He therefore has to figure out a way how to survive on Mars with the rest of his supplies and how to send a signal to Earth.

The film is an adaptation of the novel of the same title, which describes the mission in detail, including technical data. In this problem, you will investigate the beginning of the mission and the journey of the spacecrafts to Mars. Consider that the Earth, or Mars respectively, move around the Sun in the same plane along circular orbits with radii $r_Z = 1.00\,\mathrm{au}$ and $r_M = 1.52\,\mathrm{au}$, respectively.

"The presupply probes for Ares-3 launched on 14 consecutive days during the Hohmann Transfer window."

a) Calculate the time T it takes for the Ares 3 supply rockets to reach Mars if they follow a Hohmann trajectory. Also calculate the heliocentric speed v_1 of the probes near Earth and v_2 near Mars.
b) Calculate the magnitude of the Sun-Earth-Mars angle α at which the Hohmann Transfer window occurs. How often does this window occur?

"Vogel checked the position and orientation of Hermes against the projected path. It matched, as usual. In addition to being the mission's chemist, he was also an accomplished astrophysicist. Though his duties as navigator were laughably easy."

In the film, the spaceship Hermes is powered by ion engines working at constant acceleration $2\,\mathrm{mm\,s^{-2}}$. However, for the purpose of the problem, let us assume that Hermes left the Earth's gravity in the direction of the Earth's orbit with a velocity of $v = 11.5\,\mathrm{km\,s^{-1}}$ with respect to the Earth and did not accelerate further along its trajectory.

c) Calculate the heliocentric velocity v_H of Hermes at the moment of its approach to Mars and angle φ, at which the trajectories of Hermes and Mars will intersect.
d) Calculate the speed v_∞ of Hermes during its approach to Mars relative to Mars. Ignore the gravitational influence of Mars.

"If you think turbulence is rough in a jetliner going 720kph, just imagine what it's like at 28,000kph."

Now assume that Hermes, whose total mass (including fuel) is $M_0 = 110\,000\,\mathrm{kg}$, ignited its (conventional) engines, slowing to the speed that Mark Watney reports in his diary.

e) Calculate the duration Δt of this ignition. The fuel flux velocity during ignition is $u = 50\,\mathrm{km\,s^{-1}}$ and the mass-flow rate is $\dot{m} = 620\,\mathrm{kg\,s^{-1}}$. Assume the spaceship has enough fuel to slow itself down.

Hint: To solve problem e), you may need the Tsiolkovsky rocket equation.[1]

[a) 258 d, 32.8 km s^{-1}, 21.6 km s^{-1}, b) 85.3°, once in 2.14 y, c) 33.1 km s^{-1}, 34.8°, d) 19.1 km s^{-1}, e) 36 s]

Conjunction CD/R/2-21

As early as the time of Johannes Kepler, astronomers could calculate the relative distances of the planets in the solar system. For a long time, however, no one could compare these distances with terrestrial scales. This was only achieved with reasonable accuracy by Jerome Lalande in 1771 by measuring the parallax of Venus as it passed across the solar disk. In this problem, you will focus on comparing the magnitudes of the phases of Venus and the Moon in their mutual conjunction.

Assume that there was a conjunction of Venus and the Moon at an elongation of λ. Elongation refers to the angular distance of an object in the sky from the Sun. For simplicity, in this problem we will assume that the orbits of all the bodies involved are circular and lie in the plane of the ecliptic.

Let us denote a_E, a_M and a_V the radii of the orbits of the Earth, Moon and Venus, respectively. Also, let d_{MS}, d_{VE} be the distances Moon–Sun and Venus–Earth at the time the elongation occurred, and let α_M and α_V be the angles Sun–Moon–Earth and Sun–Venus–Earth, respectively.

a) Draw the situation as viewed "from above" (from the north ecliptic pole) in a clear diagram, marking the triangles Earth–Sun–Moon and Earth–Sun–Venus, as well as the distances a_E, a_M, a_V, d_{MS}, d_{VE}, and the angles α_M, α_V.

For the purpose of this problem, we define the phase of an object in the sky as the angle ϕ that increases from 0° to 360°, where $\phi = 0°$ occurs at new moon, $\phi = 90°$ occurs at first quarter, $\phi = 180°$ occurs at full moon, and $\phi = 270°$ occurs at last quarter. Thus, the phase is the angle between the terminator plane and the plane tangent to the celestial sphere,

b) Express the phases ϕ_M of the Moon and ϕ_V of Venus using the angles α_M and α_V.
c) Express $\sin \alpha_V$ by using a_E, a_V and λ.
d) Find the maximum value λ_{max} of the elongation for Venus. Express the result in terms of a_E, a_V, as well as numerically in degrees.
e) Express the distance d_{MS} in terms of a_E, a_M and λ.
f) Express $\sin \alpha_M$ in terms of a_E, a_M and λ.
g) Express $\sin \phi_M$ and $\sin \phi_V$ in terms of a_E, a_M, a_V and λ.

[1] `https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation`

h) Show that for very small angles λ, the phase ratio $p = \phi_\mathrm{M}/\phi_\mathrm{V}$ does not depend on λ. (Assume that Venus is approaching an inferior conjunction with the Sun.) Express p first in terms of the ratios $u_\mathrm{M} = a_\mathrm{M}/a_\mathrm{E}$, $u_\mathrm{V} = a_\mathrm{V}/a_\mathrm{E}$ and then calculate its numerical value.

Hint: if the angle λ is expressed in radians and $\lambda \ll 1$, we have $\sin\lambda \approx \lambda$ and $\cos\lambda \approx 1 - \lambda^2/2$.

[b) $\phi_\mathrm{M} = \pi - \alpha_\mathrm{M}$, $\phi_\mathrm{V} = \pi - \alpha_\mathrm{V}$, c) $\sin\alpha_\mathrm{V} = \frac{a_\mathrm{Z}}{a_\mathrm{V}}\sin\lambda$, d) $\lambda_\mathrm{max} \doteq 46°$, e) $d_\mathrm{MS} = \sqrt{a_\mathrm{M}^2 + a_\mathrm{Z}^2 - 2a_\mathrm{M}a_\mathrm{Z}\cos\lambda}$, f) $\sin\alpha_\mathrm{M} = \frac{a_\mathrm{Z}}{\sqrt{a_\mathrm{M}^2+a_\mathrm{Z}^2-2a_\mathrm{M}a_\mathrm{Z}\cos\lambda}}\sin\lambda$, g) $\sin\phi_\mathrm{M} = \frac{a_\mathrm{Z}}{\sqrt{a_\mathrm{M}^2+a_\mathrm{Z}^2-2a_\mathrm{M}a_\mathrm{Z}\cos\lambda}}\sin\lambda$, $\sin\phi_\mathrm{V} = \frac{a_\mathrm{Z}}{a_\mathrm{V}}\sin\lambda$, h) $p \approx 0.72$]

Solar fuel CD/N/1-21

Julius Mayer (1814–1878), one of the pioneers of the law of conservation of energy, postulated that the Sun gets its radiant energy from constant impacts of asteroids and comets.

a) Give an order of magnitude estimate of how many typical bodies (for the sake of simplicity, assume that their typical size is $\sim 1\,\mathrm{km}^3$) occurring in the Oort cloud would have to hit the Sun's surface each second to generate its radiant power.
b) Also estimate the duration for which the Sun could be supplied with energy in this manner.

[a) 2000, b) 10 000 y]

The opposition of Mars CD/N/6-21

In this problem we will investigate in detail the motion of a body along an ellipse in a central gravitational field. In figure 1 you can see a general elliptical orbit of eccentricity e with a planet (black dot). The Sun is located at the focus F_1. The area of the shaded part as a function of s can be inferred as

$$S_1 = \frac{ab}{2}\left(\arccos\frac{s}{a} - \frac{s}{a}\sqrt{1-\left(\frac{s}{a}\right)^2}\right).$$

The distance s is positive in the positive direction of the x axis and negative in the opposite direction (so in figure 1 it is $s < 0$).

a) Find the area S swept out by the (Sun)–(planet) position vector starting at the moment when the planet passed through the perihelion. Express your answer in terms of a, b, s and e.

Let us suppose that the planet orbits the Sun with period T and denote by t the time that has elapsed since the perihelion passage. For the description of

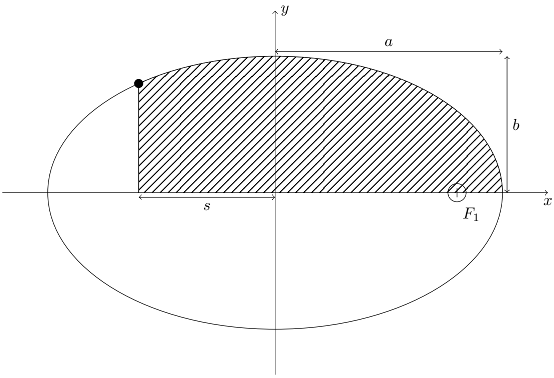


Figure 1: Elliptical orbit with shaded area S_1.

a position along the circumference of an ellipse, three different angles come up (the so-called *anomalies*). The mean anomaly M is the angle that the position vector of the planet would have swept out over time t if it was moving along a circular orbit at a constant angular velocity $\omega = 2\pi/T$.

Defining the eccentric anomaly E is somewhat more complicated. Let us construct a circle of radius a centered at the origin, and then construct a line parallel with the y axis passing through the planet. The eccentric anomaly E is then defined as the angle (perihelion)–(centre of the ellipse)–(point of intersection of the circle and the line).

