Problem Booklet
2017/18

Czech Astronomy Olympiad

Board of Organizers of the Czech Astronomy Olympiad

Prague, 2018

Czech Astronomy Olympiad

Problem Booklet 2017/18

Board of Organizers
of the Czech Astronomy Olympiad

Edited by Jakub Vošmera, Václav Pavlík, Jan Kožuško, Tomáš Gráf
Graphic design by Jakub Vošmera
Cover design by Tomáš Gráf
Cover photo by Petr Hadrava
Published by the Czech Astronomical Society, Fričova 298, 25165 Ondřejov, Czechia
First edition in Prague, 2018
Typset in LaTeX
Printed by ON tisk, s.r.o., Křesomyslova 384/17F, 140 00 Praha 4, Czechia

This publication is not for sale.

http://olympiada.astro.cz

ISBN 978-80-907341-0-4

Contents

Introduction

Foreword

Dear friends,

at the beginning of the 10 000$^{\text{th}}$ year of Czech Astronomy Olympiad, it is our pleasure to present you with a new booklet containing the previous 1 111$^{\text{th}}$ year's harvest of the Czech Astronomy Olympiad problems. Although this small binary number play is to remind us the central role of computers in today's astronomy, the problems we have selected for you are solvable purely by using pen and paper.

In the past 15 years, we welcomed over 100 000 participants in our national Olympiad. They solved 1680 multiple-choice-questions, 160 quiz-tasks such as astronomical crosswords, Sudoku or constellation puzzles, but also hundreds of challenging problems similar to those we present to you in this booklet. The Olympiad is divided into four age groups (called – from the oldest to the youngest – AB, CD, EF and GH), each having three stages. The first round takes place at school with its main objective being to attract pupils to astronomy and motivate them for further work. In the second (regional) round, participants have to solve more complex problems as well as to perform observational tasks. The best participants go forward to the national rounds held in Opava and Prague in March and May, respectively, the winners of which then meet at the selection camps for the IAO and IOAA.

Czech Astronomical Society, the main organizer of the Olympiad, celebrated its centenary in December 2017. At the time it was founded, its main goal was to build an observatory in Prague. Since then it has developed into a respected and amazing society where amateur and professional astronomers get together in a single organization. Its 600 members work in 17 branches, covering the most important areas ranging from solar astronomy over variable stars to cosmology. Aside from the research topics, the society concentrates immense effort into working with young astronomers. In addition to the Olympiad (which itself attracts about 10 000 participants annually), it organizes summer schools and camps for children and youth together with astronomical courses, clubs and more.

As the Czech language belongs to the rather difficult and less popular ones (try to read this: Strč prst skrz krk!), we hope that you pick up our offer of this English written booklet and dive into the presented problems, maybe finding further inspiration for you or your students.

We wish you dark skies and nice reading!

On behalf of Organizers of the Czech AO

Jan Kožuško

Legend and acknowledgements

Each problem presented in this booklet comes equipped with its name and ID code containing information about the place of its original use in the Olympiad. For instance, "CD/R/2" denotes the second problem in the regional round of the CD category. Finally, all problems have their answers shown in small print below their statement.

Most of the competition problems that appear in Czech Astronomy Olympiad are original work of its organizers. Credits for the problems presented in this booklet are now given:

Martin Blaschke: AB/R/3; *Stanislav Fořt:* AB/R/1, AB/N/1, AB/N/6, CD/N/2; *Tomáš Gráf*: CD/R/2, CD/R/4, CD/N/8; *Pavel Kůs:* AB/R/4, CD/R/1, CD/N/3; *Martin Raszyk:* AB/R/2, AB/N/5, CD/N/6; *Jaromír Mielec:* AB/N/2, CD/N/1; *Ondřej Theiner:* AB/N/7, CD/R/3, CD/N/7; *Jakub Vošmera:* AB/N/3, AB/N/4, AB/N/8, CD/N/4, CD/N/5; *Petra Hyklová:* EF/N/3; *Ota Kéhar:* EF/R/1, EF/R/2, EF/N/2; *Václav Pavlík:* EF/R/1, EF/R/2, EF/R/3, EF/N/1, EF/N/2, EF/N/3

The reader certainly would not be able to enjoy the problems in their present form were it not for the meticulous work of *Miroslav Randa*, *Ota Kéhar* and *Michal Švanda* who carefully reviewed all questions used in the competition rounds.

Finally, it is here that we choose to express our immense gratitude to *Jan Kožuško*, *Lenka Soumarová* and *Tomáš Gráf* for making the Czech Astronomy Olympiad happen by diligently providing organizational support.

Theoretical problems

Geometry, time and instrumentation

The Great wall of China EF/R/1

It is told that the only man-made object visible from space is the Great wall of China. But how well can we really see it? Let us imagine an astronaut on the Moon who looks towards the Earth. Let us also assume that:

- the Moon and the Earth are perfectly spherical,
- the Great wall of China is directly in the astronaut's zenith,
- the astronaut has a negligible height,
- the Great wall of China has a negligible height as well,
- the Moon is at perigee.

a) How far is the Great wall of China from the astronaut? Call the result h and give its value in whole km.

b) The Great wall of China is $l = 3500\,\mathrm{km}$ long. What is the angular size of the Wall from the point of view of the astronaut? Call it α and give its value in radians and degrees. Neglect the Earth's curvature.

Hint: You may use that $\tan\alpha \approx \alpha$ for small α, if α is in radians.

c) Compare the angular size of the Wall observed from the Moon with the angular size of some known celestial object that we can see from the Earth. Based on this knowledge, conclude if the Great wall of China is visible from the Moon.

d) Archaeologists found out that with all its branches, the Wall is 21 200 km long with up to 25 thousand towers. Assume that the towers are distributed evenly. What is the length of one section between towers? Give its value in km.

e) The astronaut carries a 200 mm Newton telescope. Calculate its resolving power for the wavelength $\lambda = 550\,\mathrm{nm}$. Give its value in radians.

f) Convert the angular resolving power of the telescope to a physical distance on Earth near the astronaut's zenith. Give its value in km.

g) What is the smallest number of sections between towers that our astronaut could resolve with its telescope? Give the result as a whole number.
h) Does the smallest length that the astronaut would be able to resolve with an unaided eye contain more or less sections between towers than the smallest length he can see with his telescope? Explain without calculation.

[a) 354 484 km; b) 0.566°; d) 0.8 km; e) 3.36×10^{-6} rad; f) 1.2 km; g) two]

Supermoon EF/N/2

Both full moons of January 2018 were *supermoons* because the Moon was near its perigee. Both events were enhanced by the fact that the Earth was in perihelion. The faintest full moon (*micromoon*) would appear if both bodies were at apocentres. Assume that the Moon orbits in the plane of ecliptic.

a) Calculate distances between the Earth and the Sun and also between the Earth and the Moon, both at pericentre and apocentre. Write your answers in meters.
b) Find the distance between the Sun and the Moon during both super and micromoon.
c) By how much (in percents) is the angular size of the Moon during a supermoon larger than that during a micromoon? Assume that you are at the centre of the Earth and that the Moon is a perfect sphere.
d) Calculate the intensity of solar radiation at the Moon's distance for both super and micromoon.
e) The Moon does not produce its own light, it only reflects the solar radiation. The luminosity L_{M} of the Moon is therefore proportional to its albedo ($A = 0.136$), cross-section and intensity of the incoming radiation. Calculate L_{M} of super and micromoon.
f) Calculate the difference in magnitudes between the super and micromoons.

