Problem Booklet
2018/19

Czech Astronomy Olympiad

Board of Organizers of the Czech Astronomy Olympiad

Prague, 2019

Czech Astronomy Olympiad

Problem Booklet 2018/19

Board of Organizers
of the Czech Astronomy Olympiad

Edited by Jakub Vošmera, Václav Pavlík, Jan Kožuško, Tomáš Gráf
Cover design by Tomáš Gráf and Václav Pavlík
Published by the Czech Astronomical Society, Fričova 298, 25165 Ondřejov, Czechia
First edition in Prague, 2019
Typset in LaTeX
Printed by ON tisk, s.r.o., Křesomyslova 384/17F, 140 00 Praha 4, Czechia

This publication is not for sale.

`http://olympiada.astro.cz`

ISBN: 978-80-907341-1-1

Contents

Introduction

Foreword

Dear friends,

for the fourth time, we present you with a new booklet containing the last year's problems of the Czech Astronomy Olympiad. Number four has a special meaning in science and astronomy. It is the smallest composite number. We have four terrestrial and four gas giant planets in our solar system. Jupiter's four Galilean moons were the first moons of another planet ever discovered. Ancient philosophy refers to four primary elements. And last but not least, four is the square root of 16 - the age of our national olympiad.

Czech Astronomical Society, the main organizer of the Olympiad, is a respected and amazing association with more than 100 years of tradition, where amateur and professional astronomers get together in a single organization. Its 600 members work in 17 branches, covering the most important areas ranging from solar astronomy over variable stars to cosmology. Aside from the research topics, the society concentrates immense effort into working with young astronomers. In addition to the Olympiad (which itself attracts about 10 000 participants annually), it organizes summer schools and camps for children and youth together with astronomical courses, clubs and more.

The Olympiad is divided into four age groups (called – from the oldest to the youngest – AB, CD, EF and GH), each having three stages. The first round takes place at school with its main objective being to attract pupils to astronomy and motivate them for further work. In the second (regional) round, participants have to solve more complex problems as well as to perform observational tasks. The best participants go forward to the national rounds held in Opava and Prague in March and May, respectively, the winners of which then meet at the qualification camps for the IAO and IOAA.

Designing tens of challenging problems every year is hard. Very important step in this process is acquiring the initial idea, the inspiration often coming quite unexpectedly: while traveling on a train, subway or in an elevator, while talking to friends in a pub, swimming in a pool or even in a bathroom. Once the idea is there, the initial proposal follows and after a careful review process, the final version is brought to the contestants and also to you.

As the Czech language belongs to rather difficult and less popular ones (try to read this: Strč prst skrz krk!), we hope that you pick up our offer of this English-language booklet and dive into the presented problems, maybe finding further inspiration for you or your students.

We wish you dark skies and nice reading!

On behalf of the Organizers of the Czech AO

Jan Kožuško

Legend and acknowledgements

Each problem presented in this booklet comes with its name and ID code containing information about the place of its original use in the Olympiad. For instance, "CD/R/2" denotes the second problem in the regional round of the CD category. Finally, all problems have their answers shown in small print below.

Most of the competition problems that appear in Czech Astronomy Olympiad are original work of its organizers. Credits for the problems presented in this volume are now given:

Martin Blaschke: AB/R/1; *Stanislav Fořt:* AB/R/2; *Petra Hyklová:* EF/N/3; *Ota Kéhar:* EF/R/2; *Pavel Kůs:* AB/R/4, AB/R/5, AB/N/4, CD/N/2, CD/R/4; *Martin Raszyk:* AB/R/3; *Jaromír Mielec:* AB/N/8, CD/R/3, CD/N/5, CD/N/7; *Václav Pavlík:* EF/R/1, EF/R/2, EF/R/3, EF/N/1, EF/N/2; *Jiří Vala:* AB/N/6, CD/N/6, CD/N/8; *Ondřej Theiner:* CD/R/2; *Jakub Vošmera:* AB/N/2, AB/N/1, AB/N/3, AB/N/5, AB/N/7, CD/R/1, CD/N/1, CD/N/3, CD/N/4

The reader certainly would not be able to enjoy the problems in their present form were it not for the meticulous work of *Miroslav Randa*, *Ota Kéhar* and *Michal Švanda* who carefully reviewed all questions.

Finally, we want to express our immense gratitude to the director of the Prague Observatory and Planetarium, *Jakub Rozehnal*, and the vice-dean of Faculty of Philosophy and Science in Opava, *Tomáš Gráf*, for kindly providing the venue and related services for the national rounds. We also thank *Jan Kožuško*, *Lenka Soumarová* for helping make the Czech Astronomy Olympiad happen by diligently providing their support.

Theoretical problems

Geometry, time and instrumentation

Meteorite-struck EF/R/3

It is said that it is extremely rare to be hit by a meteorite on the Earth. Let us try to investigate how rare it is on the Moon.

a) Look up the radius of the Moon. Assuming that it is a perfect sphere, calculate its surface area.

About 2 800 kg of cosmic material falls onto the Moon daily. Most of it consists of microscopic particles and dust. Assume that all of them had the size of air rifle bullets with mass 0.500 g and that they cover the surface of the Moon homogeneously.

b) Calculate the surface area on which (on average) exactly one bullet falls per day. Round the result to 3 significant digits.
c) The Apollo 11 mission explored about $750\,\mathrm{m}^2$ of the Moon's surface. How many days on average should we wait until one meteorite of the size of this bullet falls onto this area?
d) What are the odds that such a body will hit an astronaut during a day on the Moon outside the landing module?

[a) 1737 km, $37.90 \times 10^6\,\mathrm{km}^2$; b) $6.77\,\mathrm{km}^2$; c) 9009 days; d) 1 : 10 000 000]

Lunar eclipse EF/N/1

During a Lunar eclipse, the Moon traverses the Earth's shadow which consists of the penumbra and the umbra. Astrophotographers sometimes make a series of nice colourful pictures with the Moon disappearing and then again reappearing. One of these series is shown in Figure 1, which covers the eclipse on January 21, 2019.

a) What type of eclipse was this?
b) While passing through the penumbra, the Moon is dark grey. What colour

does the Moon have while passing through the umbra? Explain your answer.

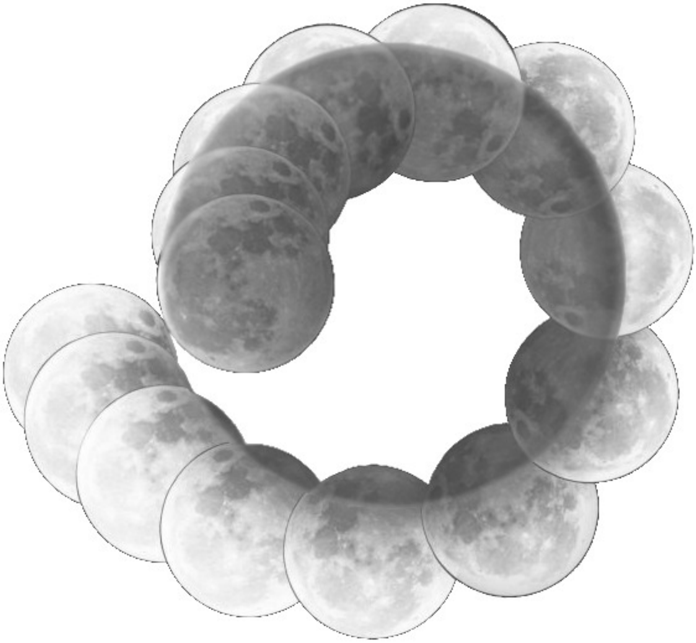


Figure 1: A series of pictures from the Lunar eclipse on January 21, 2019. (Photo: George Schmiesing)

c) How far from the Sun was the Earth on the day the eclipse occurred?
d) How long was the Earth's shadow (measured from the centre of the Earth)?

Assume that all celestial bodies are perfect spheres with equatorial radius.

e) Use Figure 1 to determine how far from the Earth the Moon during this eclipse was.

[a) total; b) brown / red / orange; c) 1.471×10^8 km; d) 1.360×10^6 km; e) 359 000 km]

Everyday astronomy CD/N/2

A wannabe astronomer constructed a telescope of Kepler type. The telescope has aperture with diameter $D = 8\,\mathrm{cm}$ and focal length of the objective $f = 100\,\mathrm{cm}$. Find the smallest possible diameter l of a crater on the surface of the Moon, such that it can be resolved by the telescope. Find the size of the image of such crater in the focal plane of the telescope. The distance of the Moon from the Earth is $a_{\mathrm{M}} = 384\,000\,\mathrm{km}$ and the diameter of the Moon is $D_{\mathrm{M}} = 3\,474\,\mathrm{km}$. The mean wavelength of visible light is $\lambda = 500\,\mathrm{nm}$.