Finally, the true anomaly ν is the angle (perihelion)–(Sun)–(planet). The eccentric anomaly E and the true anomaly ν are plotted in figure 2.

b) Re-express the result for the area S from question a) in terms of a, b, e and E.

c) Using the results from questions a) and b) and Kepler's second law, derive the Kepler's equation

$$M = E - e \sin E\,.$$

d) Derive the relationship

$$\cos E = \frac{\cos\nu + e}{1 + e\cos\nu}\,,$$

between the eccentric anomaly E and the true anomaly ν. You may find useful the equation of an ellipse in polar coordinates

$$r(\nu) = \frac{a(1-e^2)}{1+e\cos\nu}\,. \tag{2}$$

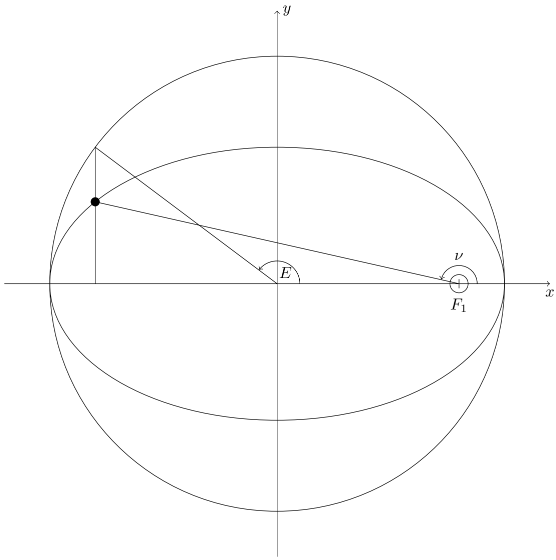


Figure 2

It gives the distance (Sun)–(planet) $r(\nu)$ as a function of the true anomaly ν.

Since the mean anomaly M depends on time t simply as $M(t) = 2\pi t/T$ and the results of problems c) and d) give us a method of computing the true anomaly ν based on the knowledge of the mean anomaly, we can now, at least in principle, reconstruct the time dependence $\nu(t)$ of the true anomaly and also the time dependence $r(t)$ of the (planet)–(Sun) distance using the formula (2).

The only problem in this procedure arises when calculating the eccentric anomaly from the mean anomaly, because the Kepler's equation does not allow for E to be expressed as a closed-form function of M. However, for a given value of M, we can always solve the Kepler's equation numerically for E by the following iterative procedure. First, let us make an initial estimate of the solution (let us call it E_1). We then obtain a more accurate estimate E_2 by substituting E_1 into the Kepler's equation and express $E_2 = M + e\sin E_1$. Repeating this procedure yields a series $\{E_n\}$ of increasingly accurate estimates, where

$$E_{n+1} = M + e\sin E_n\,.$$

When the difference between the successive estimates is less than the desired precision of the result, we can take the final estimate as the solution to the Kepler's equation.

We will now try to get our hands on this method by considering the opposition of Mars that occurred on the night of 14-15 July 2020. At first, let us assume that the orbits of Mars and Earth are circular.

e) On which day (or night) will the next opposition occur?

During the night of 14-15 October 2020, Mars was located at an ecliptic longitude of $\lambda_\mathrm{M} = 20.8°$. The ecliptic longitude of Mars' perihelion is $\psi_\mathrm{M} = -24.0°$. On the other hand, the ecliptic longitude of the Earth's perihelion is $\psi_\mathrm{Z} = 102.9°$. Let us consider, for the sake of simplicity, that the orbits of both planets around the Sun are confined withing the ecliptic plane. Also, note that the ecliptic longitude of an object located in the ecliptic plane is defined as the angle (vernal equinox point)–(Sun)–(object).

f) The date that you calculated in question d) is only an approximate date of the opposition. Now, take also into account the eccentricities of the orbits of both Earth and Mars and calculate the true ecliptic longitudes of both Earth and Mars for that date. Is the actual opposition going to occur earlier of later?

[a) $S = \frac{ab}{2}\left(\arccos\frac{s}{a} - e\sqrt{1-\left(\frac{s}{a}\right)^2}\right)$, b) $S = \frac{ab}{2}(E - e\sin E)$, e) night of Dec 3 2022, f) later]

Itokawa AB/R/5-20

On 16 September 2019, the asteroid Itokawa came to its closest to Earth in its orbit. Itokawa is a near-Earth object, crossing the Earth's orbit and could be potentially dangerous. In this problem, we will recall the Hayabusa probe that explored the asteroid.

a) Itokawa reaches a distance $r_\mathrm{p} = 0.953\,\mathrm{au}$ from the Sun in its perihelion, while its distance from the Sun in the aphelion is $r_\mathrm{a} = 1.695\,\mathrm{au}$. Calculate the semi-major axis a (in au), the eccentricity e and the orbital period T (in years) of Itokawa.

In the following questions, let us approximate the shape of the asteroid Itokawa as a sphere with radius $R_\mathrm{A} = 170\,\mathrm{m}$ and mass $M_\mathrm{A} = 3.51\times10^{10}\,\mathrm{kg}$. In 2003, the Japanese probe Hayabusa set out on its journey towards Itokawa. It arrived to the asteroid in 2005. The mission's goal was to retrieve samples of material from the surface and bring them back to Earth. Due to the low mass of the asteroid, it was decided not to park the probe in orbit. Instead, it was guided into an orbit around the Sun very close to the orbit of the asteroid.

b) Assume that the asteroid-probe system is located at a distance of 1.324 au from the Sun and that the probe hovers 1 000 m from the surface of the asteroid. Compare the acceleration of the probe caused by the Sun with the acceleration of the probe caused by the asteroid.

c) On November 25, 2005 an attempt was made to reach the surface and collect samples. Assuming that the descent was initiated at a distance of 1 000 m from the surface at zero initial speed of the probe relative to Itokawa and that the spacecraft did not perform any maneuvres during the descent, calculate the impact velocity and an approximate time of the descent. Note that since the exact calculation of the descent time would be quite demanding, a reasonable range in seconds (but not e.g. $0 < t < \infty$) supported by a calculation is acceptable.

Hint: a particle in a central gravitational field falling freely along a straight line can be thought of as moving along an infinitely elongated ellipse.

Due to technical difficulties, the probe orbited the Sun 2.5 times after collecting samples before returning to Earth.

d) Assume that the spacecraft orbited the Sun along exactly the same ellipse as the asteroid and was 20 km ahead of the asteroid at aphelion. Calculate the distance (in km) between the asteroid and the probe at perihelion. You should neglect all gravitational influence on the probe due to the asteroid.

[a) 1.324 au, 0.280, 1.523 yr, b) $a_{g,\odot} = 3.38\times10^{-3}\,\mathrm{m\,s^{-2}}$, $a_{g,A} = 1.71\times10^{-6}\,\mathrm{m\,s^{-2}}$, c) $0.153\,\mathrm{m\,s^{-1}}$, 2.83×10^{4} s, d) 36 km]

Venera 9 AB/R/1-21

45 years ago, on October 22, 1975 the lander of Venera 9 probe landed on the surface of Venus. The Soviet Venera program is considered to be a great success, as it tremendously expanded our knowledge of Venus. Specifically, the Venera 9 lander took the first photographs of the planet's surface, and the orbiter section of the probe became the first artificial satellite of Venus.

The Venera program lasted for over two decades from 1961 to 1983, during which time the probes were of course gradually improved. The Venera 9 probe was the first of the series to be carried by the more powerful Proton rocket, allowing it to be five times heavier than previous missions. The entire spacecraft weighed 4936 kg, of which 1560 kg was the lander (including the heat shield), 2231 kg was the orbital part, and the rest of the mass was fuel. The spacecraft's rocket engine was able to develop a thrust of 18 900 N.

The easiest way to move between two nearby circular orbits is the Hohmann transfer orbit[2] It is a half-ellipse that touches both circular orbits.

a) Calculate the semi-major axis and eccentricity of the Hohmann transfer orbit for a journey from Earth to Venus.

Assume that the rocket carried the probe far enough away from Earth that its gravity can be neglected. The probe is now moving around the Sun along the same orbit as the Earth.

b) Calculate by how much the probe's velocity has to be reduced so as to move it to the Hohmann transfer orbit to Venus.

During the collision course approach to Venus, the lander separates. The orbital part is then guided into orbit around Venus. At a distance of 112 200 km above the planet's surface, a rocket engine is fired to put the orbital stage into an orbit with an apocentre at that point and a pericentre at 1510 km above the surface of Venus.

c) Find the change of the probe's velocity which is required for putting it into the above-described orbit around Venus.

These two maneuvers (the transition to the Hohmann transfer orbit and the ensuing transition to an orbit around Venus) undoubtedly require a large portion of fuel. While the first change of orbit was performed by the last stage of the launch vehicle, the Venus maneuver had to be performed by the orbital section's own rocket engine. For the sake of simplicity, let us neglect the fuel used for orbital corrections during flight and assume that all fuel was consumed during this maneuver.

d) Find the required exhaust velocity of the rocket engine. Note that the change of speed of the rocket as a function of the amount of fuel consumed and the exhaust velocity is described by the Tsiolkovsky rocket equation[3] Assume that the lander itself did not carry fuel.

The lander entered Venus' atmosphere at an altitude of 125 km at speed $10.7\,\mathrm{km\,s^{-1}}$.

e) Find the total energy released as the probe descended through the atmosphere. Calculate also the energy released per unit mass of the lander. Discuss of it is possible for this energy to be converted into heat only.