[a) $1.470\,55 \times 10^{11}$ m, $1.521\,41 \times 10^{11}$ m, 3.63×10^{8} m, 4.05×10^{8} m; b) $1.474\,18 \times 10^{11}$ m, $1.525\,46 \times 10^{11}$ m; c) 12 %; d) 1402 W/m^2, 1309 W/m^2; e) 1.81×10^{15} W, 1.69×10^{15} W; f) -0.3 mag]

A floating city CD/R/3

Jules Verne, who is considered to be one of the founding fathers of the science-fiction genre in literature, would celebrate his 190$^{\text{th}}$ birthday in 2018. One of his works is titled "A Floating City" and depicts the perils of a cruise on a gargantuan steamer. You may find that the following problem takes some of its inspiration from this breathtaking story.

Consider a planet which we call Nereus and which is mostly covered by oceans. This planet orbits its star Alcyone in a circular orbit. The planet is a host to a number of floating cities whose positions are bound to a fixed latitude (i.e. they can only move along a given parallel). One of these cities is home to a young astronomer who decided to find out as much information about his planet as he can. Therefore, he started performing various measurements to complete his goal. First thing he noticed was that when the city does not move, the length of one day remains constant throughout the year. He then attempted to measure the latitude of his city by affixing a pole vertically into the ground so that the length of the pole sticking out above the ground was $l_\mathrm{t} = 0.50\,\mathrm{m}$. The pole casts a shadow of length $l_\mathrm{s} = 1.38\,\mathrm{m}$ at midday.

a) Find the latitude of the city in which the astronomer lives.
b) One day, when the city was at rest relative to the ocean, the astronomer decided to observe the "sunset" of Alcyone. Determine the duration of this sunset given that daylight on Nereus lasts 8 hours and Alcyone's angular diameter is $D = 0.5°$ when observed from Nereus.
c) On a different day, the astronomer observed the sunset again but this time the city was moving with speed $v = 70\,\mathrm{km\,h^{-1}}$ relative to the ocean such that its latitude was kept constant.[1] Given that the astronomer measured the duration of sunset as $t_1 = 110\,\mathrm{s}$, find the radius of the planet.

[a) 70°; b) 234 s; c) 463 km]

Starlight from the seabed — CD/N/1

Let us consider a terrestrial planet with radius $R = 5\,000\,\mathrm{km}$ which is entirely covered by an ocean of liquid water (refractive index $n = 1.33$). The depth $h = 1\,500\,\mathrm{km}$ of the ocean is constant over the planet's surface. There is an astronomer standing on the solid surface (seabed) at the North pole of the planet. Find the minimum altitude α above the ideal horizon in which he can observe a star. What would be its declination δ? The attenuation of light inside the water column should be neglected.

Hint: A ray which propagates from an environment with refractive index n_1 into an environment with refractive index n_2 satisfies $n_1/n_2 = \sin\alpha_2/\sin\alpha_1$, where α_1 is the angle of incidence and α_2 is the angle of refraction (both measured relative to the line perpendicular to the interface).

[12.2°, −29°]

Retrograde motion — CD/N/5

In this problem you will investigate the retrograde motion of Mars. That is, a phenomenon when the apparent motion of the outer planets of our solar

[1] The latitude was the same as in part b).

system on the sky takes place (over a certain period of time) in the direction opposite to their true orbital motion around the Sun.

Let us denote by T_{E} and T_{M} the sidereal orbital period of the Earth and Mars, respectively. Let us also denote a_{E} and a_{M} the radii of the orbits of the Earth and Mars, respectively. For the sake of simplicity let us assume that the orbits are circular.

The point of view we shall adopt is that of the heliocentric reference frame, which rotates with the same angular rate $\omega_{\mathrm{E}} = 2\pi/T_{\mathrm{E}}$ as the Earth orbits around the Sun. Let us denote this frame by $\mathsf{H_E}$.

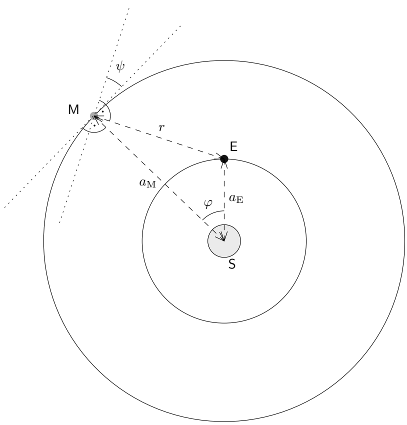


Figure 1: The points S, E, and M mark the positions of the Sun, the Earth and Mars, respectively. The diagram is not to scale.

a) Find the angular frequency ω'_{M} of Mars' orbit around the Sun relative to the reference frame $\mathsf{H_E}$. State your answer in terms of ω_{E} and the sidereal angular frequency $\omega_{\mathrm{M}} = 2\pi/T_{\mathrm{M}}$ of Mars' orbit.

Let us denote φ the angle subtended between the radius vector (i.e. the line joining an object with the Sun) of the Earth and that of Mars. See Fig. 1.

b) Find the distance r between the Earth and Mars as a function of φ. State your answer in terms of a_{M}, a_{E} and $\cos\varphi$.

Let us denote by ψ the angle between the tangent to the Mars' orbit through the position of Mars and the perpendicular to the line joining the Earth and Mars. See Fig. 1.

c) Find $\cos\psi$. State your answer in terms of a_{M}, a_{E} and $\cos\varphi$.

d) Find the transverse component v_t of Mars' orbital speed relative to an Earth-based observer in the reference frame $\mathsf{H_E}$ as a function of φ. State your answer in terms of ω_M, ω_E, a_M, a_E and $\cos\varphi$.
e) Find the angular speed ω''_M of Mars' motion on the Earth's sky relative to the background stars as a function of φ. State your answer in terms of ω_M, ω_E, a_M, a_E and $\cos\varphi$.
f) Find the two numerical values $\varphi_{1,2}$ of φ, for which ω''_M vanishes.
g) Find the duration t_ret of Mars' retrograde motion in one synodic period.