[$l = 2.9\,\mathrm{km}$, $s = 0.9\,\mathrm{cm}$]

A traveling star CD/N/5

τ Ceti is one of the closest stars with physical parameters similar to those of our Sun. It is also very likely to be an isolated star. Its coordinates are $\delta = -15^\circ\,56'\,14.93''$ and $\alpha = 1^{\mathrm{h}}\,44^{\mathrm{m}}\,4.08^{\mathrm{s}}$. Its parallax was measured as $\pi = 0.274\,2''$ while its proper motion was determined to be $\mu_\alpha = -1\,721.05\,\mathrm{mas\,yr^{-1}}$ in right ascension and $\mu_\delta = 854.16\,\mathrm{mas\,yr^{-1}}$ in declination.

a) Find the distance r of the star from the Sun.
b) Find the components v_α and v_δ of the velocity tangent to the celestial sphere corresponding to the proper motion of the star.

On the day of the national round of the Czech Astronomy Olympiad (May 9, 2019), an Earth-based spectrographical measurement of the star τ Ceti yielded the redshift $z = -0.000\,096$.

c) Find the speed v_{rZ} at which the star was approaching the Earth as the measurement was being performed.
d) Find the ecliptical coordinates $\beta_\odot, \lambda_\odot$ of the Sun on the day of the observation.
e) Find the ecliptical coordinates β, λ of the star. The inclination of celestial equator relative to the plane of the ecliptic should be taken as $\varepsilon = 23.5^\circ$. The right ascension of the northern ecliptic pole is 18^{h}.
f) Find the speed v_{r} at which the star approaches the Sun.
g) Find the speed v at which the star moves relative to the Sun.
h) Find the distance $r_{\min}$ of the closest approach of τ Ceti to the Sun.

[a) $r = 3.65\,\mathrm{pc}$; b) $v_\delta = 14.78\,\mathrm{km\,s^{-1}}$, $v_\alpha = -28.63\,\mathrm{km\,s^{-1}}$; c) $v_{\mathrm{rZ}} = -28.8\,\mathrm{km\,s^{-1}}$; d) $\beta_\odot = 0^\circ$, $\lambda_\odot = 48.3^\circ$; e) $\beta = -24.8^\circ$, $\lambda = 17.8^\circ$; f) $v_{\mathrm{r}} = -16.3\,\mathrm{km\,s^{-1}}$; g) $v = 36.1\,\mathrm{km\,s^{-1}}$; h) $r_{\min} = 3.26\,\mathrm{pc}$]

Bad day CD/R/2

The first successful measurement of Earth's circumference is usually ascribed to Eratosthenes of Cyrene who once overheard a rumor about a very deep water well in Syene (today's Aswan) whose bottom would only be illuminated at noon on the summer solstice day, the walls remaining dark. Story has it that Eratosthenes went on to stick a pole vertically into the ground in Alexandria and determined the length of its shadow at noon on the summer solstice day. This information was enough for him to determine the difference between the latitudes of Alexandria and Syene. Based on the known distance of the two cities and the fact that they lie on the same meridian, he was then able to compute the circumference of the Earth.

Let us now consider an absent-minded astronomer who lives on a desert island. One day, as he was walking around his island, he remembered that the Sun was just passing through the vernal equinox point. Unfortunately for him, this happened just at the moment when the Sun was in zenith so that he forgot to watch his step and fell into a deep and empty water well. As he was the lone inhabitant of the island so that there was nobody to help him out, he had to come to terms with the fact that the water well would become his new observation post.

Assume that the water well had a circular cross-section and that its depth and radius were $H = 15\,\mathrm{m}$ and $r = 1\,\mathrm{m}$, respectively. The astronomer's eyes are $h = 1.75\,\mathrm{m}$ above ground and he always undertakes his observations near the center of the well. Atmospheric refraction can be neglected.

a) What fraction of the sky can the astronomer see at any point in time?
b) What fraction of the sky can be observed by the astronomer over the course of one year?
c) Find the number of days during which the Sun is visible from the well (even partially) during one year. The angular diameter of the Sun is $D = 32'$. Assume that the Earth orbits the Sun along a circle.

[a) 0.14 %; b) 7.5 %; c) 48 days]

A picture of the shadows — AB/N/6

A photographer, situated at an unknown location on the Earth, placed his camera directly above a vertical pole with length $H = 1\,\mathrm{m}$ so that it captures the pole and its surroundings. As soon as the Sun crossed the southern arc of his local meridian, the photographer took a picture of the pole and its shadow. He then went on to take three more snapshots separated by 6 h. After combining all four photographs, he obtained the picture shown in Figure 2.

The lengths of the shortest and the longest shadow were measured as $d_1 = 2\,\mathrm{m}$ and $d_3 = 5\,\mathrm{m}$, respectively. Assume that the Earth orbits the Sun along a circle with period $T_{\mathrm{year}} = 365.24\,\mathrm{d}$, that the vernal equinox occurred on March 21, that the declination of the Sun does not change over a period of one day and that Earth's axial tilt is $\varepsilon = 23.44°$. Neglect atmospheric refraction.

a) Find the altitudes h_1 and h_3 of the Sun above the local horizon at the time when the shadow was longest and shortest, respectively.
b) Find the declination $\delta_\odot$ of the Sun on the above given day and the latitude ϕ of the place where the photographer was based.

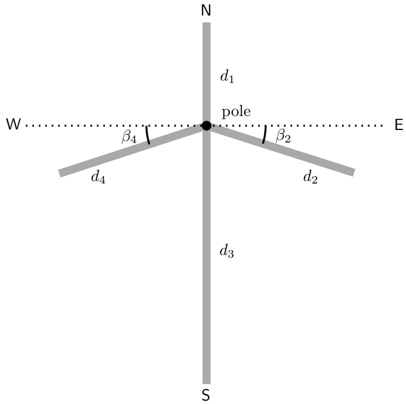


Figure 2: The four snapshots of the shadow combined into a single picture (as seen from above the pole, schematically).

c) Find the lengths d_2 and d_4 of the remaining two shadows.
d) Find the angles β_2 and β_4.
e) Determine the calendar day on which the photographs were taken.
f) Give a brief explanation why the tip of the shadow traces out a conic section (for general ϕ and $\delta_\odot$).
g) State the necessary conditions on $\delta_\odot$ and ϕ, so that the tip of the shadow describes an ellipse, parabola and hyperbola, respectively.
h) For arbitrary ϕ, find a condition on $\delta_\odot$, so that the tip of the shadow moves along a straight line. Express the distance b of this line from the pole in terms of H a ϕ.

[a) $h_1 = 26°\,34'$, $h_3 = 11°\,19'$; b) $\delta_\odot = 18°\,56'$, $\phi = 82°\,22'$; c) $d_{2,4} = 2.94\,\mathrm{m}$; d) $\beta_{2,4} = 2°36'$; e) May 15 or July 26; g) $|\delta_\odot - \phi| < 90°$, $|\delta_\odot + \phi| > 90°$ for an ellipse, $|\delta_\odot + \phi| = 90°$ for a parabola, $|\delta_\odot + \phi| < 90°$ for a hyperbola; h) $b = H|\mathrm{tg}\,\phi|$]

Solar system

‘Oumuamua EF/N/2

On October 19, 2017 the telescopes Pan-STARRS on Hawaii islands discovered the first interstellar object. It was given the name 1I/‘Oumuamua, which means “a messenger from the past”. Its light curve (i.e. its brightness as a function of time) as measured on October 27, 2017 is shown in Figure 3.

a) Find the maximum change in the object’s magnitude on that day.

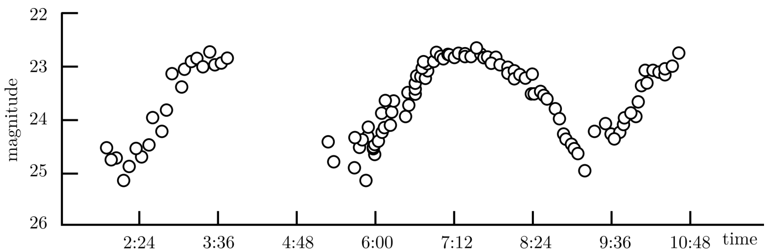


Figure 3: Light curve of 1I/'Oumuamua from October 27, 2017.

Assume that 'Oumuamua resembles a rugby ball (an ellipsoid), as illustrated in Figure 4. Assume that the rotation axis is positioned along the semi-minor axis b of the ellipsoid and that it is perpendicular to the line of sight.

b) Determine for which orientation is the brightness of the object maximal and minimal, respectively.

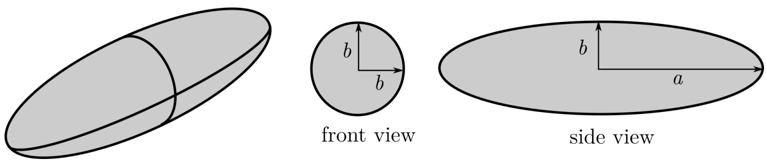


Figure 4: The shape of 'Oumuamua.

c) Using you previous results, find the ratio of semi-major axes $a : b$ (write it as a ratio of two integers).

