Problem Booklet
2016/17

Czech Astronomy Olympiad

Board of Organizers of the Czech Astronomy Olympiad

Prague, 2017

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Foreword

Dear friends,

it is our pleasure to present you with a booklet containing the previous year's harvest of the Czech Astronomy Olympiad problems. After the last year's debut, this is the second time such a compilation was made.

Czech Astronomical Society, the main organizer of the Olympiad, celebrates its centenary in December 2017. At the time it was founded, its main goal was to build an observatory in Prague. Since then it has developed into a respected society joining amateur and professional astronomers in a single organization. Its 600 members work in 17 branches, covering the most important areas ranging from solar astronomy over variable stars to cosmology. Aside from the research topics, the society concentrate immense effort into work with young astronomers. In addition to the Olympiad (which itself attracts about 10 000 participants annually) it organizes summer schools and camps for children and youth together with astronomical courses, clubs and more.

Czech Astronomy Olympiad was founded in autumn 2003 and quickly established itself as a recognized competition, comparable to the other main scientific olympiads. It is divided into four age groups (called – from the oldest to the youngest – AB, CD, EF and GH), each having three stages. The first round takes place at school with its main objective being to attract pupils to astronomy and motivate them for further work. In the second (regional) stage, participants have to solve more complex problems as well as to perform observational tasks. The best participants go forward to the national rounds held in Opava and Prague in March and May, the winners of which then meet at the selection camps for the IAO and IOAA.

After obtaining an official accreditation from the Czech Ministry of Education, Youth and Sports in spring 2006, the Olympiad received an invitation to participate at the XI$^{\text{th}}$ IAO in 2006. In 2010 we joined the IOAA and won the first gold medal in astronomy for the Czech Republic. Since then, over 40 students accompanied by 10 team leaders got hold of 6 gold, 12 silver and 21 bronze medals.

As the Czech language belongs to rather difficult and less popular ones (try to read this: Strč prst skrz krk!), we hope that you pick up our offer of this English written booklet and dive into the presented problems, maybe finding further inspiration for you or your students.

We wish you dark skies and nice reading!

Organizers of the Czech AO

Legend and acknowledgements

Each problem presented in this booklet comes equipped with its name and ID code containing information about the place of its original use in the Olympiad. For instance, "CD/R/2" denotes the second problem in the regional round of the CD category. Finally, all problems have their answers shown in small print below their statement.

Most of the competition problems that appear in Czech Astronomy Olympiad are original work of its organizers. Credits for the problems presented in this booklet are now given:

Stanislav Fořt: AB/R/2, AB/R/4, AB/N/4, AB/N/6; *Tomáš Gráf:* AB/N/8, CD/R/1, CD/N/8; *Pavel Kůs:* AB/N/5, CD/N/3, CD/N/4, CD/N/5; *Martin Raszyk:* AB/R/1, AB/R/3, AB/N/2, CD/N/1; *Ondřej Theiner:* AB/N/3, CD/R/2, CD/R/5, CD/N/6; *Jakub Vošmera:* AB/N/1, AB/N/7, CD/R/4, CD/N/2, CD/N/7; *Radek Kříček:* EF/N/1; *Václav Pavlík:* EF/R/1, EF/R/2, EF/R/3, EF/N/2;

We also gratefully acknowledge contribution from the *Russian Astronomy Olympiad*: one of their problems served as an inspiration for CD/N/7.

The reader certainly would not be able to enjoy the problems in their present form were it not for the meticulous work of *Miroslav Randa*, *Ota Kéhar* and *Michal Švanda* who carefully reviewed all questions used in the competition rounds.

Finally, it is here that we choose to express our immense gratitude to *Jan Kožuško*, *Lenka Soumarová* and *Tomáš Gráf* for making the Czech Astronomy Olympiad happen by diligently providing organizational support.

Theoretical problems

Geometry, time and instrumentation

Behold the Moon! EF/R/1

Human eye is a wonderful optical apparatus – not only the lens has no chromatic distortion but the aperture (pupil) is accommodating to the amount of light that passes into the eye (from about 1 mm in full daylight up to about 6 mm in complete darkness) in order not to saturate the retina receptors. Beware that looking directly at bright objects (such as the Sun) we can damage your sight. Even the Space telescopes have a shade against the direct sunlight, which can spoil precious detectors. The most luminous celestial object that we can observe with a naked eye is the Moon. It reflects part of the sunlight and absorbs the rest. The ratio between the reflected and absorbed light is called albedo (denoted by A). The Moon has $A = 0.13$.

a) In scientific tables, find the Solar luminosity and write it here rounded to 4 significant digits in SI units.
b) Find the intensity (total power received per unit area) of Sun's radiation at the distance of the Moon during a Full Moon? Assume that orbits of the Earth and the Moon are circular.
c) What is the maximum power that a naked eye can receive from the Moon on the surface of the Earth? Assume that the brightness of the Moon is homogeneous over its surface. When calculating the distances, neglect the dimensions of all bodies.

[a) 3.828×10^{26} W; b) $1354\,\mathrm{W\,m^{-2}}$; c) 1.016×10^{-7} W]

Airplane EF/R/3

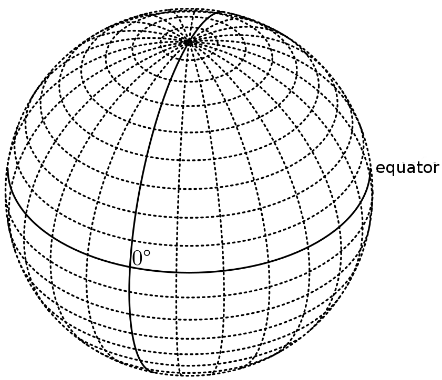
A Czech astronomer flew during the autumn equinox from the Prague–Ruzyně airport ($50°6'$ N, $14°17'$ E) to Naples to visit the observatory in Capodimonte. The airport in Naples is at $40°53'$ N and $14°17'$ E. The time of flight is usually 2 hours 5 minutes and the airplane took off exactly 3 hours to sunset.

a) In which direction does the Sun set that day?

Assume that the airplane flew the whole time parallel to the Earth's terminator.

b) What is the difference in longitudes between the terminator and the plane (i) at take-off, (ii) at landing? Round your answer to arc minutes.

c) In the figure bellow, draw the following: Prague and Naples (round their latitudes to the nearest multiple of 15 degrees), the direction to the North and South, the direction towards the Sun, the terminator and shade the night side of the Earth. Denote the difference between the longitudes of the airplane and the terminator by $\Delta\lambda$.



The astronomer sat the whole time above the left wing. From his window, he observed the shadow of the airplane on the wing.

d) How far on the wing did the shadow spread on the Prague runway? Assume that the airplane took off directly to the south. The body of the airplane has a cylindrical shape with diameter 12.6 m and length 31.2 m. The wings span 35.8 m, are attached in the middle of the body and horizontal with a negligible width. For the sake of simplicity, use only planar geometry and neglect atmospheric refraction. Round the result to the tenth of a meter.

e) What possible delay could the flight have had, so that the astronomer would have seen at least the last sun rays on the tip of the wing during the take off? Use only planar geometry and neglect atmospheric refraction. Round your answer to tens of minutes.