[a) $\omega_\mathrm{M} - \omega_\mathrm{E}$; b) $\sqrt{a_\mathrm{M}^2 + a_\mathrm{E}^2 - 2a_\mathrm{M}a_\mathrm{E}\cos\varphi}$; c) $\frac{a_\mathrm{M} - a_\mathrm{E}\cos\varphi}{\sqrt{a_\mathrm{M}^2 + a_\mathrm{E}^2 - 2a_\mathrm{M}a_\mathrm{E}\cos\varphi}}$; d) $\frac{(\omega_\mathrm{M} - \omega_\mathrm{E})(a_\mathrm{M}^2 - a_\mathrm{E}a_\mathrm{M}\cos\varphi)}{\sqrt{a_\mathrm{M}^2 + a_\mathrm{E}^2 - 2a_\mathrm{M}a_\mathrm{E}\cos\varphi}}$; e) $\frac{\omega_\mathrm{M}a_\mathrm{M}^2 + \omega_\mathrm{E}a_\mathrm{E}^2 - (\omega_\mathrm{M} + \omega_\mathrm{E})a_\mathrm{M}a_\mathrm{E}\cos\varphi}{a_\mathrm{M}^2 + a_\mathrm{E}^2 - 2a_\mathrm{M}a_\mathrm{E}\cos\varphi}$; f) $\pm 16.8°$; g) 73 d]

Spectrometer CD/N/6

An astronomer measured the solar spectrum at an observatory with latitude φ on a summer solstice. He used a very accurate spectrometer to measure the values of wavelength of a spectral line with a mean wavelength $\lambda \approx 656\,\mathrm{nm}$. He obtained two values, one measured at sunrise and the other at sunset. He found that the two values differ by $\Delta\lambda = 0.6\,\mathrm{pm}$.

Take the Earth to be of spherical shape with radius $R = 6\,378\,\mathrm{km}$ and to have axial tilt $\varepsilon = 23.44°$ relative to the plane of ecliptic. The Earth's orbit around the Sun is to be assumed circular.

a) Show that on the summer solstice the angular distance A between the point on the horizon where the Sun rises and the northern point satisfies $\cos A = \sin\varepsilon / \cos\varphi$.
b) Find the difference Δv of the radial velocities of the astronomer relative to the Sun at sunrise and sunset. State your answer in terms of ε, φ, R, T.
c) Find the latitude φ.

[b) $\frac{4\pi R\sqrt{\cos^2\varphi - \sin^2\varepsilon}}{T}$; c) $\pm 60°$]

Monument AB/R/2

Arthur C. Clark's 100th birthday is on 16th December 2017. Let us commemorate his work by this problem, which could be easily expanded into a science fiction novel.

At a remote place on Earth, participants of the astronomy olympiad had discovered a mysterious monument. It consists of two incredibly high vertical columns,[2] which are $d = |\mathsf{AB}| = 200\,\mathrm{m}$ apart, and a semicircle with diameter d and center S. The columns are touching this arc on opposite sides. Line AB goes directly from West to East.

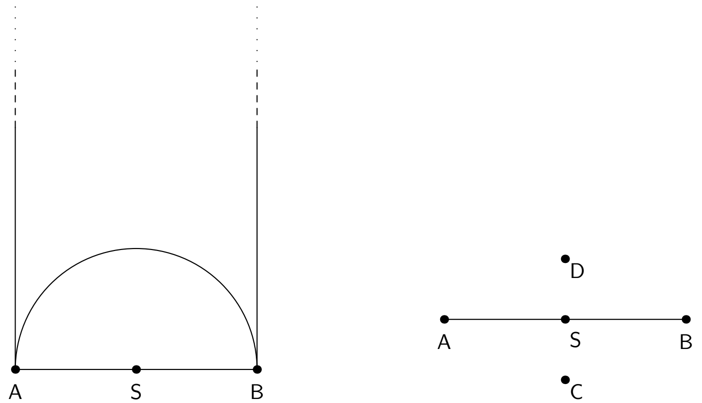


Figure 2: The monument (as seen from the front and from above).

As the explorers who had discovered this mystical place are participants of the *astronomy* olympiad, they decided to use it to determine their location. In order to do so, one of the participants (observer P) stepped in the center S between the two columns. As he looked north, observer P noticed a circumpolar star H that did not cross the arc above his head during a whole sidereal day. Declination of star H is $\delta = 55°$.

a) Based on the first two paragraphs, determine the interval of all possible latitudes of the location S, where observer P was standing.
b) Furthermore, observer P noticed that during one sidereal day, star H passes through all azimuths, including east, north and west. Based on this fact, calculate the *exact* latitude of observer P.
c) However, one of the more experienced participants noticed that observer P had confused star H with a different star. Therefore, observer P went south and stopped at location C, which is $l = |\mathsf{SC}| = 50\,\mathrm{m}$ away from center S. As he looked north from this point, he observed star H. Not only did the star stay between the two columns during a whole sidereal day, but it touched both columns as well. Given this information, calculate the corrected value of the latitude of observer P.
d) Once he knew his latitude, observer P decided to find the declination of

[2] They appear to touch in zenith.

a star H′, which culminates above the South. He went distance l north from the center of the arc (to location D) and looked south. Using his stopwatch he measured time t for which star H′ had been transiting the monument (from one column to the other). He immediately noticed that t was exactly equal to a half of one sidereal day. Help observer P determine the declination of star H′.

[a) $[35°, 55°]$; b) 55°; c) 50°; d) 38°]

Halo arc AB/N/2

Halo emerges through light refraction by small ice crystals which take the shape of a hexagonal prism. Due to the air resistance, the base of these crystals tends to be parallel to the ground. Compute the inner angular radius ρ of the Halo arc (in degrees), assuming that the speed of light in ice equals $v = 2.3 \times 10^8\,\mathrm{m\,s^{-1}}$. You may use the fact that the minimum ray deflection in a triangular optical prism is achieved when the ray inside the prism is perpendicular to the angular bisector between the input and output faces. Neglect the rays reflected inside the prism.

[21.3°]

Altazimuthal mount AB/N/5

In the Czech Republic at a site with latitude $\varphi = 50°$, an observer centered his altazimuth-mounted telescope at a star with declination $\delta = -18°$ at the moment of its culmination in the South. He then started an automated motion of the telescope along its vertical axis with a period of one sidereal day $T = 86\,164\,\mathrm{s}$. Half an hour later ($t = 1\,800\,\mathrm{s}$), he came back to the telescope and found that the star had left the field of view.

The observer is now interested to know if he can observe the star after replacing the eyepiece so that the field of view has a diameter $d = 0.5°$, and in the case that he cannot, if he could bring the star back into the field of view by moving the telescope along *exactly* one axis (either vertical or horizontal).

Note that the astronomical azimuth is measured from the South westwards.

a) Determine the horizontal coordinates $a_\mathrm{D}, z_\mathrm{D}$ (azimuth and zenith distance) of the center of the field of view of the telescope at $t = 1\,800\,\mathrm{s}$ after initiating the telescope motion. Express the result in degrees.
b) Determine the horizontal coordinates $a_\mathrm{H}, z_\mathrm{H}$ of the star at $t = 1\,800\,\mathrm{s}$ after initiating the telescope motion. Express your answer in degrees.
c) Is it possible to observe the star after replacing the eyepiece?
d) Can the observer bring the star back into the field of view by moving the telescope along *exactly* one axis? If so, along which axis (either vertical or

horizontal) does the telescope need to be moved?

Hint: For $a, b \neq 0$, the function $f(x) = a \sin x + b \cos x$ attains its minimum/maximum in the open interval $x \in (0, 180°)$ exactly at the point $x = \arctan\left(\frac{a}{b}\right)$.