[a) $\Delta m = 2.5\,\text{mag}$; c) $a : b = 10 : 1$.]

Jupiter's moons **EF/N/3**

All giant gaseous planets in our solar system appear to have a lot of natural satellites (moons). The first four such objects to be discovered (by Galileo Galilei) belong to Jupiter. We now call them Galilean moons. Table 1 lists some basic data about a number of moons.

a) Find the diameter of Jupiter in a scale $1 : 10^{10}$. Its real radius is $70\,000\,\text{km}$.
b) Write down the names of the Galilean moons and sort them by their ascending distance from Jupiter. Calculate their distance from Jupiter in the same scale (i.e. $1 : 10^{10}$).
c) Do all the moons in Table 1 belong to Jupiter? If so, write "yes", if not, write out the extra ones here.

Table 1: Some parameters of selected moons.

Name	Semi-major axis [km]	Diameter [km]
Adrastea	129 000	16
Amalthea	181 400	168
Callisto	1 882 700	4821
Elara	11 741 000	78
Europa	671 100	3122
Ganymedes	1 070 400	5262
Himalia	11 461 000	160
Io	421 800	3643
Leda	11 165 000	18
Lysithea	11 717 000	38
Metis	128 000	44
Thebe	221 900	98
Titan	1 121 865	5150
Triton	354 800	2706

d) Draw Jupiter and the Galilean moons viewed from above in a scale $1:10^{10}$. Assume that they are aligned and that their orbits are circular.

[a) 7 mm; b) Io 42 mm, Europa 67 mm, Ganymede 107 mm, Callisto 188 mm; c) Triton, Titan]

Conjunction of Mars and the Moon EF/R/1

An observer in Pisa took two pictures of Mars and the Moon before and after their conjunction (see Figure 5). The Moon was in the first quarter. Both pictures are separated by one day, with both the time in the day (18:35 CET), as well as the position of the camera being the same. Assume that the angular size of the Moon is $30'$.

a) Use the photographs to deduce the conversion between degrees and millimetres on the picture. Measure size of the longer side of the picture canvas in mm.
b) Find the length which the Moon traveled between the two pictures (in both linear and angular units). Describe the method which you have used.
c) Assuming that Moon's orbit is a circle and that Mars is stationary, use your previous results to calculate the sidereal period of the Moon.
d) The real value of the Moon's sidereal period is $T_{\rm sid} = 27.32$ days. Use it to calculate the mean motion of the Moon on the sky.

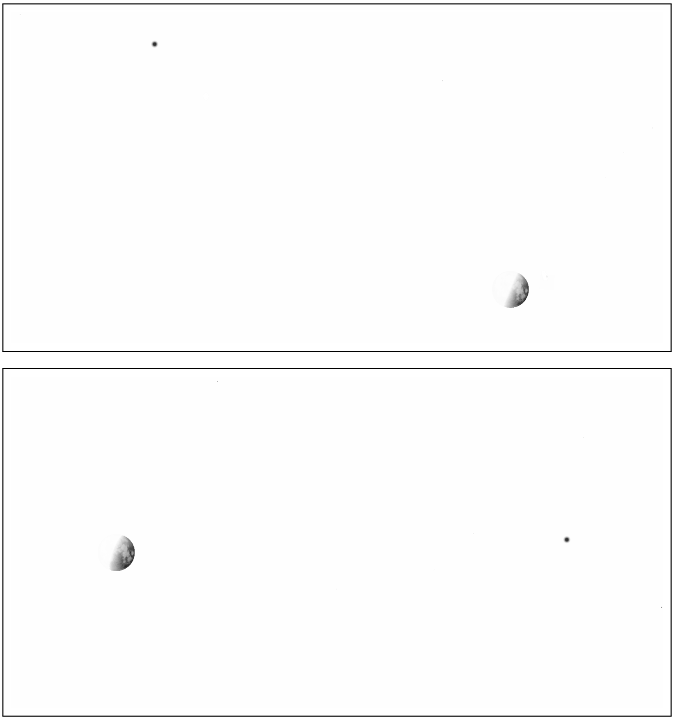


Figure 5: Photo of Mars and the Moon in inverted colours.

We are now going to explain why the value of the mean motion of the Moon as measured in part b) is different from the value calculated in part d). Below we point out two simplifying assumptions which we secretly used. Your task will be to asses the size of the errors which we picked up by using these assumptions.

e) We assumed that Mars is stationary. However, Mars, in fact, moved in a prograde fashion along the ecliptic.

f) We also assumed that the Moon orbits the Earth along a circle. This is, however, not the case and as a consequence, its orbital speed is not constant. Note that its maximum speed is 1.082 km/s while its minimum speed is 0.968 km/s.

g) Decide whether the errors incurred by using the above simplifications are sufficient to explain the difference between the measured and real values. Are there any more errors which we might have overlooked?

[a) 176 mm; b) 216 mm, 11.4°; c) 760 h $\approx$ 31.7 d; d) 13.2° d^{-1}; e) 0.52° d^{-1}; f) 1.5° d^{-1}]

An unidentified object CD/N/3

Find the sidereal orbital period T_{sid} (in days) and the semi-major axis a (in au) of an object which orbits the Sun in the plane of the ecliptic and which comes to opposition once in 290 days.

[$T_{\mathrm{sid}} = 1\,408\,\mathrm{d}$, $a = 2.46\,\mathrm{au}$]

Return of the samples CD/N/4

Find the speed v of the re-entry into Earth's atmosphere of a capsule containing samples from Jupiter's moon Europa, which was sent back along a prograde Hohmann trajectory. The orbits of both Earth and Jupiter should be considered circular. Remember that the vertices of the Hohmann ellipse are determined by the locations of the initial and target objects.

[$v = 14.2\,\mathrm{km\,s^{-1}}$]

Space agency of the town of Kocourkov CD/R/3

Citizens of the town of Kocourkov[1] decided to launch an interplanetary spacecraft on a journey to Mars. They first brought it into a circular orbit around the Earth where it waited for the right moment to set off for the journey, i.e. when the spacecraft has the maximum possible heliocentric speed. The altitude of this orbit above the Earth's surface was $h = 10\,000\,\mathrm{km}$. Assume that the Earth's orbit around the Sun is circular.

a) Determine the geocentric speed v_{G} of the spacecraft.
b) What it the maximum heliocentric speed v_{H} that the spacecraft can reach?

Having gained the maximum heliocentric speed, the spacecraft ignited its engines in order to reach remote planets. However, technicians from Kocourkov made a mistake in their calculations due to which there was only $\mu = 77\,\mathrm{kg}$ of fuel left for the maneuver. The speed at which the exhaust leaves the nozzles is $v_{\mathrm{e}} = 5\,000\,\mathrm{m\,s^{-1}}$.

c) What is the maximum achievable difference Δv of the speed before and after the maneuver? The mass of the spacecraft throughout the duration of the maneuver can be considered $m = 2\,500\,\mathrm{kg}$.

An unexpected error occurred almost immediately after completion of the maneuver. This led to a decrease of spacecraft's geocentric speed to a value of $v = 3\,\mathrm{km\,s^{-1}}$ without changing its direction. As a result, the spacecraft started orbiting the Earth along an elliptic trajectory.

[1] A fictitious town known for its mismanagement.

d) Find the theoretical distance $r_{\min}$ of the closest approach to the Earth's center.

Hint: The total mechanical energy of the body with mass m orbiting in the gravitational field of a massive body with mass M is $E = -\frac{GMm}{2a}$, where a is the semi-major axis.

The result of part d) suggests that the spacecraft will hit the Earth.

e) Find the incidence angle α of the spacecraft when it hits the Earth. Atmospheric effects can be neglected. Angle α is measured relative to the plane perpendicular to Earth's surface.

[a) $v_G = 4.9\,\mathrm{km\,s^{-1}}$; b) $v_H = 34.7\,\mathrm{km\,s^{-1}}$; c) $\Delta v = 154\,\mathrm{m\,s^{-1}}$; d) $r_{\min} = 3\,700\,\mathrm{km}$; e) $\alpha \doteq 34°$]

Visitor AB/N/2

Find the heliocentric speed v_∞ of the interstellar object 1I/'Oumuamua after it leaves the solar system. You can assume that it reached its perihelion at the distance $r_p = 0.255\,\mathrm{au}$ from the Sun as it was moving with speed $v_p = 87.7\,\mathrm{km\,s^{-1}}$. Find the impact parameter b of its orbit (the distance of the asymptote of its orbit from the Sun).