[b) $45°$, $13°45'$; d) 6.8 m; e) 80 min]

Size of celestial bodies EF/N/1

Eratosthenes of Cyrene lived in the 3$^{\text{rd}}$ and 2$^{\text{nd}}$ century BC. He worked in mathematics, astronomy a geography. One of his achievements was the measurement of the Earth's diameter. We reproduce Eratosthenes' approach with new data (e.g. from `fyzika.jreichl.com` and `antika.avonet.cz`).

a) Here, we will use the measurements from the Egyptian cities Aswan and Alexandria. Aswan is on the Tropic of the Cancer, therefore at noon of the summer equinox, the temple towers and other vertical objects do not cast any shadow. At the same moment in Alexandria, which is to the north, a 70-feet high obelisk casts a shadow of length 9 feet. Draw a diagram depicting both cities on the Earth's surface (emphasise the rays of the Sun, a shadow of the obelisk, and the angular distance of both cities).

b) Calculate the difference of latitudes of Aswan and Alexandria, rounded to degrees.

c) Eratosthenes found out from the merchants that Alexandria is 5000 stadiums from Aswan. One stadium is about 160 m. Calculate the polar radius of the Earth, rounded to hundreds of kilometres.

d) The biggest uncertainty lies in the measurement of the distance between the two cities, then in estimating the time of noon and the length of the shadow. We also assumed that the meridian was a circle. Calculate the relative uncertainty using the real value of the polar radius ($R_{\mathrm{E,p}} = 6356.8\,\mathrm{km}$). Round it to tenths of a percent.

e) Aristarchus of Samos lived some hundred years prior to Eratosthenes. He was also a mathematician and an astronomer, who derived the ratio between the distance Earth–Moon and Earth–Sun during a total Lunar eclipse. It seems that the observed shadow of the Earth is 2.6 times wider than the Lunar diameter. Draw the Sun, Earth and Moon during a total Lunar eclipse. Include also the rays that define the umbra. The drawing does not have to be to scale.

f) As the angular diameters of the Moon and the Sun are about the same, the ratio of their real diameters is the same as the ratio of their distances from the Earth. Aristarchus derived a wrong ratio due to his overcomplicated method. We shall use the real ratio which is equal to 390. Using this value and Eratosthenes' result, calculate the real diameter of the Moon, rounded to hundreds of km.

[b) 7°; c) 6500 km; d) 2.3 %; f) 1800 km]

100 years of the Czech Astronomical Society CD/R/1

In 2017, the Czech Astronomical Society (CAS) celebrates its centenary. This served as an inspiration for this problem where you will consider evolution of certain astrophysical systems for the period of CAS's existence. Your calculations should be performed for the time interval between 1st Jan 1917 00.00 UT and 1st Jan 2017 00.00 UT.

a) Find the change of the angular diameter of Crab's nebula.
b) Find by how much (in degrees) the direction to the northern celestial pole changed.
c) Count the number of minima of the Mira variable.
d) Find the angular distance by which the position of the Barnard's star changed relative to distant stars.
e) Find by how much (in kg) the mass of the Sun changed. Assume that the luminosity of the Sun is constant and reads $L_S = 3.827 \times 10^{26}$ W (20 % at 400 nm, 60 % at 550 nm and 20 % at 650 nm) and that the average speed v and density ρ of the solar wind at the distance of 1 au from the Sun are $400\,\mathrm{km\,s^{-1}}$ and 4 protons per cm^3, respectively.

[a) $32''$; b) $0.6°$; c) 110; d) $17.3'$; e) 15.8×10^{18} kg]

Bolide CD/R/5

A bolide camera in Ondřejov (latitude $\varphi_O = 49°54'37''$) captured a very bright meteor (bolide). It was determined that the bolide flared up when it was passing zenith and disappeared 2.3 s later 24° above the horizon along the southern arc of meridian. It was also determined that the angular speed of the motion of the bolide on the sky was the same both at the beginning and at the end of the bolide's track.

There is a second bolide camera situated in Hvězdov – a village located at the same longitude but different latitude compared to Ondřejov ($\varphi_H = 50°38'19''$). From the viewpoint of an observer in Hvězdov, the bolide flared up at altitude 59° above the horizon.

You should neglect atmospheric refraction, curvature of Earth's surface and deviations of bolide's trajectory from a straight line due to Earth's gravity.

a) Find the altitude H_1 above Earth's surface of the point where the bolide flared up.
b) Find the length L of bolide's track in the atmosphere and its speed v upon its entry into Earth's atmosphere assuming that the bolide's speed is constant along its track.
c) Find the altitude H_2 above the Earth, where the track of the bolide disappeared.

[a) 135 km; b) 150 km, $64\,\mathrm{km\,s^{-1}}$; c) 55 km]

Communication with a spacecraft CD/N/1

Consider an asteroid orbiting around the Sun along a circular orbit in the plane of ecliptic. A spacecraft landed on the surface of the asteroid to exchange radio signals (propagating with speed of light c) with a command centre on the Earth. Each time it receives a signal from Earth, it immediately sends it back. It is known that when the asteroid is in quadrature, the delay between the times when the signal is sent from Earth and received back is by $\Delta t = 775.6\,\mathrm{s}$ longer than in opposition. You should assume that the Earth's orbit is circular with radius a_E. Find the distance a of the asteroid from the Sun in terms of $c, \Delta t, a_\mathrm{E}$ and numerically in au.

[$\frac{8a_\mathrm{E}^2 - 4a_\mathrm{E}c\Delta t + c^2\Delta t^2}{8a_\mathrm{E} - 4c\Delta t} \doteq 2.35\,\mathrm{au}$]

Sunrise on the Moon CD/N/2

Find the duration Δt of the sunrise at $\phi = 50°$ of selenographic latitude. For the sake of simplicity, assume that the Moon orbits the Earth in the plane of the ecliptic and that its rotation axis is perpendicular to its orbital plane. The angular diameter of the Sun as observed from the Earth is $\delta = 32'$. You should give your answer numerically in hours.

[1.6 h]

Encounter CD/N/5

Consider two travellers C_1 and C_2 who met each other during their journeys through space. We will now describe their motion relative to a nearby star S, which you should assume to be identical to our Sun.

The travellers were first moving against each other along a line L, each at a constant velocity $\mathbf{v}$ with magnitude v. By coincidence their trajectory was situated in the orbital plane of two planets P_1 and P_2 which orbit the star S along circular orbits with periods $T_1 = 1560\,\mathrm{d}$ and $T_2 = 2320\,\mathrm{d}$, respectively. At the time when the travellers arrived into the system, both planets were positioned along the line L. The traveller C_2 also noticed that in her reference frame, P_1 has zero radial velocity. See Fig. 1.

The travellers' spaceships arrived to their respective planets at the same time, which we will denote as $t = 0$. Upon noticing each other's presence, traveller C_2 suddenly changed her course towards the star while C_1 continued in her original direction (the speed of the travellers remained unchanged).

Assume that neither the planets nor the star have any noticeable effect on the trajectories of the travellers and that v is much larger than the orbital speeds of the planets.

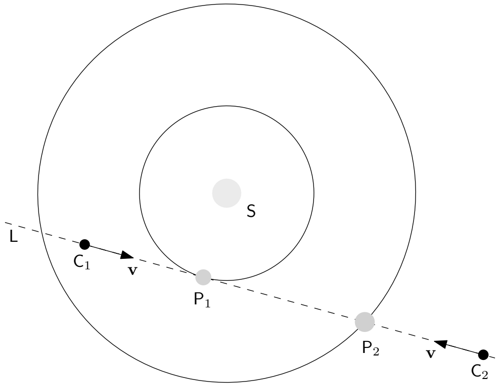


Figure 1: The relative positions of the star, the planets and the travellers as they enter the system.

a) Find the semi-major axes a_1, a_2 of the planets P_1 and P_2 (in au).
b) Find the maximum elongation ϕ of P_1 relative to P_2 (in degrees). What is the separation r of P_1 and P_2 (in au) at $t = 0$?

Let us now define $x(t) = vt/(r - vt)$.

c) Find a formula expressing the angle α between C_1 and S as they are seen by C_2 in terms of x and ϕ for $t > 0$.
d) Find a formula expressing the mutual radial speed v_r of the travellers in terms of v, x a ϕ.
e) Find the time t_min of the closest approach of the travellers to each other. Give your answer in terms of r a v. Find their minimum separation d_min in terms of r a ϕ.

[a) 2.63 au, 3.43 au; b) 50.1°, 2.20 au; c) $\sin\alpha = \frac{\sin\phi}{\sqrt{1+x^2-2x\cos\phi}}$; d) $v_\mathrm{r} = \frac{v(1+\cos\phi)(1-x)}{\sqrt{1+x^2-2x\cos\phi}}$; e) $r/(2v)$, $r\sin(\phi/2)$]

Earth's eccentricity AB/N/2

Using and accurate sundial located at the observatory in Ondřejov (latitude $\phi = 49° \, 54.548'$ and longitude $\lambda = 14° \, 46.880'$), it was determined that the length of the true solar day is $T_1 = 24.0033$ h at aphelion and $T_2 = 24.0080$ h at perihelion. It was further found that the length of the sidereal day is $T_\mathrm{s} = 23.9345$ h. Express the numerical eccentricity e of Earth's orbit in terms of T_1, T_2, T_s and find its value.