[a) 68°, 7.5°; b) 68.3°, 7.70°; c) no; d) yes, horizontal axis]

Solar system

Four seasons EF/R/2

In this question, solar activity may be considered constant during the year. It is related to the Sun's luminosity which is also to be assumed constant. The amount of solar radiation that hits the Earth's surface at a fixed latitude, however, changes during a year. If we consider a unit of surface on Earth and measure the amount of radiation from the Sun that hits it every day at noon, we would find that the value remains almost unchanged in the tropical zone but changes more rapidly in the temperate zone (smaller in winter, higher during summer).

a) What is the main cause of the change of intensity of solar radiation that hits a unit surface in the temperate zone over a year?
b) For which observers on Earth is the Sun in zenith during solstices?
c) At which location on Earth is the Sun in zenith during the equinoxes?
d) Calculate the declination of the Sun for each of the four situations described in b) and c).
e) If the observer is at latitude 50° N, what is the altitude of the Sun above the horizon in each of the four situations? Neglect the effect of atmosphere and assume that the Earth's orbit is a circle.
f) Name the quantity which gives the intensity of solar radiation incident on Earth's surface and give its value.
g) Calculate the intensity of solar radiation at latitude 50° N during equinoxes and during solstices.
h) On a graph paper, plot the values of solar intensity which you calculated in the previous parts.
i) What is the consequence of the intensity variations in solar radiation incident on Earth's surface throughout a year?

[d) +23.4°, −23.4°, 0°, 0°; e) 63.4°, 16.6°, 40°, 40°; f) $1360.8\,\mathrm{W\,m^{-2}}$; g) $1216.8\,\mathrm{W\,m^{-2}}$, $388.76\,\mathrm{W\,m^{-2}}$, $874.71\,\mathrm{W\,m^{-2}}$, $874.71\,\mathrm{W\,m^{-2}}$]

A family photo **EF/R/3**

It is very rare to find a photo with the Earth and the Moon together. In June 2015 such a picture has been captured by the spacecraft DSCOVR (NASA). Selected pictures are shown bellow (Fig. 3). These photos are even more rare than usual because they show the whole far side of the Moon illuminated.

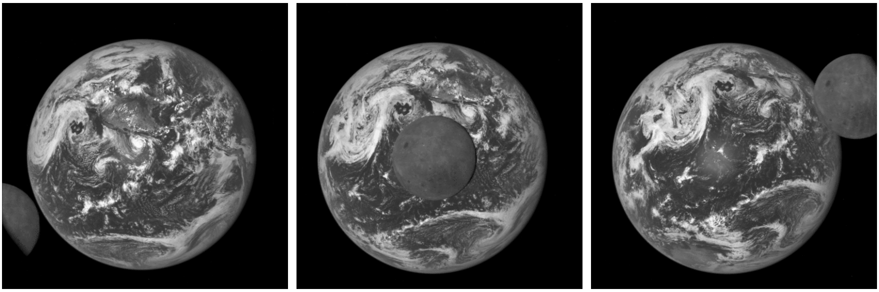
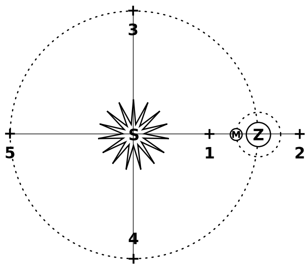


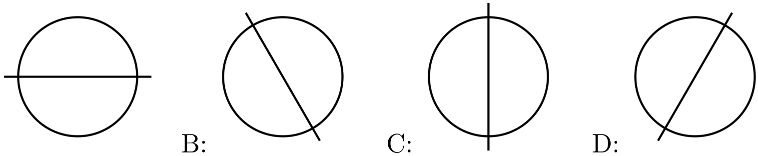
Figure 3: A sequence of images which shows the transit of the Moon over the face of the Earth.

a) The diagram on the right shows the Sun, the Earth, the Moon and five locations (1 to 5). Considering the above pictures, deduce the location of DSCOVR at the time it captured the pictures.



b) What is the Moon's phase for an observer on Earth at the time the pictures were taken?

c) Which of the possibilities bellow represent the orientation of the Earth's axis on Figure 3?



A: B: C: D:

d) Deduce from the photos how long (with a precision to one hour) did it take between the first and the last shot of the series if images in Figure 3.

[a) point 1; b) new moon; c) B; d) 5 hours]

Placing planets I **EF/N/1**

The distances between celestial bodies and their sizes can be hard to imagine. Let us try to understand them better by comparing them among each other. In the following exercises let us assume that the bodies are spherical.

a) What is the smallest distance between the surface of the Earth and the surface of the Moon when the Moon is in the apogee?
b) What is the maximum number of planets of the solar system that we can fit at once between the Earth and the Moon (without the Earth)?
c) What would change if this question was given before the year 2006?

[a) 397 000 km; b) all the remaining 7; c) Pluto (formerly a planet) would fit too]

Placing planets II **EF/N/3**

Since ancient times, every mathematician wished to find order among the planets of the solar system. Kepler tried to fit regular polyhedra between the orbits of planets. Two German astronomers, J. Titius and J. E. Bode, provided their explanation in 1766 and 1772 as well. They proposed that the planets are aligned according to the sequence

$$a = k(4 + x)\,,$$

where a is the semi-major axis of the body, $x = 0, 3, 6, 12, 24, \ldots$ for the objects of the solar system and k is a prefactor.

a) Use the definition of the astronomical unit to deduce the coefficient k.
b) Calculate the values of a from the sequence. For each value find a planet whose semi-major axis corresponds the best to a from the sequence.
c) Which planet satisfies the sequence the worst? By how many percent is the predicted value bigger than the real one?

[a) 0.1 au; c) Neptune, 29.3 %]

Tidal forces **CD/R/4**

After successfully formulating his law of universal gravity, Isaac Newton was the first to develop the theory of tidal forces which he correctly identified to be a consequence of an inhomogeneous gravitational field.

a) Find the radius a of the orbit of Jupiter's moon Io. Find the difference between the gravitational forces due to Jupiter acting on two test particles with identical masses $m_{\text{test}} = 1.0\,\text{kg}$ which are placed on the surface of the moon closest and farthest from Jupiter, respectively. Assume that the radius of Io is $R_{\text{Io}} = 1\,822\,\text{km}$, that the period of its rotation is $T = 42.5\,\text{h}$ and that the distance of Io from Jupiter is constant over time. Assume that the rotation of Io about its axis is tidally locked.

b) Compare the value for the difference of gravitational forces obtained in a) with the value computed for the system Earth–Moon. Assume that the orbit of the Moon around the Earth is circular with radius equal to the semi-major axis of the Moon's orbit.
c) Assume that the Sun collapses into a black hole with the radius of 3 km. Could an astronaut with height 190 cm observe this black hole from a distance of 300 km without being torn apart? Find the acceleration by which the astronaut is being stretched.
d) Let us go back to Jupiter (radius R_J, mean density ρ_J) and let us imagine, that a homogeneous spherical object with radius R and density ρ is radially falling onto the planet. The object is held together only by its own gravity. Derive a formula for critical distance r_crit from Jupiter where the object disintegrates due to tidal action. State your answer in terms of R_J, ρ_J and ρ. Assume that the body disintegrates when the tidal force between the centre and the surface of the object is greater than the object's gravity. Assume that the object remains spherical until the instant of disintegration. Rotation of the object is to be neglected.
e) How would the Earth–Moon separation have to change in order for the Moon to disintegrate? This time do not forget to take Moon's rotation into account.