[$v_\infty = 27\,\mathrm{km\,s^{-1}}$, $b = 0.83\,\mathrm{au}$]

Stellar astronomy

Sirius A EF/R/2

The brightest star on the night sky is Sirius (α CMa). This is mainly due to its proximity to the Sun (only 2.64 pc). Sirius is a binary star with components A (brighter) and B (fainter). They cannot be resolved with a naked eye. Here we will compare the brightness of Sirius A and the brightness of Sun.

a) Look up the apparent magnitude of Sirius A.
b) Mark the position of Sirius on the sky chart on Figure 6 and draw the IAU boundaries of Canis Major (CMa).
c) Write down the relation for the distance modulus in terms of physical distance of the object. Use it to calculate the absolute magnitude of Sirius A.
d) Provided that the absolute magnitude of the Sun is ($M_\odot = 4.83\,\mathrm{mag}$), calculate by how many times is Sirius A "brighter" than the Sun. That is, find the luminosity of Sirius A as a multiple of the solar luminosity $L_\odot$.

[a) $m = -1.47\,\mathrm{mag}$; c) $m - M = 5\log d - 5$, $M = 1.42\,\mathrm{mag}$; d) $L = 23.1L_\odot$]

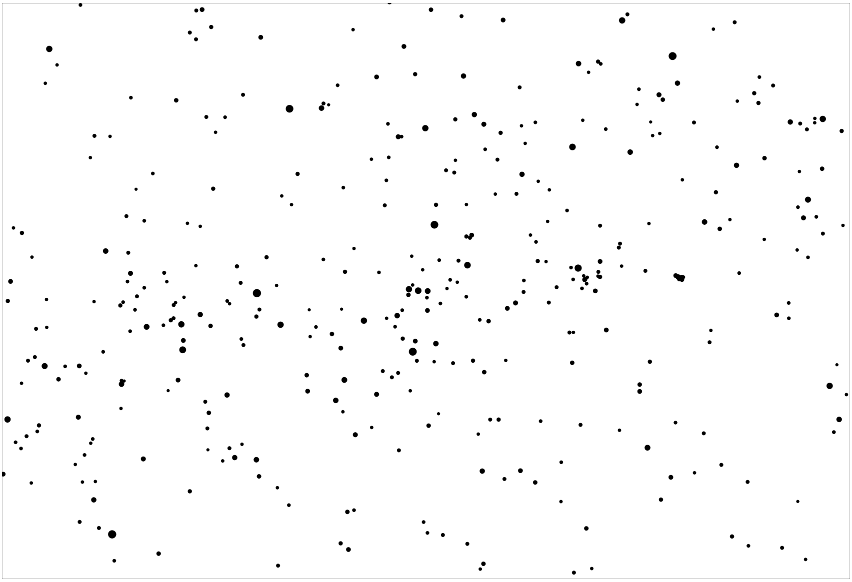


Figure 6: Sky-chart for problem EF/R/2.

Globular cluster CD/R/4

Consider the globular cluster NGC 6397 (constellation Ara) with apparent magnitude $m = 5.17\,\mathrm{mag}$ and angular diameter $\theta = 4.7'$. Scientists from the STScI (Space Telescope Science Institute) managed to measure its parallax $\pi = 0.42\,\mathrm{mas}$. Assume that all stars contained in the cluster are Sun-like.

a) Find the distance r to NGC 6397.
b) Estimate the number N of stars which the cluster contains.
c) Find the escape velocity v_{esc} from the cluster.

From now on, let us assume that all stars in the cluster are homogeneously distributed over a spherical volume of diameter θ.

d) What is the smallest diameter D of a telescope, which would (theoretically) be able to resolve individual stars in the cluster? Assume that the mean wavelength at which we are observing is $\lambda = 550\,\mathrm{nm}$.
e) Find the magnitude μ of a region of the cluster with angular area $1\,\mathrm{arcsec}^2$. Determine if the cluster can be observed from the center of Melbourne where the sky brightness reaches $18\,\mathrm{mag\,arcsec}^{-2}$.

[a) $r = 2.4\,\mathrm{kpc}$; b) $N \doteq 4 \times 10^4$; c) $v_{\mathrm{esc}} = 15\,\mathrm{km\,s^{-1}}$; d) $D \doteq 10\,\mathrm{cm}$; e) $\mu = 17\,\mathrm{mag}$]

Prossima fermata — CD/N/1

Find how many times larger (or smaller) the value of the "solar constant" on the exoplanet Proxima Centauri b is (orbital period 11 d), than it is on Earth. You can assume that the host star Proxima Centauri is 1.9 times colder, 6.5 times smaller and 8.2 times less massive than our Sun. The solar constant is defined as total power I incident on a unit surface area perpendicular to the line of sight at a given distance d from the source.

[1.3 times smaller than on the Earth]

Borderline-observable stars — CD/N/6

Let us consider a triple of stars which we will call A, B, C. It has the following properties

- star A, as seen from star B, is barely observable by naked eye,
- star B, as seen from star C, is barely observable by naked eye,
- star C, as seen from star A, is barely observable by naked eye.

The distance between the stars A, B will be denoted by d_1, the distance between the stars B, C will be denoted by d_2 and the distance between the stars C, A will be denoted by d_3. For numerical computations assume that stars A, B have absolute magnitudes $M_\mathsf{A} = 2\,\mathrm{mag}$ and $M_\mathsf{B} = 3\,\mathrm{mag}$. Assume that only stars brighter than 6 mag can be observed by naked eye.

a) Find the distance d_1 and d_2.
b) Find the interval (numerically in mag) in which the absolute magnitude M_C has to belong so that the above described configuration is allowed.

Hint: Use the triangle inequality.

c) Assuming that $M_\mathsf{C} = 4\,\mathrm{mag}$, find the largest angle γ in this stellar triangle.
d) Show that if we change the values of the three absolute magnitudes so that their differences remain the same, the angles in the triangle will not change.

[a) $d_1 = 63\,\mathrm{pc}$, $d_2 = 40\,\mathrm{pc}$; b) $M_{\mathsf{C},\min} = 4.2\,\mathrm{mag}$, $M_{\mathsf{C},\max} = 0.9\,\mathrm{mag}$; c) $\gamma \doteq 150°$]

Luminous Sun — AB/N/1

Estimate (order of magnitude in $L_\odot$) the maximum luminosity which the Sun can attain during its lifetime before it starts uncontrollably shedding its outer layers. The effective matter-radiation interaction in the photosphere can be modeled by elastic collisions of photons and objects of mass $m_\mathrm{p} = 1.67 \times 10^{-27}\,\mathrm{kg}$ with cross-section $\sigma_\mathrm{T} = 6.65 \times 10^{-29}\,\mathrm{m}^2$. For the sake of simplicity ignore the gradual decrease in solar mass.

[$10^3 L_\odot$]

A little dog **AB/N/3**

Find the absolute bolometric magnitude M_{bol} and radius R of Sirius B, a type DA2 white dwarf with apparent magnitude $m_{\mathsf{V}} = 8.4\,\mathrm{mag}$ and effective temperature $T = 26\,000\,\mathrm{K}$. Sirius has parallax $\pi = 380\,\mathrm{mas}$ and the bolometric correction for white dwarfs of given type is $BC = -3.0\,\mathrm{mag}$.

[$M_{\mathrm{bol}} = 8.3\,\mathrm{mag},\ R = 1.1R_{\oplus}$]

Binary systems and exoplanets

A dark and hot jupiter **CD/R/1**

In this problem we will focus on the very beginning of the tremendously successful career of the Kepler space telescope. Back then scientists were testing its capabilities by observing the transits of the previously identified exoplanet HAT-P-7 b. Your job will be to determine a number of parameters of this system given its light-curve (Figures 7 and 8) obtained by Kepler.

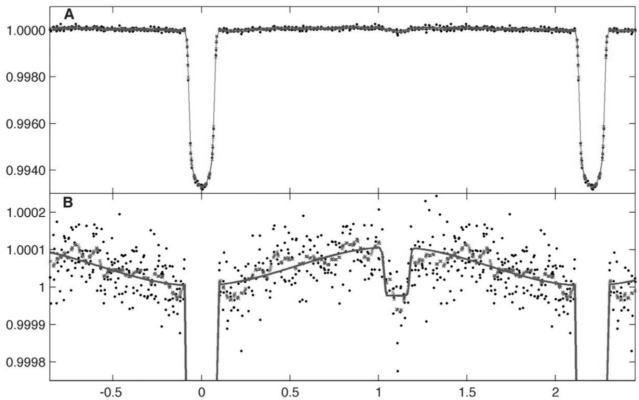
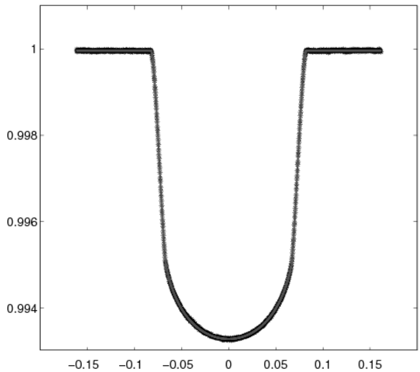


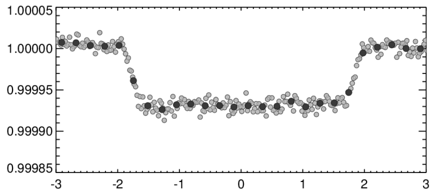
Figure 7: Light-curve of the exoplanet HAT-P-7 b (*Borucki et al.* 2009)

You may assume that the orbit of the exoplanet HAT-P-7 b around its host star is circular (which turns out to be consistent with the measured data).

a) Mark the positions of the primary and secondary minima on the light-curve. Draw diagrams depicting both situations (positions of the star and the exoplanet relative to the observer).
b) Explain the round shape of the bottom of the primary minimum. Also explain the increasing value of the observed intensity between the end of



(a) Light-curve around the primary minimum as reconstructed from data taken over a number of periods.