[a) 0.86 au, 0.163, b) $2.5\,\mathrm{km\,s^{-1}}$, c) slow down by $3.0\,\mathrm{km\,s^{-1}}$ in the direction towards Venus and gain $0.57\,\mathrm{km\,s^{-1}}$ perpendicular to the direction towards Venus, d) $7.3\,\mathrm{km\,s^{-1}}$, e) $9.10 \times 10^{10}\,\mathrm{J}$, $5.83 \times 10^{7}\,\mathrm{J\,kg^{-1}}$]

[2] `https://en.wikipedia.org/wiki/Hohmann_transfer_orbit`

[3] `https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation`

Three satellites AB/N/1-21

Over the course of one night, an observer on Earth (radius $R = 6\,378\,\mathrm{km}$), standing at latitude φ, noticed an unusual phenomenon. He observed three satellites orbiting in the equatorial plane, following the same trajectory in the sky so that at each moment he was able to see precisely one of them (that is, as the first satellite was setting, the second one was rising and so on).

a) Determine the altitude h above the Earth's surface at which these satellites are orbiting.
b) Calculate the altitude α of one of these satellites above the horizon (in degrees) at the time of its culmination.
c) Determine the period τ with which the cycle repeats.

Work numerically for observers in Prague ($\varphi = 50°\,05'$) and in Bogotá ($\varphi = 4°\,36'$).

[a) $2.12R$, $1.01R$, b) $22.7°$, $80.9°$, c) $50°\,05'$, $4°\,36'$]

Global storm AB/N/4-21

A large dust storm developed on Mars which uniformly enveloped the entire surface of the planet so that it weakened the magnitude of the Sun at zenith by $\Delta m = 0.5\,\mathrm{mag}$. Assuming that the dust grains have radius $r = 0.5\,\mathrm{mm}$ and density $\rho = 1.5\,\mathrm{g\,cm^{-3}}$, determine (an order-of-magnitude estimate is sufficient) the total mass M of dust particles in the Martian atmosphere (in kg).

[10^{13} kg]

Stellar astronomy

Classification of stars EF/N/1-21

Figure 3 shows the Hertzsprung–Russell diagram with 12 stars marked. Their luminosities and absolute magnitudes are plotted on the vertical axes, while their effective temperatures can be read off from the horizontal axis (note that hotter stars are on the left while the cooler ones are on the right), together with their spectral type. Determine which stars match the statements below. Only one star matches each statement.

___ Sun

___ Proxima Centauri, a red dwarf, the closest star to the Sun

___ Mira, the coolest star of the selection, a long-period variable giant

___ three bright blue stars from

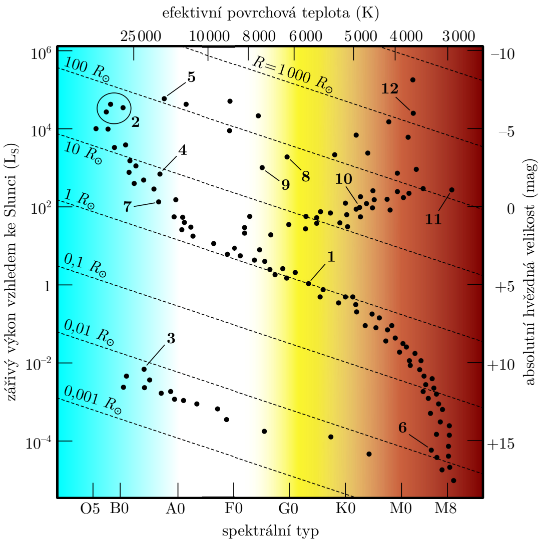


Figure 3: The HR diagram

Orion's belt

___ Delta Cephei, G0 giant, prototype of Cepheid variable stars

___ Sirius B, white dwarf

___ Arcturus, orange giant

___ Rigel, the brightest star of the selection

___ Betelgeuse, more than a thousand times the size of the Sun

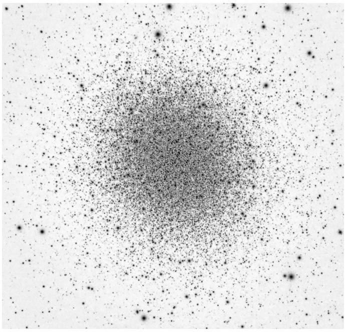
___ Alcyone (Eta Tauri), the brightest of the Seven Sisters (Pleiades) is a type B7 giant, about 10 times larger than the Sun and a thousand times more luminous

___ Regulus, a star of spectral type B7, which is on the main sequence

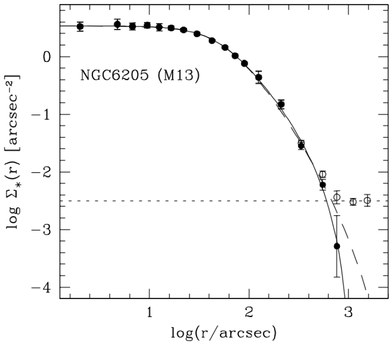
___ Polaris, also a Cepheid, but shorter period than Delta Cephei and less bright

Messier 13 CD/R/2-20

In this problem we will focus on one of the most famous objects of the Messier catalogue, the globular cluster M 13. This collection of distant (so-called deep-sky) objects, that appear in a small telescope as nebulous clouds, was created by Charles Messier in the second half of the 18th century to help with the



(a) Photography of the M 13 globular cluster (in inverted colours).



(b) Dependence of the area density of stars Σ_* on angular distance r from the centre of M 13: the points without filling represent the area density of all stars in the image, points with filling correspond only to stars belonging to M 13. The level of the galactic foreground is represented by a dashed horizontal line.

Figure 4: Data for problem CD/R/2-20 (source: Wikimedia and *Miocchi et al.* (2013)).

identification of new comets.

The globular cluster M 13 (fig. 4a) is located at a distance of $d = 6.8\,\text{kpc}$ from Earth and appears as a blurry spot in the sky with a total magnitude of $m = 5.8\,\text{mag}$.

a) Calculate the absolute magnitude M of the cluster (in mag).
b) Calculate the total luminosity L as a multiple of the solar luminosity $L_\odot$.

Figure 4b shows the dependence of the area density of stars appearing on the M 13 image (taken from Earth) on the angular distance from the centre of the cluster. Both axes are logarithmic in scale.

c) Based on the data in fig. 4b determine as accurately as possible the total number of stars N_* in the cluster (round to the nearest ten thousand).

Hint: divide the star cluster into several concentric shells.

For simplicity, let us assume from now on that all stars in the M 13 cluster are identical.

d) Calculate the luminosity L_0 (as a multiple of $L_\odot$) and the absolute magnitude M_0 (in mag) of the individual stars that form the cluster M 13.

Based on knowledge of the area density profile $\Sigma_*(r)$ it is in principle possible to reconstruct the dependence of volume density of stars $\rho_*(x)$ in the cluster on the radial distance x from the center of the cluster. But since this calculation is quite complicated, from now on we will model M 13 as a homogeneous sphere with radius R and a constant volume density of stars ρ_*.

e) Express the area density $\Sigma_{*,0} \equiv \Sigma_*(0)$ in the centre of the cluster in terms of R, ρ_* and d.
f) Express also the total number of stars N_* in the cluster in terms of R and ρ_*.
g) Determine the values of R (numerically in pc) and ρ_* (numerically in pc^{-3}) for the globular cluster M 13.

A space explorer has parked his spaceship exactly at the centre of the M 13 cluster.

h) What is the mean value of the apparent magnitude m_0 of the brightest star in the sky for an observer on the spaceship?
i) Calculate the combined magnitude m_{tot} of all stars that the explorer can observe (i.e. the magnitude of an object that would be created by moving all stars in the cluster to be located in one direction on the explorer's sky).
j) Can he read his favorite book given the illumination which is generated only by the stars in his sky?

[a) $-8.4\,\mathrm{mag}$, b) $1.9\times10^5 L_\odot$, c) 110 000, d) $1.7 L_\odot$, 4.2 mag, e) $\Sigma_{*,0} = 2R\rho_* d^2$, f) $N_* = \frac{4}{3}\pi R^3 \rho_*$, g) 4.1 pc, $380\,\mathrm{pc}^{-3}$, h) -7.2mag, i) -11.3 mag, j) no]

Amateur one — AB/N/2-21

An amateur astronomer from Opava heard about the discovery of a new triple star system. Thus, he took his Cassegrain telescope (with the primary mirror of diameter $D_1 = 203\,\mathrm{mm}$ and the secondary mirror of diameter $D_2 = 64\,\mathrm{mm}$) and traveled to the Beskydy Dark-Sky park. The triple star is known to have components of visual magnitude 14^{m}, 14.5^{m} and 16^{m}, and that they are so close to each other that the astronomer's telescope cannot resolve them. Find the limiting magnitude m_{D} of the astronomer's telescope, then the visual magnitude m_* of the triple star system and based on your results, decide if the astronomer was able to observe the triple star with his telescope.

[$m_{\mathrm{D}} = 13.5\,\mathrm{mag}$, $m_* = 13.4\,\mathrm{mag}$]

Binary systems, clusters and exoplanets

Elliptical galaxy CD/R/1-21

Since 1926, astronomers have used a scheme created by Edwin Hubble to classify galaxies according to their shape. This is usually represented in the shape of a tuning fork and divides galaxies into three basic categories: elliptical, spiral and barred spiral galaxies.