Hint: you may find useful the fact that $(1+x)^n = 1 + nx$, when $x \ll 1$.

[a) 422 000 km; 0.012 N; b) Earth–Moon 250 times weaker; c) 1 900 g; d) $R_\mathrm{J}\sqrt[3]{2\rho_\mathrm{J}/\rho}$; e) $R_\mathrm{E}\sqrt[3]{3\rho_\mathrm{E}/\rho_\mathrm{M}}$, $1.7R_\mathrm{E}$]

Roadster in space AB/N/6

Elon Musk has decided to bring his first electric car into space. In this question, we are going to compute its trajectory through the solar system. Assume that the orbits of all planets of the solar system are circular and coplanar.

We use a simplified model of the launch of Falcon Heavy. We assume that a body with mass $m = 1\,400\,000$ kg (the model of the rocket) was launched with a velocity v_0 tangentially to the Earth's surface on the equator eastwards at zero altitude. You may neglect the effect of the atmosphere but not the effect of the Earth's rotation. The length of one sidereal day is $T_\mathrm{s} = 23.93$ h. The Earth's mass and equatorial radius are $M_\mathrm{E} = 5.97 \times 10^{24}$ kg and $R_\mathrm{E} = 6\,378$ km, respectively.

a) What is the required launch velocity v_0 so that the apogeum lies at the altitude $h = 300$ km above the Earth's surface. Express the result in terms of $h, R_\mathrm{E}, M_\mathrm{E}, T_\mathrm{s}$ and the Newtonian constant of gravitation G, and only then numerically in $\mathrm{km\,s^{-1}}$.

b) Compute an approximation of the launch velocity v_0 under the assumption that the altitude above the Earth's surface at the apogee is negligible compared to the Earth's radius, i.e., $h/R_\mathrm{E} \ll 1$. You may use the fact that $(1+\varepsilon)^n \approx 1 + n\varepsilon$ for $|\varepsilon| \ll 1$. Express the result in terms of $G, M_\mathrm{E}, h, R_\mathrm{E}, T_\mathrm{s}$, and only then numerically in $\mathrm{km\,s^{-1}}$.

After a series of firings of the second-stage engines, the car broke out of the gravitational influence of the Earth. Assume that it is currently revolving along a circular orbit around the Sun with radius $a_\mathrm{E} = 1\,\mathrm{au}$. A short burn of the rocket engine boosted the car by Δv parallel to the direction of the orbit. Assume that the orbital radius of Mars is $a_\mathrm{M} = 1.524\,\mathrm{au}$ and that the solar mass is $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$.

c) What is the minimum required increase of speed Δv so that the apohelium lies at the distance of Mars from the Sun? Express the result in terms of $G, M_\odot, a_\mathrm{E}, a_\mathrm{M}$, and only then numerically in $\mathrm{km\,s^{-1}}$.

The onboard computer of the second stage made a mistake and let the engine burning longer than necessary to reach the orbit of Mars along the Hohmann trajectory. Instead of Δv, the car was boosted by $p\Delta v$, where $p > 1$.

d) What is the minimum value p_A of the parameter p, for which the car would reach the asteroid belt, i.e., a distance at least $a_\mathrm{A} = 2.5\,\mathrm{au}$ from the Sun? Express the result in terms of $a_\mathrm{A}, a_\mathrm{M}, a_\mathrm{E}$, and then numerically.

Let us assume that the car was boosted by $p\Delta v$, where $p = p_\mathrm{A}$.

e) At what speed v_M and under what angle φ_M is the car going to cross the trajectory of Mars? Express the result numerically in $\mathrm{km\,s^{-1}}$ and degrees.

[a) $7.44\,\mathrm{km\,s^{-1}}$; b) $7.44\,\mathrm{km\,s^{-1}}$, i.e. the same as before (for the given numerical accuracy); c) $2.95\,\mathrm{km\,s^{-1}}$; d) $\left(\sqrt{\frac{2a_\mathrm{A}}{a_\mathrm{A}+a_\mathrm{E}}} - 1\right) / \left(\sqrt{\frac{2a_\mathrm{M}}{a_\mathrm{M}+a_\mathrm{E}}} - 1\right) \doteq 1.97$; e) $25.6\,\mathrm{km\,s^{-1}}$, $24°$]

Collision AB/R/4

There are two bodies orbiting a planet with mass $M = 6 \times 10^{24}\,\mathrm{kg}$ and radius $R = 6\,380\,\mathrm{km}$. First body is on a circular orbit with period $T_1 = 2.81\,\mathrm{h}$, second body has an elliptical orbit with period $T_2 = 2.23\,\mathrm{h}$. The orbital planes as well as the directions in which the bodies revolve are the same. Rotation of the planet is to be neglected. An observer is standing on the planet such that the apoapsis of the elliptic trajectory of the second body is in his zenith. Suddenly, the bodies collide. The altitude of both bodies just before collision (relative to the observer) was $h = 30°$.

a) Determine the radius a_1 of the first body's circular orbit and the semi-major axis a_2 of the second body's orbit. Calculate both results numerically in terms of the planet's radius.

b) Calculate distance l between the observer and the collision. Express the result in terms of R, a_1, h and numerically in terms of the planet's radius.
c) Find the numerical eccentricity of the second body's orbit before collision.

[a) $1.59R$, $1.36R$; b) $\sqrt{a_1^2 - R^2\cos^2 h} - R\sin h \doteq 0.830R$; c) 0.20]

Stellar astronomy

Receding star CD/R/1

Consider a star in a distance of $r_0 = 10\,\text{pc}$ from the Earth which moves with speed v in the direction away from an observer on the Earth who found that it has a redshift of $z = 0.1$.

a) Find the speed of the star.
b) Given the redshift z, find the ratio E_0/E, where E_0 is the energy of the radiated photon (that is the energy we would measure in the rest frame of the star) and E is the energy of the photon detected by the Earth-based observer. Find also the ratio T/T_0, where T is the time interval between two consecutive detections of a photon on Earth and T_0 is the interval between the emissions of the same two photons by the star.
c) Find the time period t over which the star will have to continue receding from the Earth in order for its magnitude to increase by $\Delta m = 0.1\,\text{mag}$.

[a) $0.1c$; b) both 1.1; c) 15.4 years]

Luminosity CD/R/2

Spectroscopic measurements of a star show that it radiates most of its energy on the wavelength $\lambda_{\max} = 365\,\text{nm}$. Interferometric measurements, on the other hand, yield its angular radius $\alpha = 0.0017''$. Assume that the distance to the star is $r = 10\,\text{ly}$.

a) Find its luminosity.
b) Find its absolute bolometric magnitude.
c) Find its bolometric magnitude for an observer based on Earth.

[a) $4.50L_\odot$; b) 3.11 mag; c) 0.54 mag]

A variable star CD/N/3

Consider a star with radius R and effective temperature T. Show that if we change its radius by ΔR and its temperature by ΔT, then its luminosity changes by ΔL, where

$$\frac{\Delta L}{L} \approx 2\frac{\Delta R}{R} + 4\frac{\Delta T}{T}\,.$$

Assume that the changes are small.

Find by how much would the effective temperature change if the radius was increased by 1 % such that the luminosity remains constant.