(b) Light-curve around the secondary minimum as reconstructed from data taken over a number of periods.

Figure 8: Detailed light-curve of HAT-P-7 b around the primary and secondary minimum.

the primary minimum and the beginning of the secondary minimum.

c) Find the orbital period P of the exoplanet.

d) Find the ratio $x = R_{\mathrm{p}}/R_*$ of the radii of the exoplanet and the host star.

Asteroseismological models gave the mass and radius of the host star HAT-P-7 as $M_* = (1.51 \pm 0.05)M_\odot$ and $R_* = (2.00 \pm 0.02)R_\odot$, respectively. By combining the data from photometric and spectrometric measurements, we can also infer the mass of the exoplanet as $M_{\mathrm{p}} = (1.74 \pm 0.03)M_{\mathrm{J}}$.

e) Find the radius a of exoplanet's orbit, radius R_{p} of the exoplanet in multiples of R_{J} and also the mean density ρ_{p} of the exoplanet in multiples of the mean density ρ_{J} of Jupiter.

f) Find the geometric albedo A_{g} of the exoplanet HAT-P-7 b. Compare your result with geometric albedo of the gas giants of our solar system.

Hint: The geometric albedo A_{g} is a measure of exoplanet's surface reflectance. It is defined by the relation

$$I = I_0 A_{\mathrm{g}} \frac{R_{\mathrm{p}}^2}{r^2} ,$$

where R_{p} is the radius of the exoplanet, r is the distance of the host star from the observer, I_0 is the intensity incident on the exoplanet and I is the intensity of the reflected light as measured by the observer at zero phase angle.

[c) $P = 2.2\,\mathrm{d}$; d) $x = 0.077$; e) $a = 0.038\,\mathrm{au}$, $R_{\mathrm{p}} = 1.5R_{\mathrm{J}}$, $\rho_{\mathrm{p}} = 0.5\rho_{\mathrm{J}}$; f) $A_{\mathrm{g}} = 0.2$]

Triple star AB/R/2

Let us consider a gravitationally bound multiple star system consisting of a close pair of stars orbiting along circular trajectories at a separation a, and a "test-particle" star of negligible mass, which orbits the central pair on a circular trajectory at a distance $\rho \gg a$. For simplicity let us assume that the two stars forming the central pair have equal masses $M_1 = M_2 = M$. The mass μ of the test particle satisfies $\mu \ll M$.

We will assume that the orbits of the central pair and the test-particle star are coplanar. Gravitational effects on the central pair due to the third star as well as relativistic effects should be neglected. You will find it useful to assume that for $|\varepsilon| \ll 1$ the *binomial approximation* $(1+\varepsilon)^n \approx 1 + n\varepsilon + \frac{1}{2}n(n-1)\varepsilon^2$ holds. That is, for instance for $a \ll \rho$, we have

$$(a+\rho)^{-3} \approx \rho^{-3}\left(1 - 3\frac{a}{\rho} + 6\frac{a^2}{\rho^2}\right) .$$

The angle between the line from the barycenter to M_1 and the line from the barycenter to μ (as measured from M_1 to μ), will be denoted θ.

a) Find the orbital period P_c and orbital frequency Ω_c of the central pair in terms of G, M, a.

b) Using the binomial approximation, show that the distances L_1 and L_2 from μ to M_1 and M_2, respectively, satisfy

$$L_{1,2} \approx \rho\left(1 \mp \frac{a}{2\rho}\cos\theta + \frac{1}{8}\frac{a^2}{\rho^2}\sin^2\theta\right) .$$

You should neglect all higher-than-quadratic terms in a/ρ.

c) Determine the component of the gravitational force on μ due to the central pair along the line from μ to the barycenter. Express your answer as

$$F_\mathrm{r} \approx \frac{2GM\mu}{\rho^2}\left(1 + \frac{a^2}{\rho^2}\,f(\theta)\right) ,$$

where $f(\theta)$ is a function of θ. Use the same approximation as in the previous part, working to the same order in a/ρ.

d) Express $f(\theta)$ as $f(\theta) = \alpha + \beta\cos 2\theta$. Find the numerical values of α and β. *Hint:* $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

By realizing that the orbital period of the central pair is much smaller than the orbital period of μ together with the fact that $\langle\cos 2\theta\rangle = 0$, the average gravitational force acting on μ can be expressed as

$$\frac{2GM\mu}{\rho^2}\left(1 + \alpha\,\frac{a^2}{\rho^2}\right) .$$

e) Using the binomial approximation, find the orbital period p and orbital frequency ω of μ as it orbits the central pair at a distance ρ from the barycenter. Express your answer in terms of G, M, a, ρ and α. Compare your results with the orbital period p_0 and orbital frequency ω_0 which we would obtain had we replaced the central pair with a single star of mass $2M$ placed at the barycenter.

[a) $P_\mathrm{c} = \sqrt{2\pi^2 a^3/GM}$, $\Omega_\mathrm{c} = \sqrt{\frac{2GM}{a^3}}$; c) $f(\theta) = \frac{3}{4}(\cos^2\theta - \frac{1}{2}\sin^2\theta)$; d) $\alpha = 3/16$, $\beta = 9/16$; e) $\frac{\omega}{\omega_0} \approx 1 + \frac{\alpha}{2}\frac{a^2}{\rho^2}$, $\frac{p}{p_0} \approx 1 - \frac{\alpha}{2}\frac{a^2}{\rho^2}$]

Eclipsing binary I: light-curve AB/R/4

Let us introduce the following notation: the radius and the temperature of the hotter one of the two components of an eclipsing binary will be denoted by R_1 and T_1, respectively, and similarly, R_2 and T_2 for the colder component. The depth of the primary minimum will be denoted by Δm_1, the depth of the secondary minimum by Δm_2 (that is $\Delta m_1 \geqslant \Delta m_2$). Limb darkening, interstellar extinction and reflected light can be neglected.

a) Express the luminosities L_1 and L_2 of the two components in terms of R_1, R_2 and T_1, T_2.
b) Find the observed fluxes J_1 and J_2 of the two components in terms of T_1, T_2 and the angular radii α_1, α_2 of the disks of the two components.
c) Find the fluxes j_1 and j_2 per unit solid angle in terms of T_1 and T_2.
d) Give a justification to the statement that during the primary minimum, the colder star needs to be passing in front of the hotter one (independently of R_1 and R_2).
e) Express Δm_1 and Δm_2 in terms of $\rho = R_2/R_1$ and $\tau = T_2/T_1 < 1$. Consider separately the cases when $\rho \leqslant 1$ and $\rho \geqslant 1$.
f) Are the quantities ρ and τ uniquely determined by Δm_1 and Δm_2?

[a) $L_i = 4\pi R_i^2 \sigma T_i^4$; b) $J_i = \alpha_i^2 \sigma T_i^4$; c) $j_i = \frac{\sigma}{\pi} T_i^4$; e) $\Delta m_1 = 2.5 \log \frac{1+\rho^2\tau^4}{1-\rho^2+\rho^2\tau^4}$, $\Delta m_2 = 2.5\log(1+\rho^2\tau^4)$ for $\rho \leqslant 1$, $\Delta m_1 = 2.5\log(1+\rho^{-2}\tau^{-4})$, $\Delta m_2 = 2.5\log\frac{1+\rho^2\tau^4}{1-\tau^4+\rho^2\tau^4}$ for $\rho \geqslant 1$; f) no]

Eclipsing binary II: radial velocities AB/R/5

Spectroscopical observations revealed the absorption lines belonging to each of the two components of an eclipsing binary. It was found that the wavelengths of the lines oscillate about their mean values with period P. The relative amplitude z of these oscillations can be defined as $z = \Delta\lambda/\lambda$, where $\Delta\lambda$ is the maximum shift of a line with mean observed wavelength λ. The quantity z has identical value (z_1 and z_2) for all absorption lines belonging to the given component (1 and 2, respectively).

a) Express the orbital speeds v_1 and v_2 of the two stars in terms of z_1, z_2.
b) Find the orbital radii r_1 and r_2 of the two stars in terms of z_1, z_2 and P.
c) Express the total mass M of the system in terms of z_1, z_2 and P.
d) Find the ratio $\mu = M_1/M_2$ of the masses M_1 and M_2 in terms of z_1 a z_2.
e) Express the masses M_1 and M_2 of the two stars in terms of z_1, z_2 and P.