In this problem you will consider an elliptical galaxy of type E0 with mass M – meaning that the galaxy will be spherical. You will also assume that matter in the galaxy is distributed uniformly with some density ρ.

a) Express the acceleration $\mathbf{a}(\mathbf{r})$ of a point mass at distance r from the centre of the galaxy in terms of G, ρ and r (where G is Newton's gravitational constant).
b) Considering the behavior of the harmonic oscillator, determine the shape of the trajectories of point particles moving inside the galaxy.
c) Show that the orbital periods of all objects inside the galaxy are the same. Express this universal orbital period in terms of ρ and G.
d) Determine the numerical value of ρ (in $\mathrm{kg\,m^{-3}}$) assuming that $P = 10^8$ y.

Let us further assume that 80 % of the mass of the galaxy is dark matter and 20 % Sun-like stars.

e) Determine the number of stars n per cubic parsec of the volume of the galaxy.

f) Determine the total number of stars N that can be seen with an unaided eye by an observer at the centre of the galaxy. The limiting visual magnitude for naked-eye observations is 6 mag.

[a) centripetal acceleration with magnitude $a(r) = \frac{4}{3}G\pi\rho r$, b) ellipses, c) $\sqrt{\frac{3\pi}{G\rho}}$, d) $1.4 \times 10^{-20}\,\mathrm{kg\,m^{-3}}$, e) $0.041\,\mathrm{pc^{-3}}$, f) 900]

Double star I CD/N/2-21

The binary star has an annual parallax of $\pi = 0.06$". The two components are identical and each have an effective surface temperature $T = 7\,000\,\mathrm{K}$. Find the minimum separation of the components (in au) so that they can be resolved on images taken using an optical telescope with an aperture diameter of 8 cm. Assume that the telescope is equipped with a detector which can be characterized by a uniform spectral response function, i.e. that it has the same sensitivity at all wavelengths in the optical domain. Find the required the diameter of such a telescope so that it shows the two stars as discs (and not as point-like sources) given that we observe the binary system as an object with combined magnitude $m = 5.0\,\mathrm{mag}$. Consider the disc as having been resolved provided that we can image it with a resolution of at least 4×4 pixels. The bolometric correction and interstellar extinction should be neglected.

[1 050 m]

Double star II CD/N/5-21

A visual binary star (R.A. $\alpha = 19^{\mathrm{h}}\,12^{\mathrm{m}}\,33^{\mathrm{s}}$, Dec. $\delta = 34^\circ\,52'\,2''$), that lies at an unknown distance d from Earth consists of two identical components of unknown mass $M_1 = M_2 \equiv M$. Long-term astrometric observations show that the stars orbit around a common center of mass with period $P = 1.2\,\mathrm{y}$ along a circular orbit, whose radius r appears to the astronomers on Earth as subtending an angular distance $\rho = 0.01''$ on the sky.

On April 10, 2021 at 12:00 CEST one of the components was measured to have its proper motion $\mu_\alpha = 0.003^{\mathrm{s}}\,\mathrm{y}^{-1}$ in right ascension and $\mu_\delta = 0.03''\,\mathrm{y}^{-1}$ in declination. At the same moment the wavelength of the line H_α in the spectrum of this component was measured by a spectroscopic observation as $\lambda = 656.26\,\mathrm{nm}$, while the laboratory wavelength of H^α is $\lambda_0 = 656.28\,\mathrm{nm}$.

The proper motion of the binary's centre of mass and the effect of annual parallax are not considered in this problem.

a) Calculate numerically the magnitude v_{r} of radial velocity of the component in $\mathrm{km\,s^{-1}}$ at the time of measurement.
b) Calculate numerically the proper motion μ of the component on the Earth's sky in angular seconds per year at the moment of measurement.
c) Find a relationship between the quantities d, ρ, P, μ and v_{r}. It may help if you first express the magnitude v of the velocity of the components relative to the center of mass of the binary in various ways.
d) Calculate the distance d of the binary from the Earth in pc.
e) Determine the mass M of each component in units of the solar mass.

[a) $9.14\,\mathrm{km\,s^{-1}}$, b) $0.05''\,\mathrm{y}^{-1}$, c) $\frac{v_{\mathrm{r}}}{d} = \sqrt{\left(\frac{2\pi\rho}{P}\right)^2 - \mu^2}$, d) 88 pc, e) $M \doteq 1.9 M_{\mathrm{S}}$]

Weighing the exoplanets AB/R/2-20

In this problem we will calculate the mass of two exoplanets (51 Pegasi b and HAT-P-15 b) using the method of radial velocities. Let us denote the mass of the exoplanet and its parent star by M_{p} and M_*, respectively. Both components orbit around a mutual centre of mass with a period, which we denote by P.

Let us first assume that the orbit of the exoplanet is circular. This is (approximately) the case for the exoplanet 51 Pegasi b, which stood at the heart of M. Mayor's and D. Queloz's groundbreaking discovery from 1995, and whose radial velocity curve can be seen in Fig. 5. Mass of the parent star 51 Peg should be taken as $M_*^{51\text{ Peg}} = 1.05 M_\odot$.

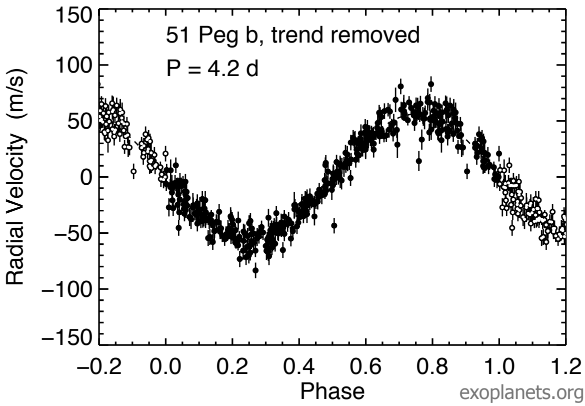


Figure 5: Radial velocity curve of the star 51 Pegasi.

a) Express the masses M_* and M_p in terms of G (gravitational constant), P and orbital speeds v_* and v_p of the star and the exoplanet, respectively, relative to the centre of mass of the system .

b) Express the maximum value K_* of the observed radial velocity of the star in terms of v_* and the inclination i (which is defined as the angle between the line of sight and the direction normal to the orbital plane).

From this point on, let us assume that $M_* \gg M_\mathrm{p}$.

c) Express the mass parameter $M_\mathrm{p} \sin i$ of an exoplanet in terms of G, P, M_* and K_*.

d) From Fig. 5, read off the value of K_* for the system 51 Pegasi. Calculate the value of the parameter $M_\mathrm{p} \sin i$ (in Jupiter's masses) for the exoplanet 51 Pegasi b.

Let us now focus on the exoplanet HAT-P-15 b. The radial velocity curve of its parent star is shown in Fig. 6. It is known that transits occur in this particular system, so that we are justified to assume that the line of sight lies in the exoplanet's orbital plane (that is, we will take the inclination to be approximately equal to 90°). Assume the mass of the parent star HAT-P-15 is $M_*^{\text{HAT-P-15}} = 1.01 M_\odot$.

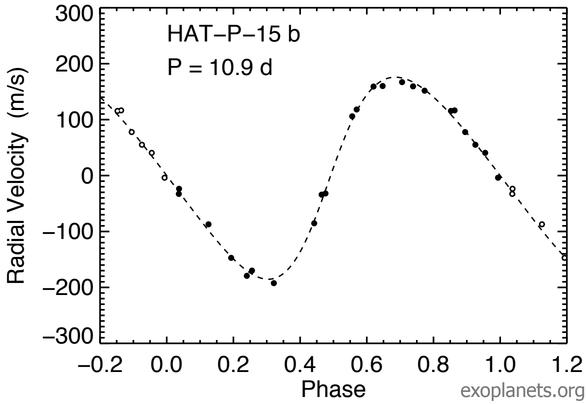


Figure 6: Radial velocity curve of the star HAT-P-15.

e) Draw (schematically) the shape of the orbit of the parent star HAT-P-15 around the centre of mass of the system. In your diagram, please mark the direction to the Earth, the direction of orbital motion, as well as the points at which the radial velocity observed on Earth is minimal, maximal, and zero.

Let us denote the observed values of the magnitude of the radial acceleration of the parent star in the periapsis and apoapsis as A_*^+ and A_*^-, respectively.

f) From Fig. 6, read off the values of A_*^+ and A_*^- (in $\mathrm{m\,s^{-2}}$) for the star HAT-P-15.

g) Express the eccentricities e_* and e_p of the orbits of the star and the exoplanet, respectively, as functions of A_*^+ and A_*^-. Find their numerical values for the exoplanetary system HAT-P-15.

h) Express the orbital speed $v_*(\theta)$ of the star as a function of the angle θ subtended between the position vector of the star and the direction to the periapsis of the orbit. Your function should have parameters G, P, M_*, M_p and e.

Hint: The distance of a point on the circumference of the ellipse from the focal point as a function of the angle θ is given by the *polar equation of ellipse*

$$r(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta}\,,$$

where a denotes the semi-major axis.

i) Show that the angle α subtended between the direction normal to the position vector and the direction tangent to the trajectory satisfies

$$\cos\alpha(\theta) = \frac{1+e\cos\theta}{\sqrt{1+2e\cos\theta+e^2}}\,, \qquad \sin\alpha(\theta) = \frac{e\sin\theta}{\sqrt{1+2e\cos\theta+e^2}}\,.$$

j) Also, show that the radial velocity $v_*^{\rm rad}$ of the star measured by an observer located in the direction of the periapsis satisfies $v_*^{\rm rad}(\theta) = K_*\sin\theta$, where you should express the parameter K_* in terms of G, P, M_*, $M_{\rm p}$ and e.

k) From Fig. 6, read off the value of K_* for the HAT-P-15 system. Calculate the mass of the exoplanet HAT-P-15 b (in Jupiter's masses).