[0.0165]

Polar orbit **AB/R/1**

This problem will discuss satellites orbiting the Earth on polar orbits. Polar orbits are defined so that their planes contain Earth's rotational axis. Thus the satellites which are put on polar orbits fly over the Earth's poles. In the following we will restrict ourselves on circular orbits.

a) At a certain location on the Earth, two observations were made of a satellite on a polar orbit passing directly through zenith. Those two observations were separated by one sidereal day. Find the smallest possible altitude of the satellite above Earth's surface.

You should assume that the Earth is a perfect sphere with radius $R = 6378\,\mathrm{km}$ and mass $M = 5.97 \times 10^{24}\,\mathrm{kg}$. The duration of one sidereal day is $T_\mathrm{s} = 86\,164\,\mathrm{s}$ and Newton's gravitational constant is $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. Effects due to air drag should be neglected.

b) Now let us assume that the two above-mentioned observations were made on Earth's equator and that the satellite passed through zenith after one half of a sidereal day (approx. 12 h) as well. Again, find the smallest possible altitude of the satellite above Earth's surface.
c) Consider the satellite with orbit as determined in the previous part provides communication for sailors. Find the maximum distance (measured along the ocean's surface) of two ships so that they can communicate via the satellite assuming that a direct connection between the ships and the satellite always needs to be maintained.
d) Finally consider a system of communication satellites orbiting the Earth at the altitude determined in b). Find the minimum number of these satellites so that given any fixed location on the Earth, at least on of the satellites is always visible above horizon.

Hint: The surface area of a spherical cap satisfies $S = 2\pi Rv$, where R is its radius and v its height. The communication satellites are not confined to the polar orbits.

[a) 2.60×10^5 m; b) 5.52×10^5 m; c) 5.13×10^6 m; d) 26]

Solar system

Observing the solar system **EF/N/2**

In January of a certain year, the dwarf planet Ceres was in aphelion of its orbit and simultaneously in its quadrature. In the same night the asteroid (3424) Nušl (named after František Nušl, the co-founder of the observatory at Ondřejov and the former president of the Czech Astronomical Society) in opposition and also in aphelion.

Hint: In the calculations bellow, neglect seeing, atmospheric refraction, phases of the observed bodies and their inclination to the ecliptic. Assume that all albedos are equal to one.

a) At the time in question, plot the Sun, the Earth, Ceres and Nušl in the plane of the ecliptic as viewed from the ecliptical pole. Mark all the important angles in this configuration. The plot does not have to be to scale but should not be obviously wrong. The parameters of Ceres can be found in the official tables of the AO, Nušl's semi-major axis is $a_{\mathrm{N}} = 2.547\,\mathrm{au}$.
b) What is the distance between the Earth and the Sun?

Hint: Beware of the date and eccentricity of Earth's orbit.

c) What is the angular diameter of Ceres as seen from the Earth? Give your result in radians and arc seconds (both rounded at 3 significant digits).

Hint: Focus on the mutual position of the bodies and work only in the plane of the ecliptic.

d) Show by calculation whether it is possible to observe Ceres by a naked eye. To determine the observed magnitude of bodies in the solar system (denoted as m), we use the equation

$$m = H + 5\log\left(l_{\mathrm{S}} l_{\mathrm{E}}\right),$$

where H is the absolute magnitude of the body, l_{S} is its distance from the Sun and l_{E} is its distance from the Earth. The values l_{S} and l_{E} must be in astronomical units. The absolute magnitude of Ceres is $H_{\mathrm{C}} = 3.34\,\mathrm{mag}$.
e) Find the distance of (3424) Nušl from the Earth (semi-major axis $a_{\mathrm{N}} = 2.547\,\mathrm{au}$ and eccentricity $e_{\mathrm{N}} = 0.073$)? Round your answer to mil. of km.
f) In order to observe Nušl, we would like to use a telescope whose angular resolution in the visible light (i.e. $\lambda = 550\,\mathrm{nm}$) was one fifth of the angular diameter θ_{C} of Ceres. What is the diameter d of this telescope?
g) Can we observe (3424) Nušl with that telescope? The asteroid has an absolute magnitude $H_{\mathrm{N}} = 12.7\,\mathrm{mag}$. Justify your answer by a calculation.

[b) 147×10^6 km; c) 2.23×10^{-6} rad $= 0.460''$; d) 8 mag, not visible; e) 262×10^6 km; f) 1.50 m; g) yes]

Journey to the Moon — CD/R/2

Assume that the mass and the radius (measured at its equator) of the Earth are $M_{\mathrm{E}} = 6.00 \times 10^{24}\,\mathrm{kg}$, $R_{\mathrm{E}} = 6.38 \times 10^3\,\mathrm{km}$, that the mass and the radius of the Moon are $M_\mathrm{M} = 1.23 \times 10^{-2} M_\mathrm{E}$, $R_\mathrm{M} = 2.73 \times 10^{-1} R_\mathrm{E}$ and that the Earth–Moon separation is $d = 60 R_\mathrm{E}$. The Newton's gravitational constant is $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

Let us first ignore that the Moon and the Earth orbit around their common centre of mass. We will also ignore gravitational effects of all other bodies. Consider now a rocket with mass m (where $m \ll M_\mathrm{M}, M_\mathrm{E}$) launched perpendicularly to the surface of the Earth towards the Sun with a lift-off speed v_1. The motion of the rocket is influenced by Earth's and Moon's gravity only.

a) Find the minimum lift-off speed v_1 such that the rocket reaches the Moon.

We will now add the orbital motion into our considerations. For the sake of simplicity we will, however, assume that centre of mass of the Earth–Moon system exactly coincides with the centre of Earth (this is a good approximation since $M_\mathrm{E} \gg M_\mathrm{M}$). From now on, we will take the viewpoint of an observer co-rotating with the system. Such observer is non-inertial and a centrifugal force acts on him.

You can further assume that

1. the work done against the centrifugal force, when a mass m is displaced from a distance r_1 from the rotation axis to r_2 (where $r_1 > r_2$) in a system which rotates with angular speed ω, can be written as $E_\mathrm{rot} = (1/2) m\omega^2 (r_1^2 - r_2^2)$,
2. the distance from the centre of the Moon to a place[1] along the Earth–Moon line, where the net force acting on a point particle is zero, can be expressed as $d_\mathrm{L1} \approx d\,\sqrt[3]{M_\mathrm{M}/(3M_\mathrm{E})}$, where d is the Earth–Moon separation.

Assume that this time the rocket is launched perpendicularly from the surface of the Moon with a lift-off speed v_2.

b) Find the minimum lift-off speed v_2 needed for the rocket to reach the Earth. This time do not forget to include the influence of the Earth–Moon orbital motion. Is it fine to fire the rocket directly towards the Earth?

[a) $11.09\,\mathrm{km\,s^{-1}}$; b) $2.32\,\mathrm{km\,s^{-1}}$]

Orbital resonance CD/N/4

Consider two particles orbiting a central body with sidereal periods P_1 a P_2 (where $P_1 < P_2$). We say that the orbits are in a resonance whenever their mutual synodic period P_syn satisfies $P_\mathrm{syn} = mP_1$, where m is an integer.

[1] Such a place is called the Lagrangian point L1.

a) Assuming that the pairs of orbits of Ganymede with Europa and Europa with Io, respectively, are both resonant with the same value of $m = 2$, find the ratios of sidereal periods for all three orbits.
b) Assuming that the semi-major axis of Ganymede is $a_G = 168R_E$, where R_E is the Earth's radius and that the sidereal period of Ganymede is longer than that of Io by 5.34 d find the numerical value for the mass of Jupiter in M_E (where M_E is the Earth's mass).

[a) $4:2:1$; b) $320M_E$]

Missile test — AB/N/5

Consider a distant Earth-like planet with mass M and radius R. The planet is inhabited by a civilisation who, at one point, decided to experiment with advanced ICBM technology. The firing polygon lies at latitude $\phi_1 = 55°$ and unknown longitude λ_1.

During one such test, the launch sequence has failed and although the missile was given correct speed (which was to be equal to the orbital speed in a circular orbit with radius equal to that of the planet), it was fired under a wrong angle relative to the ground. As a result of this mishap, the missile landed near a city with coordinates $\phi_2 = 55°$, $\lambda_2 = 0°$. The total duration of flight was $\Delta t = 1\,\mathrm{h}\,8\,\mathrm{m}$.