For the eclipsing binary YZ Cassiopeiae, the following values were measured (*Lacy et al.* 1981): $z_1 = (2.45 \pm 0.01) \times 10^{-4}$, $z_2 = (4.19 \pm 0.02) \times 10^{-4}$ and $P = 4.467\,223\,5\,\mathrm{d}$. Assume that the measurements of z_1 and z_2 were independent. Neglect the uncertainty of P.

f) Find the values of M_1 and M_2 for YZ Cassiopeiae.
g) Find the values of M and μ for YZ Cassiopeiae including their errors.

[a) $v_i = cz_i$; b) $r_i = cz_iP/2\pi$; c) $M = \frac{c^3}{2\pi G}P(z_1+z_2)^3$; d) $\mu = z_2/z_1$; e) $M_1 = \frac{c^3}{2\pi G}Pz_2(z_1+z_2)^2$, $M_2 = \frac{c^3}{2\pi G}Pz_1(z_1+z_2)^2$; f) $M_1 = 2.31 M_\odot$, $M_2 = 1.36 M_\odot$; g) $M = (3.65 \pm 0.04) M_\odot$, $\mu = 1.71 \pm 0.01$]

Cosmology and relativity

Lost in space AB/R/1

After passing through a wormhole, a spaceship reappeared at an unknown point in space and time. Having taken several measurements, an astrophysicist who was present on the ship reported the following data to the bridge: the new CMB temperature is $T = 0.3\,\mathrm{K}$ and the curvature of the universe continues to be (to a great precision) zero. You can assume that the total density of a flat universe is equal to the critical density ρ_{crit} which satisfies

$$\rho_{\mathrm{crit}} = \frac{3H^2}{8\pi G},$$

where H is the Hubble–Lemaître parameter. Also assume that the current value of the Hubble–Lemaître parameter is $H_0 = (67.7 \pm 0.5)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, that the current value of the CMB temperature is $T_0 = 2.7\,\mathrm{K}$ and that currently 28 % of total density of the universe can be accounted for by matter (baryonic and dark). You may find it useful to assume that the density of matter is proportional to the inverse third power of the scale factor, while the dark energy density remains constant in time. Let us first assume that the spaceship did not leave our universe.

a) Did the spaceship move into the past or into the future?
b) Is it possible (using the above given data) to determine the distance which the spaceship traveled?

c) Find the scale factor of the universe at the epoch when the spaceship escaped from the wormhole.

Further measurements yielded the new value $H = (57 \pm 4)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ of the Hubble–Lemaître parameter.

d) Verify that the measured data is consistent with the assumption that the ship remained in our universe.
e) Find by how much the spaceship moved in time.

Hint: Scale factor during dark-energy domination is proportional to e^{Ht}.

[a) future; b) no; c) $a = 9$; e) $30 \times 10^9\,\mathrm{yr}$]

Scale factor AB/N/5

In this question you will be looking into various aspects of the evolution of our universe from early times up until the present epoch (which will be denoted by t_0). Expansion of the universe is most conveniently described by the *scale factor* $a(t)$: it is defined as the ratio of cosmological distances at some time t relative to their present values (that is $a(t_0) = 1$).

The present values of the density parameters $\Omega_\mathrm{m}(a) \equiv \rho_\mathrm{m}(a)/\rho_\mathrm{c}(a)$, $\Omega_\mathrm{r}(a) \equiv \rho_\mathrm{r}(a)/\rho_\mathrm{c}(a)$, $\Omega_\Lambda(a) \equiv \rho_\Lambda(a)/\rho_\mathrm{c}(a)$ are $\Omega_{\mathrm{m},0} = 0.308\,9$, $\Omega_{\mathrm{r},0} = 9.236 \times 10^{-5}$, $\Omega_{\Lambda,0} = 0.691\,1$, the value of the Hubble–Lemaître parameter in the present epoch is $H_0 = 67.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the present value of the CMB temperature is $T_0 = 2.73\,\mathrm{K}$. You can further assume that

- matter, radiation and dark energy densities satisfy $\rho_\mathrm{m}(a) = \rho_{\mathrm{m},0}a^{-3}$, $\rho_\mathrm{r}(a) = \rho_{\mathrm{r},0}a^{-4}$, $\rho_\Lambda(a) = \rho_{\Lambda,0}$, where $\rho_{\mathrm{m},0}$, $\rho_{\mathrm{r},0}$, $\rho_{\Lambda,0}$ are their values at time t_0,
- the critical density $\rho_\mathrm{c}(a)$ and the Hubble–Lemaître parameter $H(a)$ are related as
$$\rho_\mathrm{c}(a) = \frac{3H(a)^2}{8\pi G}\,,$$
- the total density of the universe is equal to the critical density (that is, our universe is flat), so that
$$H(a) = H_0\sqrt{\Omega_{\mathrm{m},0}a^{-3} + \Omega_{\mathrm{r},0}a^{-4} + \Omega_{\Lambda,0}}\,.$$

a) Find the scale factors $a_\downarrow$ and $a_\uparrow$ of the universe at the time of recombination and reionization, respectively. The CMB temperature at recombination was $T_\downarrow = 3\,000\,\mathrm{K}$, while the beginning of the reionization era corresponds to redshift $z_\uparrow = 20$.
b) Find the scale factors $a_{\text{m-r}}$ and $a_{\Lambda\text{-m}}$ of the universe at matter-radiation equality and matter-dark energy equality, respectively.

c) Find the scale factors a_r and a_Λ such that $\Omega_\mathrm{r}(a) < 0.01$ for $a > a_\mathrm{r}$ and $\Omega_\Lambda(a) > 0.01$ for $a < a_\Lambda$, respectively. Compare these with $a_\downarrow$, $a_\uparrow$ and $a_\text{m-r}$, $a_{\Lambda\text{-m}}$.

For $a > a_\mathrm{r}$ we can roughly assume that the universe contains only matter and dark energy. Then it can be derived that the scale factor satisfies

$$a(t) = \left[\sqrt{\frac{\Omega_{\mathrm{m},0}}{\Omega_{\Lambda,0}}}\sinh\left(\frac{3}{2}\sqrt{\Omega_{\Lambda,0}}H_0 t\right)\right]^{\frac{2}{3}} . \qquad (\heartsuit)$$

d) Use equation ($\heartsuit$) to estimate the present age of the universe.

e) Show that for $a \gg a_{\Lambda\text{-m}}$ we have $a(t) \approx C_1 e^{H_\infty t}$, while for $a \ll a_{\Lambda\text{-m}}$ we have $a(t) \approx C_2 t^p$. Find C_1, C_2, p and H_∞ in terms of $\Omega_{\mathrm{m},0}$, $\Omega_{\Lambda,0}$ and H_0.

Hint: We define $\sinh x \equiv (e^x - e^{-x})/2$. For $x \ll 1$ we have

$$e^x \approx 1 + x + \frac{1}{2}x^2 .$$

f) Give a physical interpretation for H_∞ and find its numerical value.

[a) $a_\downarrow = 0.0009$, $a_\uparrow = 0.0476$; b) $a_\text{m-r} = 0.0003$, $a_{\Lambda\text{-m}} = 0.7646$; c) $a_\Lambda = 0.165$, $a_\mathrm{r} = 0.030$; d) $t_0 = 13.8\,\mathrm{Gyr}$; e) $C_1 = [\Omega_{\mathrm{m},0}/(4\Omega_{\Lambda,0})]^{1/3}$, $H_\infty = \sqrt{\Omega_{\Lambda,0}}H_0$, $C_2 = [(3/2)\Omega_{\mathrm{m},0}^{1/2}H_0]^{2/3}$, $p = \frac{2}{3}$; f) $H_\infty = 56.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$]

Pions in the air — AB/N/4

Neutral pion π^0 is an unstable particle with rest mass $m_0 = 135.0\,\mathrm{MeV/c^2}$ and mean lifetime $\tau' = 8.4 \times 10^{-17}\,\mathrm{s}$. It decays into two photons as $\pi^0 \to \gamma + \gamma$. Consider one particular pion which originated as a result of interaction of cosmic rays with the atoms in upper atmosphere. Relative to an Earth-based observer, this pion has energy $E = 1\,\mathrm{GeV}$. You should neglect interactions of the pion with atoms in the atmosphere.

a) Find the speed v of the pion relative to the observer and the path l which it travels before it decays.

b) Find the maximum possible difference ΔE of energies of the two photons relative to the observer.

[a) $v = 0.991c$, $l = 10^{-7}\,\mathrm{m}$; b) $\Delta E = 0.991\,\mathrm{GeV}$]

Practical problems

View from a window CD/N/8

An astronomer situated at an unknown location on the Earth frequently observed the sunset from his window which was 1.5 m wide and tall. He was also in a possession of a small laser pointer which he placed horizontally at the level of the lower edge of the window at a distance of 1 m from the center of the lower edge. On the vernal equinox day, the astronomer undertook to record the Sun's trajectory by pointing the pointer at the center of Sun's disk and marking the point at which the laser beam hit the window. This he repeated every 20 min. Choosing the origin of his reference frame in the lower left corner of the window, he measured the coordinates x a y of all recorded points. The data he obtained is show in Table 2. Atmospheric refraction is to be neglected throughout the problem.