[a) $M_* = \frac{P}{2\pi G}(v_* + v_{\rm p})^2 v_{\rm p}$, $M_{\rm p} = \frac{P}{2\pi G}(v_* + v_{\rm p})^2 v_*$, b) $K_* = v_*\sin i$, c) $M_{\rm p}\sin i = \left(\frac{PM_*^2}{2\pi G}\right)^{\frac{1}{3}} K_*$, d) $0.45 M_{\rm Jup}$, f) $A_*^+ \doteq 1.74\times10^{-3}\,{\rm m\,s^{-2}}$, $A_*^- \doteq 0.81\times10^{-3}\,{\rm m\,s^{-2}}$, g) $e_* = e_{\rm p} = (\sqrt{A_*^+} - \sqrt{A_*^-})/(\sqrt{A_*^+} + \sqrt{A_*^-})$, h) $v_*(\theta) = M_{\rm p}\left(\frac{2\pi G}{PM_*^2}\right)^{1/3}\sqrt{\frac{1+2e\cos\theta+e^2}{1-e^2}}$, j) $K_* = M_{\rm p}\left(\frac{2\pi G}{PM_*^2}\right)^{1/3}\sqrt{\frac{1}{1-e^2}}$, k) $K_*^{\text{HAT-P-15}} \doteq 180\,{\rm m\,s^{-1}}$, $M_{\rm p}^{\text{HAT-P-15}} \doteq 1.94 M_{\rm Jup}$]

Greenhouse effect AB/N/6-21

Consider an exoplanet of radius $R_{\rm e}$, which orbits its central star along a circular orbit at a distance d. Let us denote luminosity of the central star by L_*. Assume that the exoplanet is a perfect sphere with surface albedo A and has no internal heat source.

a) Find the surface temperature $T_{\rm e1}$ of the exoplanet assuming that its rotation period is short enough so that we can consider the temperature to be uniformly distributed. Express the result in terms of the quantities given above and the Stefan-Boltzmann constant σ.

Until now, we have assumed that the exoplanet has no atmosphere. We will now discuss how the result changes if we assume that an atmosphere is present. Let us model the atmosphere as a thin spherical layer above the planet's surface, which has different *transmission coefficients*[4] for radiation originating from the star (either directly incident on the atmosphere or reflected from the surface of the exoplanet) and black-body radiation that is emitted by the exoplanet. Let us denote these transmission coefficients by t_* and $t_{\rm e}$, $t_* \neq t_{\rm e}$,

[4] A transmission coefficient t of a thin layer is a real number taking values in the range $[0,1]$ and expresses the fraction of radiation intensity that passes through the layer. The fraction of radiation intensity that is reflected back is usually denoted by r, where $t+r=1$, meaning that the layer does not absorb heat.

respectively. You should also assume that their value is independent of the angle of incidence of the incoming radiation and that both coefficients are generally non-zero.

b) Show that for our simple model of the atmosphere, the equation of thermal equilibrium can be written in the form

$$\frac{t_*(1-A)}{1-A(1-t_*)}\frac{L_*}{4\pi d^2}\pi R_\mathrm{e}^2 = \frac{t_\mathrm{e}}{1-A(1-t_\mathrm{e})}4\pi R_\mathrm{e}^2 T_\mathrm{e2}^4\sigma\,.$$

Hint: You may find it useful that for $0 < |q| < 1$, we have

$$\sum_{n=0}^{\infty} aq^n = \frac{a}{1-q}\,.$$

c) Find the equilibrium temperature T_e2 of the exoplanet in the presence of an atmosphere described by the simple model discussed above.

d) Find the relationship between the transmission coefficients t_* and t_e such that the greenhouse effect can be observed (meaning that the equilibrium temperature increases compared to a setting in the absence of an atmosphere).

e) Find the surface temperature when $t_* = t_\mathrm{e}$.

[a) $T_\mathrm{e1} = \sqrt[4]{\frac{L_*(1-A)}{16\pi d^2\sigma}}$, c) $T_\mathrm{e2} = \sqrt[4]{\frac{L_*(1-A)}{16\pi d^2\sigma}\frac{t_*[1-A(1-t_\mathrm{e})]}{t_\mathrm{e}[1-A(1-t_*)]}}$, d) $t_* > t_\mathrm{e}$, e) we get T_e1 again]

Cosmology and relativity

Cosmic shower — AB/R/2-21

This year marks 85 years since Austrian physicist Victor F. Hess won the Nobel Prize for his discovery of cosmic rays. Since then, we have learned a lot about cosmic rays, thanks to projects like the H.E.S.S (High Energy Spectroscopy System), a system of several telescopes in Namibia which observe Cherenkov photons produced in cosmic ray showers.

Cosmic shower is a phenomenon where a high-energy particle penetrates the Earth's atmosphere, where it creates a cascade of particles and electromagnetic radiation. These showers are detected on the ground in various ways, allowing us to learn more about the particles that caused the showers and also the sources of these particles. Assuming a purely electromagnetic shower created by, for example, an ultra-relativistic electron, the following interactions may subsequently occur

$$
\begin{aligned}
e^- &\longrightarrow e^- + \gamma\,, \\
\gamma &\longrightarrow e^+ + e^-\,, \\
e^+ &\longrightarrow e^+ + \gamma\,.
\end{aligned}
$$

In other words, the primary electron is slowed down by interactions with atoms in the Earth's atmosphere. In doing so, it emits bremsstrahlung photons γ until it stops (first equation). The bremsstrahlung photons produce electron-positron pairs (second equation). The positrons then interact similarly to electrons and produce bremsstrahlung until they stop completely (third equation).

The shower reaches its maximum intensity[5] when the probability of creating electron-positron pairs is the same as the probability of ionizing the atoms in Earth's atmosphere. This occurs at the so-called critical energy, which is equal to $E_\mathrm{c} = 80\,\mathrm{MeV}$. One interaction (any of those described above) takes place on average over a distance of X. This distance is called the radiation length and is equal to $X = 304\,\mathrm{m}$.

a) Assume that an electron with energy $E_\mathrm{in} = 1\,\mathrm{TeV}$ enters the Earth's atmosphere and that the primary interaction occurred at altitude $H_0 = 15\,\mathrm{km}$ above the Earth's surface. Based on the above assumptions, estimate at what altitude the shower reaches its maximum intensity.

If any of the particles produced exceeds the speed of light in the atmosphere, Cherenkov radiation is produced. Imagine that we want to build an observatory consisting of several telescopes (similar to the H.E.S.S. experiment) in such a way that one shower could be observed by multiple telescopes at the same time so that we could obtain a stereoscopic image.

b) Roughly estimate the diameter of the circular area on the Earth that makes sense to be covered with telescopes, so that the telescopes are as far apart as possible (for a better stereoscopic effect), but the area is not unnecessarily large (a larger area covered means more telescopes and more costs for building the observatory). Justify your answer and support it with a calculation. Take the shower parameters from the previous problem, assume that the shower barely continues in the atmosphere after it reaches its maximum and that most of the particles produced are focused close to the axis of the shower. It is also known that Cherenkov photons are radiated isotropically along the entire path of the shower. Take the refractive

[5] The stage at which most particles are produced.

index of the atmosphere at the altitude where the shower normally occurs to be $n = 1.000\,1$.

Hint: Cherenkov radiation is emitted into a cone with a well-defined opening angle.

Telescope A, designed to observe the Cherenkov photons produced in the shower, captured the following event (see figure 7). The image captures 5° in the direction of the vertical axis. Using the image from Telescope B, which observed the shower from a nearby location, it was determined that the axis of the shower was perpendicular to the Earth's surface and passed at a distance $b = 100\,\mathrm{m}$ from the centre of Telescope A's mirror.

c) Estimate the linear dimension of this shower (i.e. the distance from the first interaction of the primary particle to the point of the shower's extinction).

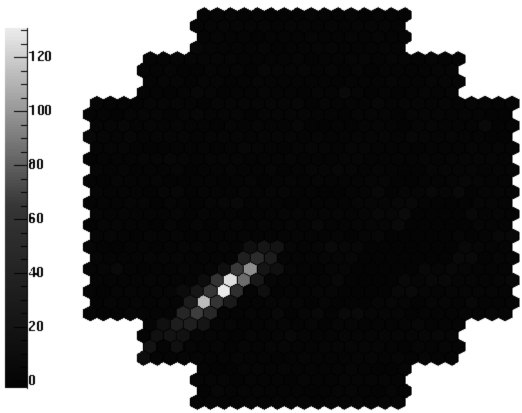


Figure 7: A snapshot of the shower from the Cherenkov telescope. The greyscale indicates the intensity of radiation detected by a given imaging element (expressed in terms of the number of photoelectrons produced). Adapted from an image available at `https://www.mpi-hd.mpg.de/hfm/HESS/`.

d) Estimate the uncertainty in determining the linear dimension of the shower.

If the primary particle is a hadron, π mesons are produced. These decay further and produce muons. Assume that the energy of the produced muon is $E_\mu = 4.8\,\mathrm{GeV}$ and its lifetime (measured in the muon's rest frame) is $\tau = 2.2 \times 10^{-6}\,\mathrm{s}$.

e) Find the lifetime of the muon as measured by an observer on Earth standing directly on the axis of the cosmic shower.

f) Find the maximum altitude at which the muon could have been created so that it can still be detected on Earth.