Neglect all effects due to atmosphere in all calculations below. You should assume that the planet's rotation about its axis is very slow, i.e. $\Delta t \ll T$, where T is the period of the planet's rotation.

a) Find the semi-major axis a of the missile's trajectory. Give your answer in multiples of the radius R of the planet.
b) Find the elevation angle α under which the missile was fired (in terms of $G, M, R, \Delta t$ and numerically). Determine the maximum altitude H the missile has attained during its flight. Give your answer in multiples of R.
c) Find the longitude λ_1 of the launch site (in degrees). Give all possible solutions.

Hint: A spherical triangle satisfies $\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$, where a, b, c are sides of the triangle and α is the angle between b, c (all quantities in radians).

d) Find the maximum altitude δ (in degrees) the missile reaches as seen by an observer with coordinates $\phi_3 = 30°$, $\lambda_3 = \lambda_1/2$.

[a) $a = R$; b) $\arcsin\left[\frac{1}{2}\left(\sqrt{GM/R^3}\Delta t - \pi\right)\right] \approx 73°$, $\frac{1}{2}\left(\sqrt{GM/R^3}\Delta t - \pi\right) \doteq 0.956$; c) $\pm 61°$; d) $37°$]

Light of distant Earth AB/N/6

Assume that Mars and the Earth orbit the Sun along circular coplanar orbits. The radius of Mars' orbit is $a_{\mathrm{M}} = 1.52\,\mathrm{au}$, the radius of Mars is $r_{\mathrm{M}} = 3390\,\mathrm{km}$, its visual geometric albedo and Bond albedo are $\alpha_{\mathrm{M}} = 0.15$ and $\beta_{\mathrm{M}} = 0.25$, respectively. The radius of Earth's orbit is $a_{\mathrm{E}} = 1.00\,\mathrm{au}$, the radius of Earth is $r_{\mathrm{E}} = 6371\,\mathrm{km}$, its visual geometric albedo and Bond albedo are $\alpha_{\mathrm{E}} = 0.37$ and $\beta_{\mathrm{E}} = 0.31$, respectively. Consider a situation when Mars and Earth are at opposition. Visual magnitude of the Sun as observed from the Earth is $m_{\odot} = -26.7\,\mathrm{mag}$, the temperature of the photosphere is $T_{\odot} = 5778\,\mathrm{K}$ and radius of the Sun is $r_{\odot} = 695\,700\,\mathrm{km}$.

The visual geometric albedo α of a spherical body is defined so as

$$I = \alpha I_0 \frac{R^2}{r^2} ,$$

where R is radius of the body, r is distance from the body to an observer, I_0 is the intensity of light incident on the body and I is the intensity of the light reflected by the body and subsequently received by the observer at zero phase angle. The Bond albedo β is defined as the reflected-to-incident ratio of the total radiative energy.

a) Find the total luminosity $L_{\odot}$ of the Sun. Give your answer in terms of $r_{\odot}, T_{\odot}$ and suitable constants and also numerically in watts.

You should ignore the difference between the total (bolometric) and visual luminosity in the subsequent parts.

b) Determine the surface equilibrium temperature T_{M} of Mars. Assume that this temperature is uniform over the surface. Give your answer in terms of $T_{\odot}, r_{\odot}, a_{\mathrm{M}}$ and suitable type of albedo and also numerically in kelvins.
c) Repeat the calculation from b) for the case of the Earth.
d) What is the visual magnitude m of Mars as observed from the Earth (in opposition)? Give your answer in terms of $a_{\mathrm{E}}, a_{\mathrm{M}}, r_{\mathrm{M}}, m_{\odot}$ and suitable type of albedo and also numerically in mag. Neglect all effects due to atmosphere.
e) Part of the lights reflected by Mars and received by the Earth is reflected back towards Mars. Assuming that this is the only light emitted by the Earth, find its magnitude m' for an observer on Mars. Give your answer in terms of $a_{\mathrm{E}}, a_{\mathrm{M}}, r_{\mathrm{E}}, r_{\mathrm{M}}, m_{\odot}$ and suitable types of albedo and also numerically in mag.
f) The total power produced by all the power plants on the Earth is about $P_{\mathrm{el}} = 10^{13}\,\mathrm{W}$. Assuming that this power is converted into light irradiated

evenly in all directions, find its magnitude m_{el} as observed from Mars. Give your answer in terms of $P, L_\odot, a_{\text{E}}, a_{\text{M}}, m_\odot$ and numerically in mag.

[a) $4\pi r_\odot^2 \sigma T_\odot^4 \approx 3.84 \times 10^{26}\,\text{W}$; b) $T_\odot \left(\frac{1-\beta_{\text{M}}}{4}\right)^{1/4} \left(\frac{r_\odot}{a_{\text{M}}}\right)^{1/2} \approx 210\,\text{K}$; c) $254\,\text{K}$; d) $m_\odot - 2.5 \log\left(\alpha_{\text{M}} \frac{a_{\text{E}}^2}{a_{\text{M}}^2} \frac{r_{\text{M}}^2}{(a_{\text{M}}-a_{\text{E}})^2}\right) \approx -1.93\,\text{mag}$; e) $m_\odot - 2.5 \log\left(\alpha_{\text{M}}\alpha_{\text{E}} \frac{a_{\text{E}}^2}{a_{\text{M}}^2} \frac{r_{\text{M}}^2 r_{\text{E}}^2}{(a_{\text{M}}-a_{\text{E}})^4}\right) \approx 19.6\,\text{mag}$; f) $m_\odot - 2.5 \log\left(\frac{P_{\text{el}}}{L_\odot} \frac{a_{\text{E}}^2}{(a_{\text{M}}-a_{\text{E}})^2}\right) \approx 5.84\,\text{mag}$]

Sling-shot effect AB/R/3

Consider a spacecraft approaching Jupiter. Sufficiently far from Jupiter (so that we can neglect Jupiter's gravity), but not too far (so that we do not have to bother about changes to craft's trajectory due to the Sun's gravity), the spacecraft has heliocentric speed $v_{\text{H}} \approx 18.5\,\text{km}\,\text{s}^{-1}$ and its trajectory intersects the trajectory of Jupiter at angle $45°$. The orbital speed of Jupiter is $v_{\text{J}} \approx 13.1\,\text{km}\,\text{s}^{-1}$.

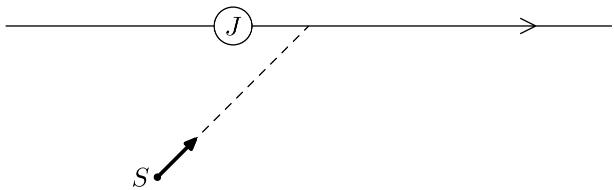


Figure 2: A small section of Jupiter's (J) trajectory and the spacecraft (S) with its velocity vector, all drawn in heliocentric reference frame before the fly-by takes place.

After the spacecraft passes by Jupiter, its velocity is aligned along the direction in which Jupiter orbits the Sun.

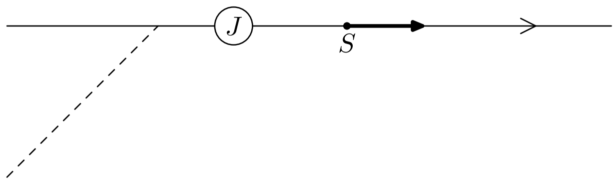


Figure 3: A small section of Jupiter's (J) trajectory and the spacecraft (S) with its velocity vector, all drawn in heliocentric reference frame after the fly-by.

a) Find the magnitude v and the direction of spacecraft's velocity $\mathbf{v}$ in Jupiter's reference frame before the fly-by.
b) Find the speed v' of the spacecraft in Jupiter's reference frame after the fly-by.
c) Find the heliocentric speed v'_{H} of the spacecraft after the fly-by.
d) Compare the values of the two speeds v_{H} and v'_{H}. Does it follow that the energy was not conserved during the fly-by?