Hint: You may use that

$$\cos\Delta = \sin h_1 \sin h_2 + \cos h_1 \cos h_2 \cos(A_2 - A_1)\,,$$

where Δ is the angular distance between two objects on the sky, h_1, h_2 are their altitudes above local horizon and A_1, A_2 are their azimuths, and that

$$\sin\delta = \sin\varphi \sin h - \cos\varphi \cos h \cos A\,,$$

where δ is the declination of an object, h and A are its azimuthal coordinates and φ is the latitude of the observer.

a) In the $[x, y]$-plane, plot the positions of the Sun as recorded in Table 2.

b) Determine the time t_s (UT) and the horizontal coordinate x_s (in cm) of the sunset.

d) For each entry in Table 2, find the altitude h of the Sun above local horizon in degrees.

e) Using your results from part b), find the corresponding azimuth A of the Sun for each entry in Table 2.

Table 2: Position of the Sun on the window as measured at various times.

Time (UT)	$\frac{x}{\mathrm{cm}}$	$\frac{y}{\mathrm{cm}}$	Time (UT)	$\frac{x}{\mathrm{cm}}$	$\frac{y}{\mathrm{cm}}$
14:00	66.5	54.3	15:40	104.6	26.2
14:20	74.2	48.9	16:00	114.3	21.8
14:40	79.8	42.1	16:20	120.1	14.5
15:00	90.1	38.3	16:40	130.2	9.1
15:20	96.5	32.0	17:00	138.8	3.1

f) Plot the positions of the Sun in the $[h, A]$ plane. Fit the data with a suitable curve.

g) Find the latitude φ and longitude λ of the observer. The equation of time around the vernal equinox day is approximately $-7\,\mathrm{min}$.

[b) $x_\mathrm{s} = 143\,\mathrm{cm}$, 17:10 UT; g) $\varphi = (49.7 \pm 0.5)^\circ$ N, $\lambda = 14.25^\circ$ E]

An unknown object AB/R/3

In this problem you will use an MS Excel spreadsheet template to compute orbital elements of an unknown object based on its observations. You can assume that the object orbits the Earth subject to Earth's gravity only.

On October 22, 2018, two observers stationed on the equator and equipped with radars sighted an unknown object O. At a reference time t_0, observer A saw the object at altitude $h_0 = 15.8^\circ$ exactly in the direction to his west, while the observer B saw the object passing through local zenith. At the same time, using his radar, observer B found that at t_0 the object had zero radial speed relative to him. The distance between the two observers is $l = 1\,481.2\,\mathrm{km}$ (as measured along the Earth's surface). After time $\Delta T = 176.12\,\mathrm{s}$, that is at the time $t_1 = t_0 + \Delta T$, the object O reached the zenith of observer A. Let us denote by Z the center of the Earth and by S and P the center and perigee, respectively, of the unknown object's orbit.

You will now use a spreadsheet template [1] to determine the orbital parameters of the object O. The computation goes roughly as follows: the inputs for the algorithm are the above-given quantities together with an estimate T for the orbital period of the object. Given these data, the spreadsheet computes the rest of orbital parameters before using the Newton's method [2] to solve the Kepler's equation $E - e \sin E = 2\pi t/T$, where e is the numerical eccentricity, E is the angle between the lines SP and SO, and t is the time at which the object passed through perigee. The spreadsheet then calculates the angle traced by the radius vector of the object between the times t_0 and t_1, which can be independently computed using the details of the setup described above. The difference between these two values constitutes a precision test for the initial estimate of T.

a) Errors were intentionally introduced into a number of spreadsheet formulae (highlighted cells). Correct these errors and determine the following orbital parameters of the object: period, semi-major axis, numerical eccentricity and inclination.
b) Using the database [3], identify the object by giving its NORAD number. Individual entries in the database [3] conform to the format TLE [4].

[1] `http://olympiada.astro.cz/zadani/AO_2018_19_2_AB_prakticka.xlsx`
[2] `https://en.wikipedia.org/wiki/Newton%27s_method`
[3] `https://www.space-track.org`
[4] `https://en.wikipedia.org/wiki/Two-line_element_set`

[b) NORAD no. 33 208]

Dark universe AB/N/7

In this problem you will use the observation data of type Ia supernovae (CfA3, CfA4 and Pan-STARRS surveys) to determine the ratio of dark energy and matter abundances in the present universe.

Let us assume that our universe is flat and that it contains dark energy and matter only (that is, the total density $\rho = \rho_{\mathrm{m}} + \rho_{\Lambda}$ of the universe is equal to the critical density $\rho_{\mathrm{c}} = 3H^2/(8\pi G)$). Let us denote by $\Omega_{\mathrm{m},0} = \rho_{\mathrm{m},0}/\rho_{c,0}$, $\Omega_{\Lambda,0} = \rho_{\Lambda,0}/\rho_{c,0} = 1 - \Omega_{\mathrm{m},0}$ the present values of the density parameters and by H_0 the present value of the Hubble–Lemaître parameter. Also recall that the magnitude m (corrected for extinction) of a source with cosmological redshift z and absolute magnitude M satisfies

$$m - \mathcal{M} = 5 \log \frac{\mathcal{D}_{\mathrm{L}}(z)}{\mathrm{km\,s^{-1}}}\,,$$

where $\mathcal{M} = M - 5 \log \frac{H_0}{\mathrm{km\,s^{-1}\,Mpc^{-1}}} + 25$ and for small z we have

$$\mathcal{D}_{\mathrm{L}}(z) \approx cz \left[1 + \left(1 - \frac{3}{4}\Omega_{\mathrm{m},0}\right) z\right]\,.$$

Table 3 shows the observation data of selected type Ia supernovae (cosmological redshift z and calibrated[2] peak magnitude m_{B} in the B band.

a) Find the value of $10^{0.2m_{\mathsf{B}}}/(cz/\mathrm{km\,s^{-1}})$ for each supernova in Table 3.
b) Plot the values of $10^{0.2m_{\mathsf{B}}}/(cz/\mathrm{km\,s^{-1}})$ against z.

[2] That is, corrected for galactic extinction and (for large-z supernovae) transformed to the B band from the instrumental band (typically R, I).

Table 3: Data of selected type Ia supernovae (*Scolnic et al.* 2018).

SN	z	$\frac{m_\mathrm{B}}{\mathrm{mag}}$	SN	z	$\frac{m_\mathrm{B}}{\mathrm{mag}}$
2422	0.26318	21.35760	80646	0.31811	21.81160
8921	0.14393	19.83575	110460	0.19958	20.61175
12898	0.08286	18.51790	370595	0.18141	20.42600
13689	0.24999	21.20115	440005	0.30549	21.69950
14212	0.20284	20.73490	440050	0.28546	21.46645
15132	0.15286	20.00240	2008bw	0.03348	16.48020
15203	0.20291	20.64165	1999cc	0.03144	16.31895
15440	0.26110	21.20360	2000cn	0.02348	15.66265
16259	0.11787	19.35185	2003cq	0.03416	16.57565
16276	0.19190	20.53150	2006cq	0.04949	17.32405
17825	0.16031	20.07605	6D4dh	0.30158	21.64245
18602	0.13679	19.76485	1997dg	0.03438	16.60710
19913	0.20532	20.73155	2001gb	0.02673	15.97105
20227	0.27540	21.44850	2006on	0.06888	18.17780
20581	0.20311	20.57340	2006te	0.03292	16.45200

In the following parts, let us assume that all type Ia supernovae have the same peak absolute magnitude.

c) Determine graphically the best fit for the parameters $\mathcal{M}$, $\Omega_{\mathrm{m},0}$ and $\Omega_{\Lambda,0}$ together with their uncertainties.
d) Assuming that the B band peak absolute magnitude of type Ia supernovae is $M_\mathrm{B} = -19.44\,\mathrm{mag}$, determine the best fit for H_0 and its uncertainty.

[c) $\mathcal{M} = -3.57 \pm 0.02\,\mathrm{mag}$, $\Omega_{\mathrm{m},0} = 0.33 \pm 0.06$, $\Omega_{\Lambda,0} = 0.67 \pm 0.06$; d) $H_0 = 67 \pm 1\,\mathrm{km\,s^{-1}Mpc^{-1}}$]

51 Pegasi b — AB/N/8

The spectroscopic measurements of the star 51 Pegasi led to the discovery of one of the first exoplanets. Table 4 shows the data (radial velocities v_r and their uncertainties) which provided a confirmation of this discovery. The mass of the host star is $M_* = 1.12\,M_\odot$, its radius is $R_* = 1.2\,R_\odot$ and its effective temperature is $T_* = 5\,800\,\mathrm{K}$.

a) Plot the data from Table 4.
b) Use your plot to determine the amplitude and period of radial velocities of 51 Peg together with their uncertainties.
c) Find the distance of the exoplanet from the host star and estimate its minimum possible mass. Do not forget to compute uncertainties.