[a) 10 863 m, b) 274 m, c) 7 710 m, d) 1 800 m, e) 1.0×10^{-4} s, f) 30 km]

Decay of the meson AB/N/3-21

A π^0 meson with rest mass m_0 decays into two photons as $\pi^0 \to \gamma\gamma$. Let θ_1, θ_2 be the angles which express the deviation of the direction of flight of the photons from the original trajectory of the meson. Given the total energy E of the meson relative to the laboratory frame (LAB), determine

a) the energies E_1, E_2 of both photons in terms of E, m_0 and θ_1, θ_2,

b) the speed v of the meson relative to LAB provided that $E_1 = E_{\max}$ and $E_2 = E_{\min}$, i.e. in the situation where one photon had the smallest possible energy, while the other photon had the largest possible energy. Express the result in terms of $E_{\max}$, $E_{\min}$ and the speed of light c.

[a) $E_{1,2} = \frac{m_0^2 c^4}{2} \frac{1}{E - \cos\theta_{1,2}\sqrt{E^2 - m_0^2 c^4}}$, b) $v = c\frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$]

Practical problems

The sunspot EF/R/2-20

The Sun is made of gases, has no solid surface, and therefore rotates differentially, i.e., for different heliographic latitudes (coordinates similar to geographic latitudes but measured on the Sun) it will have different rotation speeds. As early as the 19th century, Richard Carrington studied the rotation of the Sun. Since there is no single fixed point on the Sun, he decided to fix the meridian passing through the center of the solar disk on January 1, 1854 as the zero meridian. The first rotation was counted at 12 hours UTC on November 9, 1853, when Carrington began to create a photographic series of the solar photosphere. Each successive revolution is a “day on the Sun”. The speed at which this meridian rotates can be determined, e.g., from the observations of sunspots in the photosphere in the equatorial region. Two drawings of the Sun's photosphere from the Ondřejov Solar Patrol are shown in the figure 8.

a) The rotational axis of the Sun is not perpendicular to the plane of the Earth's orbit around the Sun (the so-called ecliptic) or parallel to the Earth's rotational axis. Instead, it wobbles in space. This means that the Sun's equator also does not lie in the plane of the ecliptic nor in the plane of the Earth's equator. The plane parallel to the plane of the Earth's equator (i.e., the plane in which the Sun's daily motion takes place) is indicated in the drawings by the letters W (west) and E (east). Based on Figure 8, find the inclination of the solar equator relative to this plane at the time of observation.
b) Find the conversion between the dimension of the drawing in Figure 8 and the actual radius of the Sun.
c) Find the time which has elapsed between the two drawings.
d) The drawings of the Sun in Figure 8 show 4 spots. Choose the one that is most suitable for determining the Carrington rotation (i.e., the motion of the Carrington meridian).

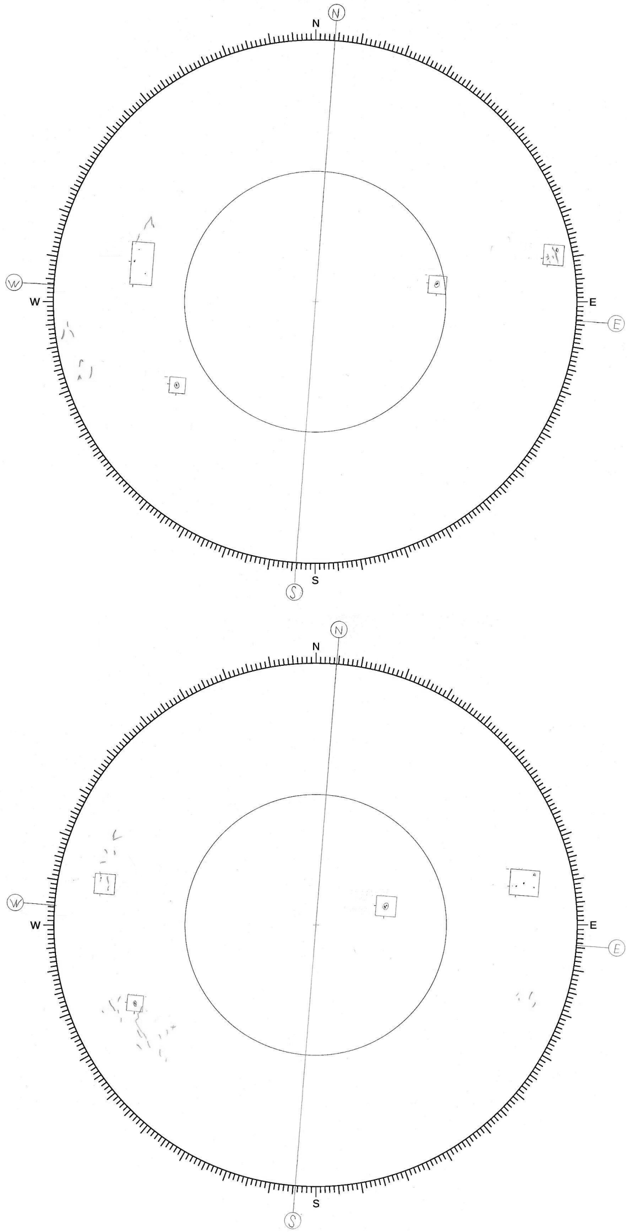


Figure 8: Drawings of the Sun on July 16, 2015 at 9:24 UTC and July 17, 2015 at 8:30 UTC with the axis of rotation highlighted.

e) In both drawings, measure the distance of the selected spot from the rotation axis of the Sun. Express this distance in cm, as well as in multiples of the Sun's radius.
f) What we see in the drawing is just the projection of the Sun onto the plane of the paper, but the Sun is actually a sphere. Calculate the angle (relative to the center of the Sun) by which the sunspot has moved between the two drawings, assuming that it has only moved within the plane of the equator.
g) Based on your measurements, find the duration of one Carrington rotation.
h) Find the Carrington rotation number corresponding to the two drawings.
i) The real Carrington rotation number at the time the drawings were taken was 2166. Explain why the calculated result differs from the real one and find the relative error we made in its determination.

[a) 4°, c) 1386 min, f) 13°, g) 26 d 15 h 42 min, h) 2215th Carrington rotation, i) 2.3 % due to measurement errors]

The Star of Bethlehem EF/R/1-21

The so-called Star of Bethlehem has decorated the Christmas sky in 2020. The conjunction of the planets Jupiter and Saturn alone is a rare phenomenon – we will have to wait until 2040 for the next one. Also, this phenomenon was probably the biblical event now known as the Star of Bethlehem in 7 BC. In 2020, moreover, it appeared in the sky during the winter solstice for the first time in more than 2 000 years.

a) How do we know if an object in the sky is a planet or a star? List at least three methods based on different principles.

Figures 9 and 10 show sky charts as seen one hour after sunset. The specific dates are unknown, but their knowledge is not necessary for the purposes of the problem (they correspond to certain two days in the past 2 028 years). We do know, however, that there is a difference of one month between the two charts. They show the stars that are visible to the naked eye (black circles – size corresponds to brightness), as well as the Sun (empty circle), the horizon (the circular arc), the sky below the horizon (shaded portion of the chart), and also the 4 planets of the Solar System (also black circles).

b) Find and circle all 4 planets in both charts.
c) Assuming the planets orbit the Sun in the same plane as the Earth (that is, in the plane of the ecliptic), draw the ecliptic as a straight line in Figure 9.
d) Find and label the planets Mercury and Mars in both Figures 9 and 10. Also, mark the position of the conjunction of the planets Jupiter and Saturn.

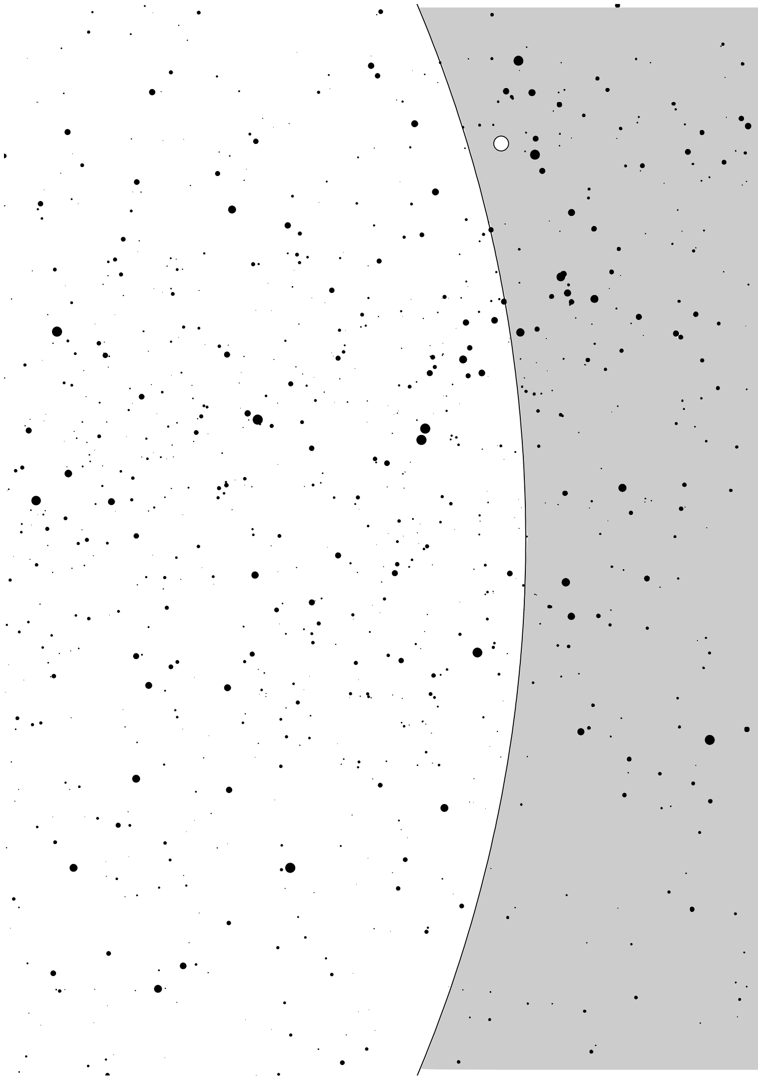


Figure 9: First sky chart.