[a) $13.1\,\mathrm{km\,s^{-1}}$; b) $13.1\,\mathrm{km\,s^{-1}}$; c) $26.2\,\mathrm{km\,s^{-1}}$]

Stellar astronomy

Pulsating star — CD/N/3

Consider an RR Lyrae type variable. Astronomers found that the period of its pulsations is $P = 12\,\mathrm{h}$, its magnitude varies over an interval of $\Delta m = 0.5\,\mathrm{mag}$ and that the ratio $x = T_1/T_2$ of its temperature T_1 when its brightness is at a maximum to the temperature T_2 when its brightness is at a minimum is $x = 1.2$. It is further known that the magnitude of the redshift (blueshift) is approximately constant over the pulsation cycle and equal to $z = 2.7 \times 10^{-5}$. Find the radii R_1 and R_2 of the star at the maximum and minimum of its brightness, respectively. Give your answer numerically as a multiple of the radius $R_\odot$ of the Sun.

[$1.75R_\odot$, $2.00R_\odot$]

Light echo — AB/N/3

During a survey, a star was discovered which developed a thin circular ring around it. The ring takes form of an ellipse on the pictures, with semi-major and semi-minor axes $\theta_a = 0.80''$ a $\theta_b = 0.57''$. It was further found that the star changes its brightness in irregular intervals. The same changes (with a delay) are observed on individual parts of the ring. The maximum observed delay is $t = 400\,\mathrm{min}$. Assuming that the ring changes its brightness as a consequence of the light emitted by the star interacting with the particles in the ring, find the distance (in pc) of the star from the Earth.

[$35.3\,\mathrm{pc}$]

Binary stars and exoplanets

Astrophysical densimeter CD/R/4

Laws of mechanics can often be used to indirectly estimate density of celestial bodies from easily measurable quantities.

First consider a spacecraft orbiting the Jupiter's moon Europa with period $P_{\mathrm{Eu}} = 115\,\mathrm{min}$. The craft orbits the moon at a low altitude so that the radius of its orbit can be taken to be equal to the radius of the moon.

a) Use this information to find the mean density ρ_{Eu} of Europa.

Now consider a contact binary, which (for the sake of simplicity) we model by two identical spherical components touching each other at one point and orbiting around the common centre of mass. We also assume that the binary is eclipsing with period $P_* = 600\,\mathrm{min}$.

b) Find the mean density ρ_* of the stars.
c) Assuming that this system lies $d = 60\,\mathrm{pc}$ away from the Earth, that the effective temperature of both stars is $T_{\mathrm{eff}} = 8000\,\mathrm{K}$ and that the maximum flux from the system as measured on the Earth is $F = 2 \times 10^{-9}\,\mathrm{W\,m^{-2}}$, find the radius R_* and the mass M_* of each component in multiples of the solar radius and mass.

[a) $3000\,\mathrm{kg\,m^{-3}}$; b) $110\,\mathrm{kg\,m^{-3}}$; c) $5.5R_{\mathrm{S}}$, $13M_{\mathrm{S}}$]

Exoplanet CD/N/6

In this question you will consider an extrasolar system containing one planet orbiting the host star along a circular orbit. The wavelength at which the host star radiates maximum energy is $\lambda = 651\,\mathrm{nm}$.

a) Find the effective temperature T_* of the host star (in K).
b) Determine the luminosity L_* of the host star (as a multiple of the luminosity $L_\odot$ of the Sun), given that its radius R_* was determined as $R_* = 0.75R_\odot$.

It was found that the star is orbited by a transiting exoplanet.

c) Assuming that the transits undergo centrally, that the depth of a primary minimum is $\Delta m = 0.210 \times 10^{-3}\,\mathrm{mag}$ and that the amount of radiation emitted by the planet itself is negligible, find the radius R_{p} of the planet (in km).

The main sequence stars satisfy the following approximate relation between their luminosity L and mass M: $L \propto M^{7/2}$. Assume that the mass of the planet can be neglected compared to the mass of the star.

d) Find (in au) the separation a between the star and the planet, assuming that analysing the light-curve, we determined the planet's orbital period as $P = 205\,\mathrm{d}$.

Using spectroscopic data, it was also inferred that the planet has no atmosphere. You can further assume that the planet rotates rapidly about its axis and that its Bond albedo is $A = 0.36$.

e) Find the equilibrium surface temperature T_p of the planet (in K).

[a) 4450 K; b) $0.20L_\odot$; c) 7260 km; d) 0.58 au; e) 218 K]

Eclipsing binary AB/R/4

A main sequence star with mass $M = 1.683M_\odot$ and radius $R = 1.281R_\odot$ is periodically obscured by its brown dwarf companion. When observed from Earth, the line of sight lies in the orbital plane of the system. The total magnitude of the system is $m_0 = 9.1012\,\mathrm{mag}$ when both stars are visible, $m_\mathrm{front} = 9.1249\,\mathrm{mag}$ when the brown dwarf is transiting in front of the star and $m_\mathrm{back} = 9.1120\,\mathrm{mag}$ when the dwarf passes behind the star. Looking at the spectrum of the system, the line Hα (laboratory wavelength $\lambda_0 = 656.28\,\mathrm{nm}$) lies at $\lambda = 656.59\,\mathrm{nm}$ and oscillates with amplitude $\Delta\lambda = 1.79 \times 10^{-3}\,\mathrm{nm}$.

Considering the transit of the dwarf across the disk of the star, the first and second contact are separated by $\Delta t = 22.122\,\mathrm{min}$ and second and third contact are $\Delta T = 126.469\,\mathrm{min}$ apart. Orbital period of the system is $P = 48.0986\,\mathrm{h}$. Assume that the disks of both the star and the dwarf are circular and uniform in brightness.

a) Find the radial velocity of the whole system relative to the Sun.
b) Find the semi-major axis a of the system (separation of the two components) and the mass m of the brown dwarf.
c) Using data inferred from the light-curve, find the ratio r/R of the radii of the two components.
d) Find the radii R of the star and r of the dwarf.

[a) $142\,\mathrm{km\,s^{-1}}$; b) $5.536 \times 10^9\,\mathrm{m}$, $1.369 \times 10^{28}\,\mathrm{kg}$; c) 0.1477; d) 892 000 km, 131 700 km]

Galactic astronomy

Galaxy cluster EF/R/2

In Table 1, we list the velocity components of ten galaxies. Each row contains the components (in x, y and z) for one galaxy. Those galaxies are part of a cluster with more than 1000 members whose coordinates are $\alpha = 12^\mathrm{h}59^\mathrm{m}48.7^\mathrm{s}$,

$\delta = 27°58'50''$ with a distance of 103 Mpc from the Earth. The cluster is 10 Mpc across. Its cosmological redshift is $z = 0.0231$.

Table 1: The velocity components of the chosen galaxies from the cluster.

i	$\frac{v_x}{\mathrm{km\,s^{-1}}}$	$\frac{v_y}{\mathrm{km\,s^{-1}}}$	$\frac{v_z}{\mathrm{km\,s^{-1}}}$
1	1990.3	765.1	2530.4
2	2235.2	410.0	746.3
3	645.6	1785.4	797.8
4	1064.0	2350.2	1576.7
5	1362.6	1172.4	1024.3
6	990.6	1708.5	1931.5
7	1816.8	1556.2	883.3
8	1851.9	1587.4	1760.5
9	1118.9	2099.5	732.1
10	698.2	844.6	654.1

a) Find the name of this cluster and cite your source.
b) Based on the listed parameters, calculate the Hubble parameter. Write your answer up to 3 significant digits in the commonly used units.
c) Using the velocity components, calculate the speed of each galaxy in Table 1 (denote v_i), i.e. v_1 to v_{10}. Then calculate the mean speed (denote μ). Finally, calculate the statistical dispersion of speeds from the mean (denote σ). You are allowed to use spreadsheet software to do the calculation (e.g. Excel), however, you are required to comment on all steps. Write σ in SI units up to 4 significant digits.
d) The mass of the cluster which is derived from the observed velocities, i.e. the dynamical mass, can be estimated using the relation $M_{\mathrm{dyn}} = (5/3)(R\sigma^2/G)$, where R is the radius of the cluster, σ is the velocity dispersion and G is the gravitational constant. What is the value of the dynamical mass of this cluster? Write down your result in SI units up to 2 significant digits.
e) Assume that we have estimated from some observations that every galaxy in this cluster has a mass of 10^{12} (one hundred billion) Solar masses. What is the total (observed) mass of this cluster? Write the result in SI units up to 2 significant digits. *Note:* This may differ from the dynamical mass.
f) In cosmology, we denote the dynamical mass M and the observed mass L. Find the ratio $\frac{L}{M}$. What is the usual explanation of this discrepancy?