[−0.5 %]

The physics of stars AB/N/4

a) Compute the difference ΔM (in mag) of the absolute bolometric magnitudes of two stars whose effective temperatures and radii differ by 10 % and 20 %, respectively (the hotter star is smaller).

b) Let us consider a simplified model of the Sun, which we will assume to be a sphere of ideal gas consisting of protons and electrons in the hydrostatic and local thermodynamic equilibrium (the gas is fully ionized but overall neutral). In that case, the virial theorem reads $\Omega + 2U = 0$, where Ω is the total gravitational potential energy of the Sun and U is its total internal energy. The Sun suddenly receives energy $\Delta E = 10^{40}$ J. Compute the approximate change $\Delta \overline{T}$ in its mean temperature (in K) and determine the sign of this change.

[a) correct answers include: 0.018 mag, 0.062 mag, −0.027 mag and −0.071 mag; b) -10^5 K]

Quark star AB/R/3

Hypothetical quark stars that are made entirely of strange quark matter (s-quarks) are noticeable for their extraordinary equation of state

$$\varrho = \begin{cases} \gamma T^2 \,, & \text{for} \quad r \leq R \\ 0 \,, & \text{for} \quad r > R \end{cases} \,,$$

where T is the surface temperature of the star, parameter γ is equal to $340\,\mathrm{g\,cm^{-3}\,K^{-2}}$, r is the distance from the star and R is its radius. In other words, the density is constant throughout the whole star and zero outside.

There is an asteroid orbiting the star at distance $a = 0.75$ au. Assume that the orbit follows Kepler's laws. One period lasts $P = 1\,300$ d. Radiative flux on the surface of the asteroid is $S = 10\,\mathrm{kW\,m^{-2}}$.

a) Calculate radius R of the star. Express the result in terms of a, P, γ, S and suitable constants, and then numerically in km.

b) Is it physically possible for a star like this to exist?

[a) 1 km; b) yes, it is]

Binary systems and exoplanets

Mizar and Alcor CD/N/2

Alcor and Mizar are a binary star in the constellation Ursa Major which can be easily resolved by an unaided eye. Visual magnitudes of the components are $m_1 = 2.04\,\mathrm{mag}$ and $m_2 = 3.99\,\mathrm{mag}$. Consider that an observer would be unable to resolve this binary (e.g. due to a low resolution of her instrument). What would be the visual magnitude m of the unresolved binary?

[1.87 mag]

Wibbly-wobbly CD/N/4

Find the orbital speed $v_\odot$ of the Sun around the centre of mass of the binary system Sun–Jupiter.

[$12.5\,\mathrm{m\,s^{-1}}$]

Binary neutron star AB/N/3

We consider a binary system of two identical neutron stars with mass M and radius R each, which is located in our Galaxy. Assume that both components revolve along circular orbits and that the line of sight lies in the orbital plane. In the spectrum of the system, we observe the absorption line O VIII Ly α (laboratory wavelength $\lambda_0 = 1.897\,\mathrm{nm}$) with a mean observed wavelength $\overline{\lambda} = 2.561\,\mathrm{nm}$. The line periodically splits into two components with a maximum separation $2\Delta\lambda = 0.007\,\mathrm{nm}$ and a period $P = 1\,\mathrm{h}$. The proper motion of the system's barycenter has been eliminated from the data. Determine the radius R in terms of $R_\mathrm{g} = 2GM/c^2$, the mass M in terms of $M_\odot$ and the mean density ϱ of the components in $\mathrm{kg\,m^{-3}}$.

Hint: The gravitational redshift can be calculated as

$$z_\mathrm{g}(r) = [1 - (2GM)/(rc^2)]^{-1/2} - 1\,,$$

where M is the mass of the gravitating body and r is the distance between the center of the gravitating body and the point at which the photon was emitted.

[$2.2R_\mathrm{g}$, $2.37M_\odot$, $3.00 \times 10^{17}\,\mathrm{kg\,m^{-3}}$]

Cosmology and relativity

Newtonian cosmology AB/N/1

Assume that the universe consists solely of gravitationally-interacting matter. We are going to model the universe as a sphere with radius R which expands radially. The speed of radial motion of matter at distance r from the center equals $v(r) = Hr$, where the value of the Hubble's constant H is constant in space and time. Let the matter in the universe be homogeneously distributed with density ϱ.

a) Determine the total mechanical energy E in a spherical layer with radius r and thickness d. Assume that $d/r \ll 1$. Express the result in terms of r, d, H, ϱ and the Newton's gravitational constant G.
b) What is the required density ϱ_{crit} of the universe so that its total mechanical energy equals zero? Express the result in terms of G and H.

Hint: If the total mechanical energy of each layer equals zero, then the total mechanical energy of the whole universe equals zero.

[a) $2\pi r^4 H^2 d\varrho - \frac{16}{3} G\pi^2 \varrho^2 r^4 d$; b) $\frac{3H^2}{8\pi G}$]

Expansion of the universe AB/R/1

Hubble parameter H gives us the rate of cosmic expansion and its current value is $H_0 = 67.8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The content of the universe is usually expressed by the ratio Ω of density of a component ϱ to the critical density of the universe ϱ_{crit} as $\Omega = \varrho/\varrho_{\text{crit}}$. It has been measured that our universe currently consists of $\Omega_{\Lambda,0} = 0.68$ dark energy, $\Omega_{\mathrm{m},0} = 0.32$ matter (both baryonic and dark) and $\Omega_{\mathrm{r},0} = 0.0001$ radiation.

Scale factor a is a ratio of the size of the universe at a given epoch to the current size, so the value of scale parameter is $a_0 = 1$ right now. The value of Hubble parameter at an epoch with scale parameter a can be determined as

$$H(a) = H_0\sqrt{\Omega_{\mathrm{r},0}a^{-4} + \Omega_{\mathrm{m},0}a^{-3} + \Omega_{\Lambda,0}}\,.$$

For a dark-energy-dominated universe the scale factor evolves as $a(t) \propto e^{Ht}$, for a matter-dominated era as $a(t) \propto t^{2/3}$ and for a radiation-dominated era as $a(t) \propto t^{1/2}$. Assume that the universe is $t_0 = 13.82 \times 10^9\,\mathrm{yr}$ old and that the current temperature of cosmic microwave background is $T_0 = 2.7\,\mathrm{K}$.

a) Determine the value of scale factor $a_{\Lambda-\mathrm{m}}$ at an epoch when the densities of dark-energy and matter were equal. You can assume that $\Omega_{\mathrm{r}} = 0$. Determine temperature $T_{\Lambda-\mathrm{m}}$ of CMB at that time.

b) Determine the value of scale factor $a_{\mathrm{m-r}}$ at an epoch when the radiation density and matter density were equal. You can ignore the dark-energy ($\Omega_\Lambda = 0$).