Table 4: The data yielded by spectrographic measurements of 51 Peg (Butler *et al.* 2006).

HJD	$\frac{v_r}{\mathrm{m\,s^{-1}}}$	$\frac{\text{error}}{\mathrm{m\,s^{-1}}}$	HJD	$\frac{v_r}{\mathrm{m\,s^{-1}}}$	$\frac{\text{error}}{\mathrm{m\,s^{-1}}}$
2450011.631285	-55.7	6.8	2450016.860336	13.1	9.1
2450011.826574	-45.6	7.0	2450017.746840	55.4	6.4
2450012.624850	21.7	7.2	2450017.858681	52.3	9.6
2450012.880313	26.0	10.0	2450018.602975	14.3	8.1
2450013.612662	52.3	6.4	2450018.754653	2.5	7.7
2450013.842350	33.0	11.0	2450018.845972	-27.0	8.1
2450014.632407	-8.8	7.4	2450019.611632	-47.6	6.5
2450014.734514	-15.6	7.4	2450019.838044	-62.2	6.0
2450014.896100	-37.6	8.8	2450020.602187	-25.3	5.6
2450015.613912	-52.5	7.2	2450020.842303	-6.0	7.5
2450015.876921	-53.8	7.8	2450020.864352	4.9	7.0
2450016.630394	-5.8	6.6	2450021.627025	60.2	6.0
2450016.750521	4.9	6.5	2450021.698472	48.9	7.6

d) Estimate the equilibrium temperature of the exoplanet (taking its albedo to be equal to 0.5).

Given the fact that the exoplanet appears to be a gas giant, the distance of the exoplanet from its host star and its equilibrium temperature which you found above should look somewhat suspicious. One possible explanation would be that 51 Peg b is in fact a brown dwarf. It is known that a gaseous object with mass above $13\,M_\mathrm{J}$ fuses deuterium and should, therefore, be classified as a brown dwarf.

e) Determine the probability that 51 Peg b is a brown dwarf. Assume that orbital inclinations of exoplanetary systems are randomly distributed and ignore the uncertainties of the quantities which you computed in previous parts.

[b) $v_\mathrm{A} = (120{\pm}10)\,\mathrm{m\,s^{-1}}$, $P = (4.2{\pm}0.2)\,\mathrm{d}$; c) $a = (0.053{\pm}0.002)\,\mathrm{au}$, $M_\mathrm{P} = (0.52{\pm}0.06)\,M_\mathrm{J}$; d) $T = 1\,000\,\mathrm{K}$; e) $p \approx 2.6\,\%$]

Cosmic Morse code CD/N/7

Photometric measurement of transiting exoplanets is one of the prime methods enabling discovery of new planetary systems. In this problem you will reconstruct the light-curve of the star WASP-47. You are given the measured values of flux F normalized to a reference level F_0. After the detection of transits, spectrographic measurements were performed which confirmed the presence of an exoplanet: the period of radial velocity changes was determined as $P = 4.16\,\mathrm{d}$ while the amplitude of radial velocities was measured to be $v_\mathrm{r} = 140\,\mathrm{m\,s^{-1}}$. Radius of the host star is $R_* = 1.16\,R_\odot$.

Table 5: Photometric data of WASP-47.

HJD	F/F_0	HJD	F/F_0
2149.84421	1.0003	2175.46567	0.9997
2149.90551	0.9977	2179.12292	0.9987
2149.94637	0.9884	2182.57585	0.9998
2149.98724	0.9874	2183.18880	0.9934
2150.02810	0.9900	2183.22966	0.9874
2151.00884	1.0000	2185.70188	0.9998
2151.96914	1.0000	2187.39769	0.9874
2154.09406	0.9887	2187.62244	1.0000
2154.87048	0.9998	2189.54300	0.9998
2156.81151	0.9998	2191.52486	0.9889
2158.36433	0.9964	2191.58616	0.9876
2157.83310	0.9994	2191.62702	0.9918
2159.87628	0.9996	2194.69175	1.0002
2161.93989	0.9996	2195.71332	0.9878
2162.40983	0.9887	2196.87792	1.0006
2166.57790	0.9880	2199.08453	1.0004
2170.72553	0.9890	2199.88136	0.9875
2170.86855	0.9996	2200.16740	1.0002
2174.89358	0.9949	2212.32421	0.9881
2174.97531	0.9885	2212.40593	0.9937

a) Find the orbital phase of the exoplanet for each entry in Table 5. Assume that zero phase occurs at HJD = 0.
b) Plot the normalized flux F/F_0 against the phase.
c) Find the depth $\Delta F/F_0$ and length ΔT of the transit.
d) Find the orbital speed v of the exoplanet. Assume that the transits are central and that the orbit of the exoplanet around its host star is circular.
e) Find the mass M_* of the star as a multiple of $M_\odot$ and the mass M_p of the exoplanet as a multiple of M_J.

[c) $\frac{\Delta F}{F_0} = 0.0125$, $\Delta T = 3.5\,\mathrm{h}$; d) $v = 142\,\mathrm{km\,s^{-1}}$; e) $M_* = 1.23\,M_\odot$, $M_\mathrm{p} = 1.28\,M_\mathrm{J}$]

National rounds in Opava

For the last 4 years, the national rounds of the Czech Astronomy Olympiad in categories AB and CD have been hosted by the Faculty of Philosophy and Science at the Silesian University in Opava. The stimulating environment which it provides, as well as the kindness of its staff, have been instrumental in fostering the talent of young Czech astronomers.

The Faculty of Philosophy and Science (FPS) in Opava started in 1990 as a Faculty of the Masaryk University in Brno and became part of the newly established Silesian University in Opava in 1991. The faculty teaches the subjects of arts (history, museology, archaeology, the Czech language, foreign languages, and library science), sciences (physics and computer science) and the subject of creative photography. Nowadays the faculty offers 14 bachelor's programmes, 12 master's programmes and 6 doctoral programmes.

As a part of the the faculty, the Institute of Physics offers its students to follow either the Theoretical Physics or Computational Physics programmes. Having acquired general knowledge in all areas of theoretical physics, students of the Theoretical Physics programme can focus on various topics in relativistic physics and astrophysics, particle physics, or quantum theory of atoms and molecules. Significant aspect of the studies is computer modeling of physical processes, which requires the knowledge of both numerical methods and specialised mathematical software, as well as the ability of creating customised computer programs. Students in the Computational Physics programme can specialise in one of the following areas: computer simulations of physical processes, graphical and multimedia processing of computer simulations, or processing and utilization of experimental and observational data.

Graduates of both study programmes will be ready to work in various areas covering application of computational methods, mathematics, physics and computer science, also in industry, services, and administration. This qualification also represents a perfect building ground for further scientific work within doctoral studies, enabling PhD graduates to work at universities or research institutes.

More information is available online at `www.physics.cz`.

100 years of IAU and 100 years of Czech astronomer Luboš Perek

July 26, 1919 *– Luboš Perek was born in Prague, Czech Republic.*
July 28, 1919 – The International Astronomical Union (IAU) was founded. Its mission has been to promote and safeguard the science of astronomy in all its aspects, including research, communication, education and development, through international cooperation.
1922 – IAU has been calling their General Assemblies approximately every 3 years, the first one having taken place in Rome.
1946 *– Luboš Perek – Doctor rerum naturalium, Masaryk University, Brno, CZ*
1958 – IAU started publishing the biannual Information Bulletin to keep all members up to date with activities of the Union. IAU Circulars were a series of postcards informing on recent discoveries that required prompt dissemination.
1961 *– Luboš Perek – Doctor of Science, Charles University, Prague, CZ*
1967 *– Luboš Perek (with Luboš Kohoutek) – best publication „Catalogue of galactic planetary nebulae", cited 777 times (as of 2019)*
1973 – The General Assembly convened in Sidney, Australia – the first time an important meeting of this kind was held in the southern hemisphere.
until 1980 *– Perek's Career: Director of the Astronomical Institute 1968–1975. In IAU: Commission for Galactic Structure and Dynamics (Vice-President 1961–1964, 1970–1973, President 1973–1976), Executive Committee (Assistant General Secretary 1964–1967, General Secretary 1967–1970). Chief of Outer Space Affairs Division in United Nations 1975–1980.*

1994 – IAU inaugurated its official webpage and, in the 90s, started publishing their Bulletins in a digital form.
2006 – IAU issued the first electronic IAU Newsletters, containing short and urgent announcements.
2009 – IAU helped celebrate the International Year of Astronomy, coinciding with the 400th of Galileo's first telescope observations. This event reached out to over 800 million people from 148 countries.
2015 – IAU actively participated in the International Year of Light.
2019 – *Luboš Perek* & IAU – 100th birthday!

ISBN 978-80-907341-1-1
9 788090 734111