[a) Coma cluster; b) $67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$; c) $\mu_x \approx 1377\,\mathrm{km\,s^{-1}}$, $\mu_y \approx 1428\,\mathrm{km\,s^{-1}}$, $\mu_z \approx 1264\,\mathrm{km\,s^{-1}}$, $\sigma = 1000\,\mathrm{km\,s^{-1}}$; d) 7.7×10^{45} kg; e) 2.0×10^{44} kg; f) 0.026, dark matter]

Cosmology and relativity

Flashes from outer space AB/N/1

A spaceship whose length at rest is $L_0 = 500\,\mathrm{m}$ has on its front end attached a planar rearview mirror. The plane of the mirror is perpendicular to the direction of the flight. The ship moves at a speed $0 < v < c$ away from the Earth. At some point in the centre part of the ship (i.e. in the distance $L_0/2$ from the mirror), the captain brings about a bright flash propagating in all directions. Find the time interval T' between the two flashes received by an observer on the Earth (in terms of L_0, v, c and then numerically in μs for $v = 9c/10$). You should neglect all effects due to gravity.

[$\sqrt{(c+v)/(c-v)}(L_0/c) \approx 7.3\,\mu\mathrm{s}$]

Distant galaxy AB/N/4

The most distant observed galaxy GN-z11, which lies in the constellation Ursa Major, has redshift $z_\mathrm{g} = 11.09$. According to some estimates, this galaxy originated at the beginning of the reionisation period and its stars were only $\tau = 40 \times 10^6$ let old at the time the observation was made. Find the redshift z_h at which the creation of the first stars in this galaxy would be seen. You can assume that the universe is $T = 13.82 \times 10^9$ years old. First express your answer in terms of z_g, τ, T and then find its numerical value.

[$\left((1+z_\mathrm{g})^{-3/2} - \tau/T\right)^{-2/3} - 1 \approx 12.18$]

Black holes and their temperature AB/R/2

Black holes belong to the simplest objects in our universe. A non-rotating, uncharged black hole (Schwarzschild black hole) is fully described just by its mass M.

a) Using Newtonian physics, find the radius R with mass M such that the escape velocity from its surface is equal to c.
b) Find the surface area A of a black hole with mass M.
c) Find the average density of a black hole with mass M. Comment on its behaviour as M increases.

Theoretical physicists Jacob Bekenstein and Stephen Hawking found out that black holes are probably not entirely black. The following parts will explore some details about the thermal radiation of black holes.

Classical thermodynamics defines a quantity called entropy, which can be thought as a measure of disorder and is intimately related to temperature. Entropy is denoted S and its SI-derived units are $\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$. The value of entropy is proportional to the logarithm of the number of ways how to reorganize the microstructure of a system so that macroscopically it still looks the same. For a black hole, it was predicted that its entropy is directly proportional to its surface area. Let us define a coefficient η so that $S = \eta A$.

d) Assuming that $\eta = c^\alpha G^\beta k^\gamma \hbar^\delta$, where c is the speed of light, G is the Newton's gravitational constant, $\hbar$ is the reduced Planck's constant, k is the Boltzmann's constant and α, β, γ and δ are real numbers, use dimensional analysis to fix η (up to a dimensionless rescaling). Hence, find the entropy S of a black hole with mass M.

In order to infer the temperature of a system, all we have to do is to look at the change in its entropy when we change the total energy of the system a little. Assume that the total energy of the black hole is $E = Mc^2$. For small ΔM, the thermodynamic temperature is then defined as

$$T = \frac{\Delta E}{\Delta S} = \frac{E(M + \Delta M) - E(M)}{S(M + \Delta M) - S(M)}\,.$$

e) Find the thermodynamic temperature of a black hole with mass M. Comment on its behaviour as M is increased and determine the temperature of black holes with masses m_e, $M_\odot$ and $10^8 M_\odot$ (m_e is the rest mass of electron).
f) Find the luminosity of a black hole in terms of M.
g) Find the time τ which it takes for a black hole to evaporate due to Hawking's radiation. Give your answer in terms of M. Assume that during the evaporation process, the black hole absorbs no energy from its surroundings. How long does it take for black holes with masses m_e, $M_\odot$ and $10^8 M_\odot$ to evaporate?

[a) $2G\frac{M}{c^2}$; b) $16\pi\frac{G^2}{c^4}M^2$; c) $\frac{3}{32\pi}\frac{c^6}{G^3M^2}$; d) $16\pi\frac{Gk}{c\hbar}M^2$; e) $\frac{c^3\hbar}{32\pi Gk}\frac{1}{M}$; f) $\frac{1}{2^{16}\pi^3}\frac{c^8\sigma\hbar^4}{G^2k^4}\frac{1}{M^2}$; g) $2^{16}\pi^3\frac{G^2k^4}{c^6\sigma\hbar^4}M^3$, 10^{-104} s, 10^{70} yr, 10^{95} yr]

Practical problems

Mars orbiter — CD/N/7

A probe orbits Mars (mass $M = 6.417 \times 10^{23}$ kg, sidereal period of rotation $P_{\mathrm{sid}} = 24\,\mathrm{h}\,37\,\mathrm{min}$) in the plane of its equator. All engines of the probe are down so that the probe's motion is influenced only by Mars' gravity. Once per each $\Delta t = 30\,\mathrm{min}$, the probe takes a picture of the region of Mars' surface directly *beneath* the probe (in nadir). Times t at which the photographs were captured are summarised in Table 2 together with the longitudinal coordinate λ at the centre of each photograph. Denote $\Delta\lambda$ the change of the coordinate λ between two consecutive captures.

a) Find the values $\omega_{\mathrm{syn}} = \Delta\lambda/\Delta t$ (in $\deg\,\mathrm{h}^{-1}$) for each interval Δt between consecutive captures and assign them to the values t_{c} of the centres of the corresponding intervals.

b) Plot the values ω_{syn} into a graph against t_{c}.

NB: You do not have to plot all data points. It is acceptable to plot only such data point which are suitable for determining the quantities required in parts c) and e). You should, however, plot at least 25 data points.

c) Determine the orbital period P (in hours) and semi-major axis a (in km) of the probe.

d) Write down a formula expressing the sidereal angular speed ω_{sid} of the probe relative to the centre of Mars in terms of ω_{syn}.

Further, you are given that $r_{\mathrm{p}} v_{\mathrm{p}} = r_{\mathrm{a}} v_{\mathrm{a}}$, where $r_{\mathrm{p}}, v_{\mathrm{p}}$ and $r_{\mathrm{a}}, v_{\mathrm{a}}$ are the distance of the probe from the centre of Mars and its orbital speed at periapsis and apoapsis, respectively.

e) Find the numerical eccentricity e and r_{p} with r_{a} (in km).

[c) 12.8 h, 13 200 km; d) $\omega_{\mathrm{sid}} = \omega_{\mathrm{syn}} + \frac{360^\circ}{P_{\mathrm{sid}}}$; e) 0.40, 7900 km, $r_{\mathrm{a}} \doteq 18\,500$ km]

Atmospheric extinction — CD/N/8

Speaking of atmospheric extinction, what we have in mind is the decrease of intensity of radiation coming from space towards Earth's surface due to scat-

Table 2: Times and centre coordinates of individual captures.