The radiation, which is nowadays called CMB, began to propagate at an epoch that can be observed with a cosmological redshift $z_{\mathrm{CMB}} = 1\,100$. The following questions are to determine the age of the universe at that time.

c) Determine the value of scale factor a_{CMB} at the time of CMB emission. What was the temperature of CMB T_{CMB} back then?

d) Unrealistically assuming the dark-energy-domination during the whole time, estimate the age of the universe t_{CMB} at the moment of CMB emission. Due to a false premise, you should expect a physically nonsensical outcome.

e) And now for a more realistic solution; assume that for $1 > a > a_{\Lambda-\mathrm{m}}$ the universe behaves as if it only contained dark-energy, for $a_{\Lambda-\mathrm{m}} > a > a_{\mathrm{m-r}}$ as matter-dominated, and for $a_{\mathrm{m-r}} > a$ as radiation-dominated. Determine the age of the universe t_{CMB} at the moment of CMB emission.

[a) 0.78, 3.5 K; b) 0.000 3; c) 0.000 9, 3000 K; d) -90×10^9 years; e) 400×10^3 years]

Practical problems

Minor planet 221 235 CD/N/7

In this problem you will analyse the data from observations of the minor planet designated as 221 235. Table 1 shows the measured geocentric ecliptic coordinates of this minor planet and the Sun over a period of time.

a) Given this data, find, for each observation, the angle δ between the projections of radius vectors Earth–Sun and Earth–minor planet on the plane of ecliptic. The angle δ takes values in the range from $0°$ to $360°$, where $0°$ gives the conjunction.
b) Plot the angle δ against the JD. Estimate (as accurately as possible) the synodic orbital period T_{syn} of the asteroid.
c) Given your calculated value for the synodic period, find the sidereal period T_{sid} of the minor planet and its semi-major axis.

Assume that the minor planet's orbit lies precisely in the plane of ecliptic. Table 2 shows the ecliptic longitude of the minor planet and the cartesian components of its velocity relative to the Earth on the day of the national round of the Czech Astronomy Olympiad. The table also shows the cartesian components of the Earth's velocity relative to the Sun.

d) Find the magnitude v of the minor planet's velocity relative to the Sun and its distance r from the Sun on this particular day.
e) Draw the position of the Earth and the minor planet relative to the Sun as they were situated on 10^{th} May 2018, as seen from the direction of the North ecliptic pole. You may assume that the Earth's orbit is circular and that the vernal equinox day in 2018 occurred on 20^{th} March.

[b) 1.27 years; c) 4.7 years, 2.8 au; d) $18.88\,\mathrm{km\,s^{-1}}$, 2.6 au]

Exoplanets CD/N/8

In this problem you will extract some information about three exoplanets orbiting three different stars given their light-curves shown on Fig. 4.

Table 1: Measured data for minor planet 221 235. Angular coordinates are given in degrees. Ecliptic longitude and latitude are denoted by λ and β, respectively.

	Minor planet		Sun	
JD	λ_p	β_p	λ_s	β_s
2456743.50	357.28	0.01	6.24	0.00
2456788.50	15.90	0.02	50.24	0.00
2456833.50	32.47	0.02	93.38	0.00
2456878.50	44.73	0.03	136.35	0.00
2456923.50	48.45	0.04	179.90	0.00
2456968.50	40.97	0.05	224.49	0.00
2457013.50	34.89	0.04	270.04	0.00
2457058.50	40.45	0.04	315.84	0.00
2457103.50	53.66	0.03	1.04	0.00
2457148.50	70.23	0.02	45.16	0.00
2457193.50	87.96	0.02	88.38	0.00
2457238.50	105.60	0.02	131.33	0.00
2457283.50	121.92	0.02	174.77	0.00
2457328.50	134.80	0.02	219.24	0.00
2457373.50	140.04	0.02	264.71	0.00
2457418.50	133.73	0.01	310.52	0.00
2457463.50	126.26	0.00	355.83	0.00
2457508.50	130.00	0.00	40.08	0.00

Table 2: Data for the minor planet and the Earth as recorded on 10$^\mathrm{th}$ May 2018. Ecliptic longitude is given in degrees. Velocities are given in $\mathrm{km\,s^{-1}}$.

	Minor planet			Earth	
Date	λ_p	$\frac{v_{\mathrm{p},x}}{\mathrm{km\,s^{-1}}}$	$\frac{v_{\mathrm{p},y}}{\mathrm{km\,s^{-1}}}$	$\frac{v_{\mathrm{e},x}}{\mathrm{km\,s^{-1}}}$	$\frac{v_{\mathrm{e},y}}{\mathrm{km\,s^{-1}}}$
10$^\mathrm{th}$ May 2018	336.01	−8.26	32.59	22.01	−19.65

a) For each plot 4a to 4c estimate (as accurately as possible) the scale of the time axis, that is, find how many days correspond to 1 mm of the horizontal axis.

b) Estimate (as accurately as possible) orbital periods of all three exoplanets. You should perform 10 measurement of the orbital period for each exoplanet and then calculate the respective average values.

Table 3 shows typical masses, radii and effective temperatures of main sequence stars as they change with the spectral class.

Table 3: Dependence of the mass, radius and effective temperature of main sequence stars on spectral class.

Spectr. class	A0	A5	F0	F5	G0	G5	K0	K5	M0
$\frac{\text{Mass}}{M_\odot}$	3.5	2.2	1.8	1.4	1.07	0.93	0.81	0.69	0.48
$\frac{\text{Radius}}{R_\odot}$	2.63	1.78	1.35	1.20	1.05	0.93	0.85	0.74	0.63
$\frac{\text{Temperature}}{\text{K}}$	9 700	8 100	7 200	6 500	6 000	5 400	4 700	4 000	3 300

c) Assuming that the host stars belong to the main sequence, use the values shown in Table 3 to find their probable masses, radii and effective temperatures.
d) Find the semi-major axes of the three exoplanets' orbits
e) Assuming that the exoplanets can be modelled as black bodies, find their surface temperatures. Is it possible for any of these exoplanets to sustain liquid water on their surface?

[b) 74.9 d, 204.3 d, 154.6 d; d) 0.39 au, 0.63 au, 0.44 au; e) 551 K, 263 K, 190 K]

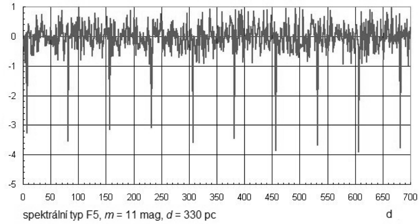
Neutrino mass AB/N/7

In 1987, one could observe a supernova explosion, which later became known as SN1987A. The astronomers observed this phenomenon not only optically, but they also managed to detect electron neutrinos from the explosion in three neutrino experiments. This allowed to gather more information about the explosion of the supernova and also about the properties of neutrinos. The distance of the supernova from the Earth is $d = 150\,000\,\text{ly}$. Tables 4, 5 and 6 contain data from various neutrino detection experiments. Time is measured from the detection of the first neutrino in each experiment.

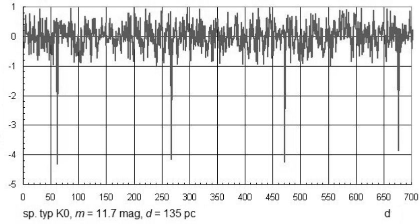
a) Assuming that the time difference between the first neutrino detection in individual experiments is negligible, plot the neutrino energy against time for all experiments into a single diagram.