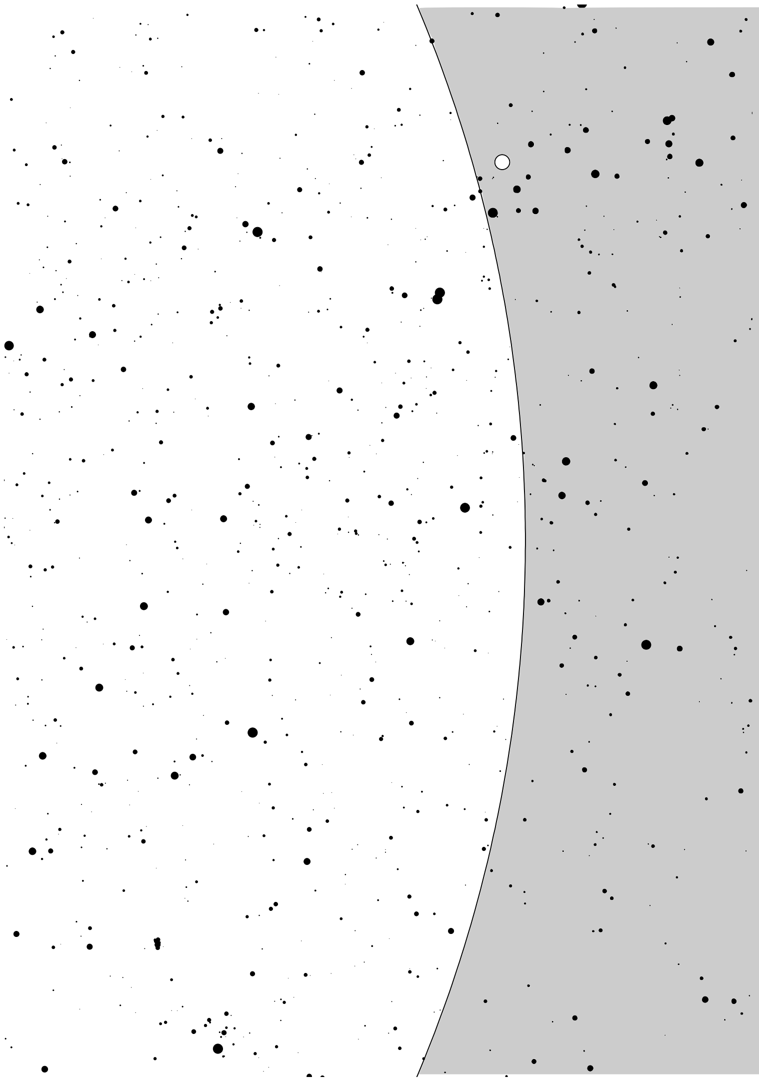


Figure 10: Second sky chart.

Hint: The planets Uranus and Neptune are not visible to the naked eye. Also, the solution can be found very quickly with a straightforward justification.

e) On days that correspond to the two charts, would one see Venus as Evening Star or Morning Star? Explain briefly.

Observation of a star EF/R/3-21

Pick a bright star of you choosing and use a sky chart to verify that it is actually a star (and not, for example, a planet). Choose a suitable observation site so that your star passes behind a lightning rod, mast or other narrow object.

a) Write down the address or GPS coordinates of the observation site and the observing conditions.
b) Write down the name/designation of the star.
c) Record the exact time at which the star passes behind the selected object. Be sure to note the time zone too. Then repeat the measurement after a few days.
d) Give a concise description of what you observed and explain the cause of the phenomenon.

Sky map EF/N/3-21

Figure 11 below shows two blind maps of the night sky (without solar system bodies) to which the following questions relate.

a) Determine which map of the two shows the northern sky.
b) Mark with a square (□) and the corresponding number (1 to 9) the position of the following stars:

1. Aldebaran
2. Alpha Centauri
3. Antares
4. Betelgeuse
5. Canopus
6. Castor
7. Deneb
8. Mizar
9. Regulus

c) Mark with a cross (×) and the corresponding number (1 to 8) the position of the following deep-sky objects:

1. Andromeda Galaxy (M31)
2. Lagoon Nebula (M6)
3. Pleiades (M45)
4. double cluster h and χ Per
5. Orion Nebula (M42)
6. Large Magellanic Cloud
7. Ptolemy Cluster (M7)
8. globular cluster 47 Tuc

d) The boundary of the constellation Canis Major looks like a rectangle. Mark it on both charts.
e) Using a triangle (△), mark the position of the radiant of the Perseid meteor shower.
f) Mark the position of the Sun on 31. 12. 2020 using a circle (◯).

Star-trails CD/R/3-20

Your job in this problem will be to take picture of the arcs that the stars trace out in the sky as a result of the Earth's rotation around its axis (the so-called star-trails). To do this, you can either take a number of short exposures which you then combine into a single image using suitable software, or you can just take one very long exposure.

a) Take a picture of star-trails so that it includes the North Celestial Pole. Aim for the longest possible arcs: the corresponding hour angle should reach at least $0^{\mathrm{h}}10^{\mathrm{m}}$. Describe you image processing procedure in detail. Record the time and place of the observation, the time of each exposure and camera parameters.
b) Describe the shape of the curves traced out by the stars in the picture depending on the declination δ_* of the given star and also on the declination δ_{c} of the center of the image. Justify your answer thoroughly.

Picture of Auriga AB/R/3-20

In this problem, your will take a photograph of a region in the winter sky in which you will identify as many objects as possible. Also, you will use your photograph to measure the angular distances of several pairs of stars. For the purpose of problem task, we do not recommend using a "fish-eye" lens.

a) Take a photograph of the region around the constellation of Auriga and so that all the objects that are mentioned below are contained in the photograph. Make as large a print-out of your photograph as possible (at least A4). Use inverted colours.
b) Record the date, time and the location where the picture was taken, as well as the parameters of your camera: brand, type, focal length and aperture.
c) In your photograph, mark the positions of the objects listed in the table below and label them by their appropriate number. Also, for each object, complete the required data (V-magnitude and name) in the table (make use e.g. of the SIMBAD database[6]).
d) Formulate and describe a method of determining the relative angular distances by measuring the positions of the objects in the photograph using

[6] http://simbad.u-strasbg.fr/simbad/

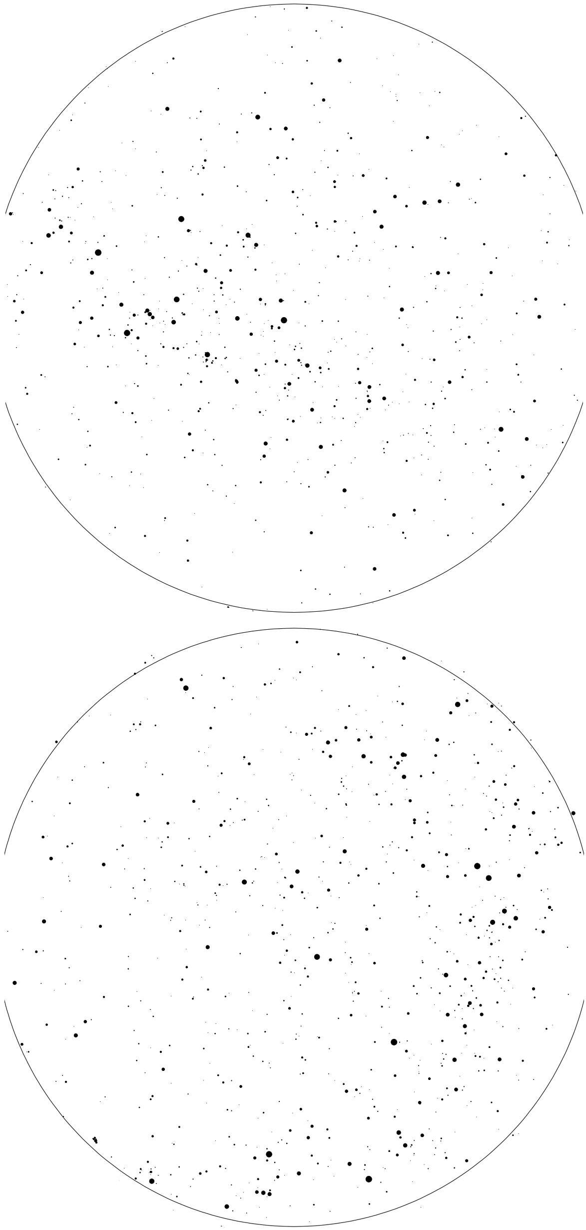


Figure 11: Sky maps for problem EF/N/3-21.