$\frac{t}{\mathrm{h}}$	$\frac{\lambda}{\mathrm{deg}}$	$\frac{t}{\mathrm{h}}$	$\frac{\lambda}{\mathrm{deg}}$	$\frac{t}{\mathrm{h}}$	$\frac{\lambda}{\mathrm{deg}}$	$\frac{t}{\mathrm{h}}$	$\frac{\lambda}{\mathrm{deg}}$
0.0	–159.09	6.0	–20.01	12.0	13.14	18.0	110.98
0.5	–159.03	6.5	–6.32	12.5	12.57	18.5	135.99
1.0	–158.38	7.0	2.71	13.0	12.29	19.0	154.60
1.5	–156.88	7.5	8.53	13.5	12.45	19.5	167.22
2.0	–154.17	8.0	12.19	14.0	13.25	20.0	175.50
2.5	–149.73	8.5	14.36	14.5	14.96	20.5	–179.18
3.0	–142.76	9.0	15.48	15.0	17.97	21.0	–175.87
3.5	–132.05	9.5	15.86	15.5	22.84	21.5	–173.94
4.0	–115.98	10.0	15.73	16.0	30.45	22.0	–172.99
4.5	–93.33	10.5	15.27	16.5	42.10	22.5	–172.73
5.0	–65.98	11.0	14.60	17.0	59.45	23.0	–172.94
5.5	–39.98	11.5	13.85	17.5	83.35	23.5	–173.45

tering and absorption processes in the Earth's atmosphere. The magnitude of this extinction depends mainly on the thickness of the layer through which the radiation has to pass, i.e. it is lowest near the zenith and highest near the horizon. The values for the extinction also depend on the wavelength λ of the radiation. The measurements in different photometric filters are therefore affected differently by atmospheric extinction. In the following, we will consider measurements in filters B (blue) and V (visible).

Let us define the air-mass X as the ratio of the optical depth of the layer through which the radiation has to pass to the optical depth of the layer towards zenith. The magnitude m_λ of a star as observed outside of the atmosphere can then be found using the magnitude $m_{\mathrm{inst},\lambda}$ measured on the ground (both in a filter at wavelength λ) as

$$m_\lambda = m_{\mathrm{inst},\lambda} - K_\lambda X \,, \tag{1}$$

where K_λ is the extinction coefficient, which can be determined by measuring the brightness of a star (not a variable one though) during the course of one night at a number of different altitudes above the horizon.

Photometric data (taken at a fixed observatory over one night) in filters B and V for one such star are shown in Table 3. The values of air-mass for each observation are also included.

a) Plot the data from Table 3 into two graphs $m_{\mathrm{inst},\lambda}$ against X (one plot for each filter B and V).

Table 3: Photometry for one star over one night (for different X).

filter B				filter V			
X	$\frac{m_{\text{inst,B}}}{\text{mag}}$	X	$\frac{m_{\text{inst,B}}}{\text{mag}}$	X	$\frac{m_{\text{inst,V}}}{\text{mag}}$	X	$\frac{m_{\text{inst,V}}}{\text{mag}}$
1.224	17.303	1.223	17.309	1.228	16.302	1.227	16.302
1.977	17.477	1.430	17.351	2.030	16.415	1.422	16.312
1.949	17.474	1.591	17.373	2.006	16.408	1.561	16.351
1.235	17.287	1.609	17.372	1.228	16.278	1.581	16.337
1.233	17.288	1.857	17.435	1.227	16.280	1.918	16.383

b) Graphically find the extinction coefficients K_B and K_V in filters B and V.
c) Find the exoatmospheric magnitudes m_B and m_V. Find the ground-observed color index $(B - V) = m_\mathsf{B} - m_\mathsf{V}$.

[b) 0.23, 0.15 c) 17.0 mag, 16.1 mag, 0.9 mag]

Detection of gravitational waves AB/N/7

At the beginning of the year 2016, a little more than 100 years since Einstein formulated the principles of general relativity, the first direct detection of gravitational waves was announced by the LIGO collaboration. This important discovery opens up new opportunities for exploration of high energy phenomena in our universe.

Both the so far detected events (GW150914 a GW151226) are thought to correspond to a collision of two inspiralling components in a binary: the emitted gravitational waves carry away energy, consequence of which is the decrease in size of the orbits, eventually resulting in a collision. The frequency of the emitted waves is always twice the orbital frequency. The goal of this problem will be to determine the so-called chirp mass of GW151226 and infer further information about the system.

Working to the leading order in the post-Newtonian expansion of Einstein's field equations, one can show that the frequency f of gravitational waves emitted by an inspiralling binary just before collision satisfies

$$\frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t-t_0)+\frac{3}{8}f(t)^{-8/3}=0\,, \tag{2}$$

where t_0 is the epoch of collision, G is Newton's gravitational constant, c is the speed of light and $\mathcal{M}$ is the chirp mass, where

$$\mathcal{M} := \frac{(m_\mathrm{A} m_\mathrm{B})^{3/5}}{(m_\mathrm{A}+m_\mathrm{B})^{1/5}} \tag{3}$$

with masses of the two components denoted as m_{A} and m_{B}.

a) Pick 10 suitable points on Fig. 4 for which you read off their values of f and t.

You will now determine the chirp mass $\mathcal{M}$ of GW151226 by reading it off the plot $F(f)$ against t for a suitable F.

b) Choose $F(f)$ such that you can easily determine $\mathcal{M}$ from the plot $F(f)$.
c) Plot the values of $F(f)$ into a graph against t.
d) Graphically determine the chirp mass $\mathcal{M}$ (in units of solar mass).

Further assume that the components of GW151226 have equal masses.

e) Find the masses of the components of GW151226 (in units of solar mass).
f) Estimate (you can use Newtonian mechanics) the separation of the components just before they collide (in km).
g) Can something be said about the nature of the colliding objects?

[b) $f^{-8/3}$; d) $10 M_{\mathrm{S}}$; e) $12 M_{\mathrm{S}}$; f) 110 km; g) black holes]

Rotation of Mercury — AB/N/8

In the year 1965, a series of short radio-pulses with frequency $f = 430\,\mathrm{MHz}$ was sent by the Arecibo antenna in the direction to Mercury. These pulses were reflected by the planet's surface and returned back to the Earth, where they were detected widened in both frequency and time. Assume that the Mercury's rotation axis is perpendicular to the line joining the Earth and Mercury.

In Fig. 6, five radar reflections with different time delay at detection are displayed. Assume that the corresponding pulses were reflected off the Mercury's surface in its equatorial plane. The data were corrected for the orbital motion of both Earth and Mercury. The situation is depicted in Fig. 5.

a) Determine the component V_0 of the rotation speed of Mercury in the direction towards Earth in terms of $c, f, \Delta f$.
b) Find x, y in terms of $R, c, \Delta t$.
c) Find P in terms of $R, c, \Delta t,\ f$ a Δf.
d) From Fig. 6, read off the frequency shifts Δf and their uncertainties.
e) Find x, y, V_0 and P numerically for individual time delays. Find the mean value $\bar{P}$ for the period of rotation and its uncertainty (in days).
f) On 17/8/1965, Mercury was situated $a_{\mathrm{M}} = 0.3977\,\mathrm{au}$ from the Sun, Earth was situated $a_{\mathrm{E}} = 1.0116\,\mathrm{au}$ from the Sun and the angle Sun–Earth–Mercury was $\alpha = 4°$. The time delay between emission and reception of the reflected pulse was $\tau = 616.125\,\mathrm{s}$. Deduce the value of 1 au in km.

[a) $\frac{c\Delta f}{2f}$; b) $R - \frac{c\Delta t}{2}$, $\sqrt{R^2 - \left(R - \frac{c\Delta t}{2}\right)^2}$; c) $\frac{4\pi f}{c\Delta f}\sqrt{R^2 - \left(R - \frac{c\Delta t}{2}\right)^2}$; f) 149 503 000 km]