Let us now focus on the decreasing part of the plot from the previous subquestion and assume that all those neutrinos were emitted by the supernova at the same moment. We also know (e.g. from cosmological limits) that the rest energy m_0c^2 of electron neutrino is certainly by orders of magnitude less than 1 MeV.

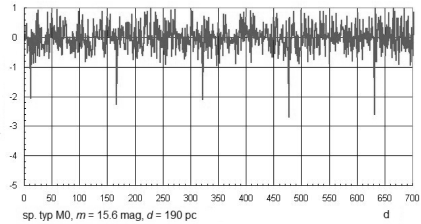
b) Show that the time difference $\Delta t_{21} = t_2 - t_1 > 0$ between detections of two neutrinos with energies E_1 and E_2 (where $E_1 > E_2$), which were emitted



(a) Exoplanet 1



(b) Exoplanet 2



(c) Exoplanet 3

Figure 4: Measured light-curves of all three exoplanets. The vertical axis shows relative flux while the horizontal axis shows time in days. Spectral type, magnitude and distance of their host stars is shown below each light-curve.

at the same moment, can be approximately written as

$$\Delta t_{21} \approx \frac{1}{2} d m_0^2 c^3 \left(\frac{1}{E_2^2} - \frac{1}{E_1^2} \right) .$$

Hint: You may use the fact that $(1+x)^r \approx 1+rx$ for $x \ll 1$. The total energy E of a particle which is moving with a velocity v equals $E = \gamma m_0 c^2$, where $\gamma = (1 - v^2/c^2)^{-1/2}$.

c) Assuming that all neutrinos on the decreasing part of the plot were emitted at the same moment, determine the rest energy of electron neutrino from the plot as accurately as possible. Express the result in eV.

d) Comment on the legitimacy of the assumption that all neutrinos were emitted at the same moment. How would your plot of particle energy against time of detection change if all neutrinos were *actually* emitted at the same moment? Is your approximation an upper or lower limit for the actual rest energy of electron neutrino?

[c) 15 eV]

Event ID	Time / s	Energy / MeV
1	0.00	20.0
2	0.11	13.5
3	0.30	7.5
4	0.33	9.2
5	0.51	12.8
6	0.69	6.3
7	1.54	35.4
8	9.22	8.6
9	10.43	13.0
10	12.44	8.9

Table 4: Registered neutrino events in the experiment Kamiokande.

Event ID	Time / s	Energy / MeV
1	0.00	38
2	0.41	37
3	0.65	28
4	1.14	39
5	1.56	36
6	2.68	36
7	5.01	19
8	5.58	22

Table 5: Registered neutrino events in the experiment IMB (Irvine-Michigan-Brookhaven).

Event ID	Time / s	Energy / MeV
1	0.00	12.0
2	0.44	18.0
3	7.69	17.0
4	9.10	20.1

Table 6: Registered neutrino events in the experiment Baksan.

O–C Analysis AB/N/8

The so-called O–C analysis, i.e., a comparison between the observed and calculated data, can be used in particular to make the periods of eclipsing variable stars and transiting exoplanets more precise. In this question, we are going to employ the method to make the orbital period and reference transit time of the exoplanet TrES-2b more precise.

Table 7 shows the results of photometric measurements of the system TrES-2. The quantity n (first column) is called epoch and counts the number of transits since the reference transit, which occurred at $t_0 = 2\,453\,957.6348\,\mathrm{HJD}$. The value $n = 0$ thus corresponds to the time t_0. The second column shows the observed transit times in the units of HJD $-$ 2 450 000 and the third column shows their uncertainties. At the time of observation, the orbital period of TrES-2b was estimated as $\overline{P} = (2.470\,621 \pm 0.000\,020)\,\mathrm{d}$.

n	observed transit time HJD − 2 450 000	$\frac{\text{uncertainty}}{\text{s}}$
4	3967.5180	37
13	3989.7529	25
15	3994.6939	27
34	4041.6358	26
140	4303.5209	26
142	4308.4613	39
274	4634.5828	26
276	4639.5232	27

Table 7: Observed transits of the exoplanet TrES-2b (Rabus et al. 2009). Epoch since reference transit $t_0 = 2\,453\,957.6348$ HJD is denoted as n.

a) Use the value of $\overline{P}$ and the reference transit time t_0 to calculate the predicted transit time (ephemeris) for each value of n from Table 7.
b) For each value of n from Table 7, determine the difference $O - C$ between the observed transit time and the calculated transit time from the previous subquestion.
c) Plot the differences $O - C$ against n including error bars.
d) Fit a line $O - C = \alpha + \beta n$ through the data, whose parameters α, β should be calculated by the least squares method. Estimate the error of the parameters α, β from the plot.

Least squares method: When fitting a line $y_i = \alpha + \beta x_i$ (where x_i are with respect to the x-axis and y_i are with respect to the y-axis) through the data (x_i, y_i) for $i = 1, \ldots, N$, then the mean values $\overline{\beta}$, $\overline{\alpha}$ of the parameters β, α of the line are given by the following formulas:

$$\overline{\beta} = \frac{N\sigma_{xy} - \sigma_x\sigma_y}{N\sigma_{xx} - \sigma_x^2} \qquad \text{and} \qquad \overline{\alpha} = \frac{1}{N}\left(\sigma_y - \overline{\beta}\sigma_x\right) ,$$

where $\sigma_x = \sum_i x_i$, $\sigma_y = \sum_i y_i$, $\sigma_{xy} = \sum_i x_i y_i$, $\sigma_{xx} = \sum_i x_i^2$, $\sigma_{yy} = \sum_i y_i^2$.

e) Use your results to calculate more accurate values $\overline{P}'$ and t_0' of the orbital period of TrES-2b and its reference transit time, including their uncertainties.

[d) $\alpha = (20 \pm 10)\,$s, $\beta = (-0.9 \pm 0.1)\,$s; e) $(2\,453\,957.6351 \pm 0.0001)\,$HJD, $(2.470\,611 \pm 0.000\,002)\,$d]

120 years of Ondřejov observatory

1898 – On 21st January, a plot on a hill near Ondřejov was bought by the astronomer Josef Frič. This is considered to be the founding date of the observatory.

1905 – Construction works commenced based on a project by the architect Josef Fanta.

1911 – The West dome was completed. Later in 1920, an astrograph with apertures 16/72 cm, 21/95 cm and a 13.5/117 cm pointer was installed in the dome.

1912 – The Central dome is completed and fitted with an 8-inch Clark achromatic object glass (20.8/283 cm) which was later replaced by a 0.65m reflector.

1928 – The observatory becomes a property of the Czechoslovak state with a provision that it is to be at Charles University's disposal. Prof. František Nušl is appointed as its first director.

1952 – The observatory is incorporated into the newly established Czechoslovak Academy of Sciences. Construction of solar laboratory with a 6m Zeiss dome begins.

1967 – The 2m Perek Telescope starts operating. At the time of its commisioning this 80 ton instrument from Carl Zeiss-Jena was the 7th largest telescope in the world and it remains the largest telescope in the Czech Republic. Its dome is on the front cover.

present times – Ondřejov observatory is part of the Astronomical Institute of the Czech Academy of Sciences. The institute is well integrated into wider European structures and organizations, such as ESO and ESA.

ISBN 978-80-907341-0-4