#	label	$\frac{V}{\text{mag}}$	name	#	label	$\frac{V}{\text{mag}}$	name
1	SAO 40186			6	Cr 71		
2	HR 1791			7	TYC 2391-1446-1		
3	Mel 38			8	HD 32630		
4	BD+22 739			9	WDS J05020+4349AB		
5	34 Aur			10	HIP 21421		

a ruler. You can assume the knowledge of one reference angular distance of two objects in the image.

Hint: Remember that the camera projects the sky on a detector which is planar. In the case of photographing objects "at infinity", we can approximately replace the lens system of the objective with a thin lens of a given focal length.

e) Apply your method to determine the angular distances of the following pairs of objects in the table (including an estimate of the uncertainties): objects 2 and 7, 1 and 10, 4 and 5, 5 and 10. Take the angular distance of objects 1 and 8 (which is $5°6'$) as a reference.
f) Calculate the actual angular distances of these pairs of stars and compare them with your measured values.

Size of the Moon AB/R/3-21

In this problem, you will measure the angular size of the Moon, as well as its changes over time, using a camera on a tripod. You will then use your measurements to determine the eccentricity of the Moon's orbit around the Earth.

a) Write down the type of camera you will use.
b) For the measurement purposes, set a fixed focal length of the lens (zoom) and the resolution of the pictures. Choose these parameters appropriately so that the effect of the change in angular size of the Moon between perigee and apogee can be measured. Record the values of these parameters in your solution.
c) Take several pictures of the Moon as close to the perigee and apogee as possible (use the Internet to find the times of perigee and apogee). For each photograph, record the time and location from where it was taken.
d) Determine the size of the Moon in each photograph in pixels.
e) Determine the average size D_p and D_a of the Moon in pixels at perigee and apogee, respectively. Estimate the uncertainties of the values of ΔD_p and ΔD_a.
f) Based on the data obtained, determine the numerical eccentricity e of the

Moon's orbit around the Earth and its uncertainty Δe.

g) Describe a method of correcting for the error introduced into the result by neglecting the finite size of the Earth.

V838 Monocerotis AB/N/7-21

At the start of the year 2002, the star V838 Mon exploded and immediately became the focus of astronomical interest. The explosion reached its peak on February 6, 2002. In the following months, the Hubble Space Telescope took several photographs of the light echo around the star. This very rare phenomenon can be used to estimate the distance to the object.

A light echo is created when a strong flash from a source gets reflected on interstellar dust. If we assume that each photon is reflected just once, we conclude that the illuminated dust must satisfy the equation $r + l = d + ct$, where r is the distance of the dust from the source, l is the distance of the dust from the observer, d is the distance between the source and the observer, and t is the time which elapsed since the flash was observed. From this equation it is easy to see that the illuminated dust has to lie on an ellipsoid with foci at the source and the observer.

In the case of V838 Mon, let us assume that the star was surrounded by several spherical dust shells on which bright rings were formed as a result of the light-echo effect. The radius r_e of such a ring which formed at time t after the observed explosion (and which corresponds to a spherical shell of radius r_0) satisfies $r_\mathrm{e} = \sqrt{2r_0ct - c^2t^2}$. This can be used to derive the rate of change of the radius r_e as

$$v = \frac{r_0c - c^2t}{\sqrt{2r_0ct - c^2t^2}}.$$

Figure 12 shows two pictures of the light echo taken with the Hubble Space Telescope. The upper one was taken on 30 April 2002 in the B filter, while the lower one is from 20 April 2002 and was created by combining the pictures in the B, V and I filters. Your job will be to identify the rings in both pictures that correspond to the same dust shell, to measure the angular speed of the ring expansion, and, finally, to estimate the distance to the star using a suitable relation between the angular speed of expansion and the angular radius of the rings.

a) Express the angular speed of expansion of a single ring in terms of its angular radius θ, the time t since the explosion was observed, the distance d of the star from the observer, and the speed of light c.

b) Identify at least five pairs of rings in both pictures that belong to the same shell. Measure the radii of these rings at at least five points. Using these

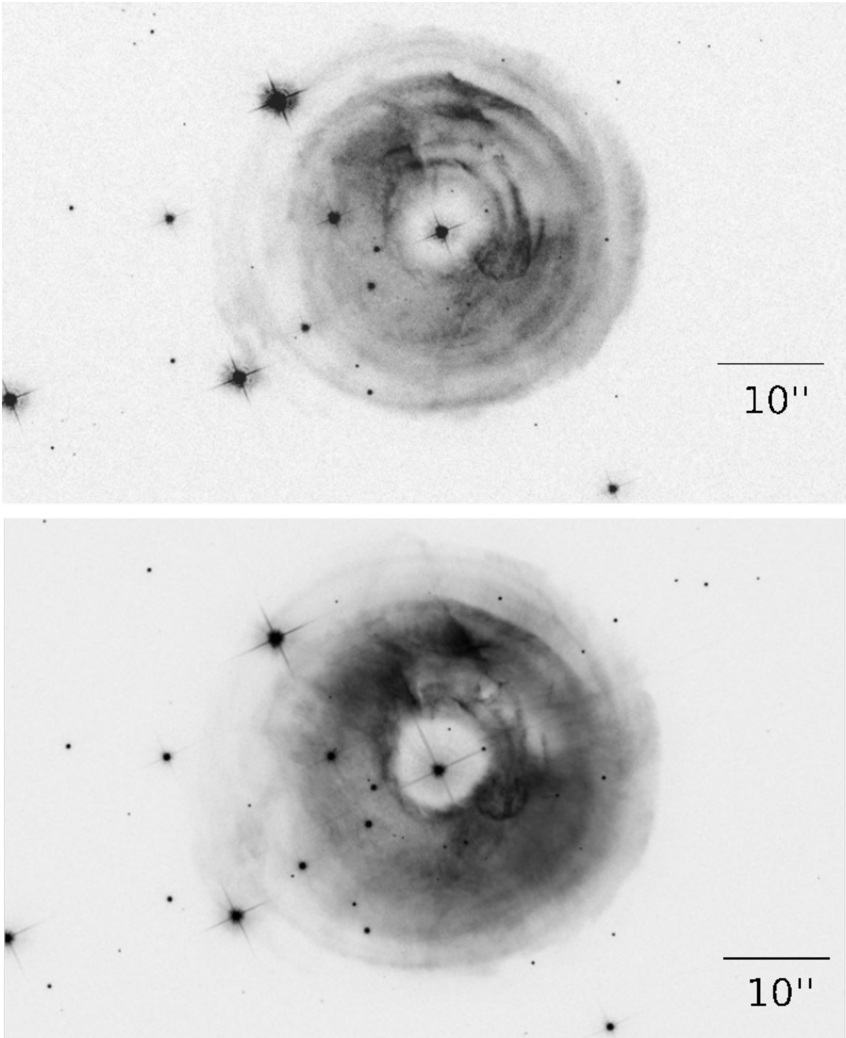


Figure 12: Top: picture of V838 Mon taken on April 30, 2002, bottom: picture taken May 20, 2002. Source: E. Bond et al. (2003)

measurements, determine the average radius of each ring and calculate the angular speed of expansion.

c) Plot the dependence of $\omega\theta$ on θ^2 for $t = 93\,\mathrm{d}$. For the angular radius take its average value.

d) Using the method of least squares, fit a suitable linear dependence through the points in your plot.

Method of least squares: given data (x_i, y_i) for $i = 1, \ldots, N$ a line $y_i = \alpha + \beta x_i$ can be fitted, where the expected values $\overline{\beta}$, $\overline{\alpha}$ of the parameters β, α of this line can be found as

$$\overline{\beta} = \frac{N\sigma_{xy} - \sigma_x\sigma_y}{N\sigma_{xx} - \sigma_x^2} \qquad \text{and} \qquad \overline{\alpha} = \frac{1}{N}\left(\sigma_y - \overline{\beta}\sigma_x\right) \, ,$$

where $\sigma_x = \sum_i x_i$, $\sigma_y = \sum_i y_i$, $\sigma_{xy} = \sum_i x_i y_i$, $\sigma_{xx} = \sum_i x_i^2$ a $\sigma_{yy} = \sum_i y_i^2$.

e) By reading off a suitable quantity from your plot, determine the distance of the star V838 Mon.

[a) $\frac{\theta}{2t} - \frac{c^2 t}{2d^2\theta}$, e) $8\,\mathrm{kpc}$]

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Bolid Příbram
60 years since the impact (1959 – 2019)

On **7 April 1959, at 20:30.20 local time (19:30.20 UTC)**, a very bright bolide passed over the territory of former Czechoslovakia. Its flight was recorded by the cameras of the meteor observation network. The so-called „*Příbram meteorite*“ was the first meteorite in the world to be found on the basis of images recording its trajectory in the atmosphere. The main credit for this achievement goes to *Zdeněk Ceplecha* (1929–2009), who proposed methods for calculating the „dark flight trajectory“, a new scientific term he also introduced. He had already proposed a method to calculate the path of a meteor in the atmosphere from the luminous trajectory recorded in an image, as well as the path of the original body – a meteoroid – in the Solar System.

By accurately observing and calculating its trajectory, it was possible to estimate its mass at entry into the Earth‘s atmosphere at 1300 kg, its final mass before splitting at 53 kg, its initial velocity at 21 km/s and its altitude before splitting at 12 km. Based on the predicted impact site east of Pribram, a total of 4 pieces of the original body were recovered, named after the villages near the finding sites: Luhy – 4.48 kg, Velká – 0.8 kg, Hojšín – 0.42 kg, Drážkov – 0.1 kg. These meteorites are now exhibited in the National Museum in Prague. It was also the first time that the origin of a meteorite in the main belt of asteroids has been demonstrated.

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