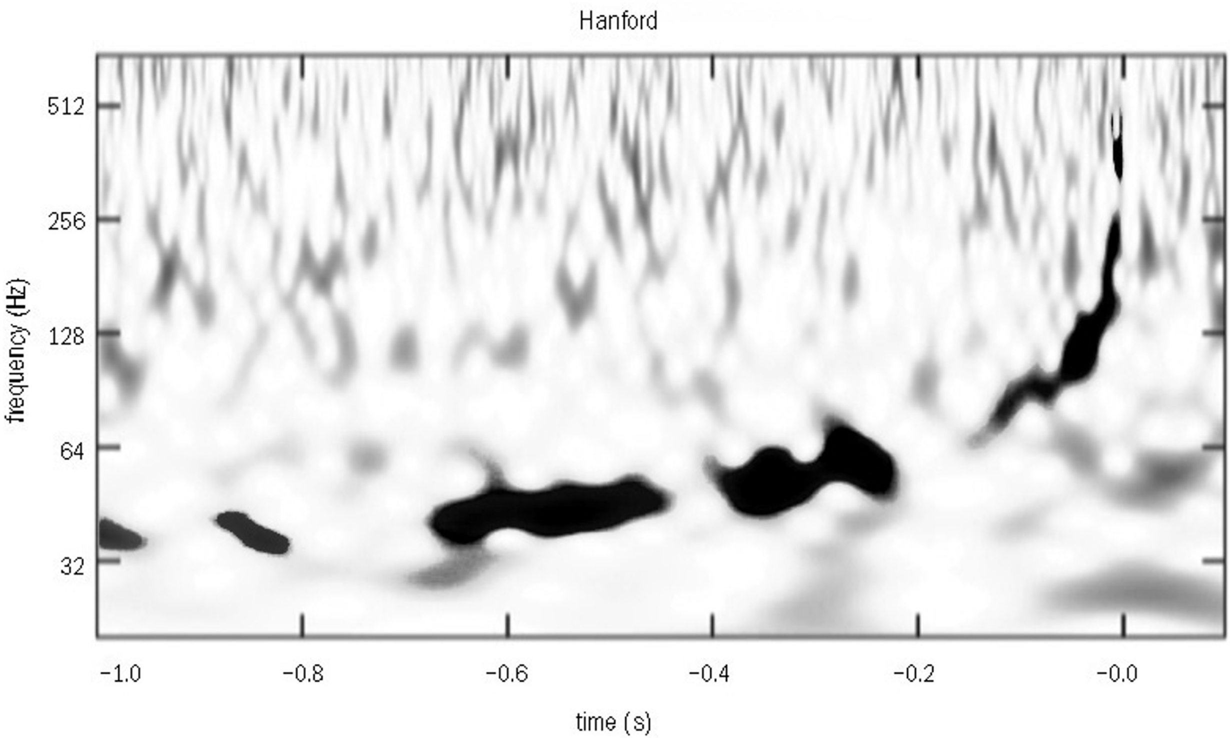


Figure 4: Plot of the frequency of the gravitational waves against time as detected by LIGO at the Hanford site during the event GW151226. Credit: LIGO collaboration.

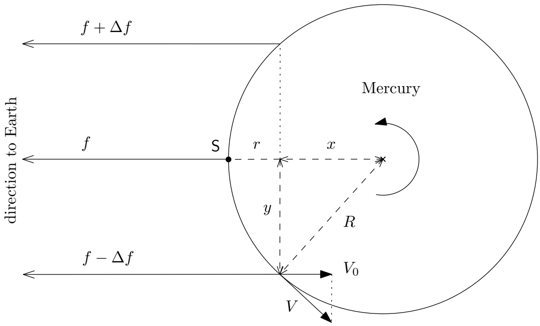


Figure 5: Reflection of a pulse from Mercury's surface. S is the sub-radar point, V is the speed of Mercury's rotation along its equator.

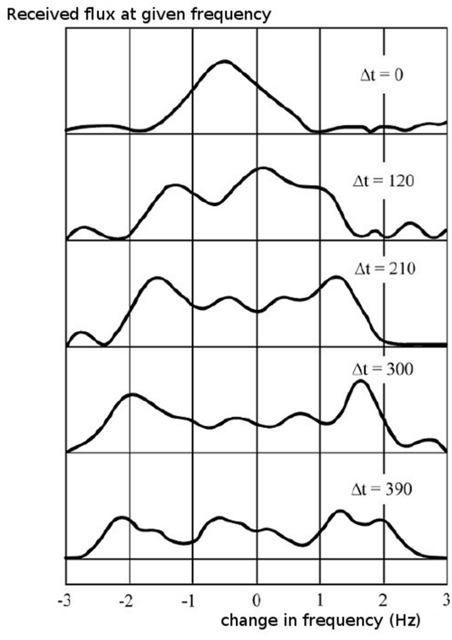


Figure 6: Radar pulses reflected from mercury (Δt is in microseconds). The measurement was performed on 17/8/1965 at Arecibo, Puerto Rico. Frequency of the pulse sent towards Mercury was 430 MHz. Source: M. Zejda, *Sbírka praktických a laboratorních úloh – Základy astronomie I* (in Czech).

2^{nd} IWAA, Prague

The 2^{nd} International Workshop on Astronomy and Astrophysics (IWAA) was held from 3^{rd} to 10^{th} July 2017 at the Faculty of Mathematics and Physics in Prague, Czech Republic. It was attended by nearly 40 students from the Czech Republic, Estonia, Hungary, Poland and Slovakia, accompanied by their leaders. The programme of the workshop contained seminars (called "training sessions") along with public lectures, observations, planetarium session and excursions. The participants were also given an opportunity to taste competitive atmosphere during an IOAA-style competition.

Figure 7: The theoretical round is about to begin.

This year's event was intended as a continuation of the camp, which was organised in 2016 at the Toravere observatory in Estonia. The training sessions were supervised both by local organizers and guest team leaders. In general, main emphasis was put on developing problem solving skills ranging over a large span of areas of interest in astronomy and astrophysics, covering both theoretical and practical aspects. While the language of instruction was English, all competition problems were translated into national languages.

Apart from attending the training sessions, students were offered some insight into different areas of modern astronomy, physics and computer science through a series of evening public lectures. Rhys Taylor of the Astronomical Institute of the Czech Academy of Sciences gave a talk on the problem of missing galaxies, which keeps plaguing the existing theories of their formation. Stanislav Fořt, the absolute winner of the 5^{th} IOAA in Poland and now a PhD student at Stanford University, introduced the audience to the secrets of modern artificial intelligence methods. Finally, Michal Švanda from the Astronomical Institutes of the Charles University explained how to use the propagation of seismic waves to investigate stellar structure.

Figure 8: The planetarium round was held in Hradec Králové.

The competition part of the workshop consisted of four rounds: theory with 5 short, 5 medium and 3 long questions, data analysis with 3 questions, sky map and planetarium, which was held in the newly constructed dome in Hradec Králové featuring the sky of Phuket, the host of the 11^{th} IOAA.

The absolute winner of the 2^{nd} IWAA competition was Martin Okánik of

Figure 9: The absolute winner Martin Okánik from Slovakia being presented with his certificate by the President of the Czech Astronomy Olympiad, Dr. Jan Kožuško.

Slovakia, who also achieved the best result in data analysis and sky map. Second place went to Jindřich Jelínek from the Czech Republic, third place was seized by Richard Luhtaru form Estonia. Best theory was awarded to Taavet Kalda from Estonia, while the Czech student Martin Orság performed best in the planetarium round. The prizes were presented to the students by Prof. Steven N. Shore, an editor of Astronomy & Astrophysics, and Dr. Jan Kožuško, President of the Czech Astronomy Olympiad. Estonia and the Czech Republic used ranking of their students to select their teams for the 11$^{\text{th}}$ IOAA.

The 2$^{\text{nd}}$ IWAA would not take place were it not for a generous contribution from the Czech Ministry of Education, Youth and Sports. The local organisational committee, recruited form the ranks of the organisers of the Czech Astronomical Olympiad, would also like to express gratitude to the Faculty of Mathematics and Physics, Charles University for providing lecture rooms, as well as to acknowledge smaller contributions from many individuals.

The hopes are high that this is a beginning to a tradition of similar events, which will continue to inspire students in the years to follow: positive feedback obtained from this year's participants suggests that the idea of such workshops certainly merits preservation.

History of the Czech Astronomical Society

1898 – 1906 The industrialist dr. Josef Jan Frič built an astronomical observatory in Ondřejov, which later housed the Astronomical Institute of the Czechoslovak (now Czech) Academy of Sciences. Its subsequent history was intimately bound with that of the Czech Astronomical Society.

1917 The Czech Astronomical Society was established in Prague. Its founding session was attended by fifty people, among whom dr. Frič, prof. Nušl, prof. Zdeněk.

1920 The first issue of the journal Říše hvězd (Realm of Stars) appeared, with the main objective to promote astronomy and related sciences to the general public. Its successor Astropis continues to be published until now.

1928 Štefánik Observatory (on the front cover), the first public observatory in Prague, was opened on Petřín hill.

1967 Perek Telescope was commissioned in Ondřejov. With its 2m mirror, it is the largest telescope ever to be built in the Czech lands.

2004 Czech Astronomy Olympiad was founded. It is jointly organised by the Ministry of Education, Youth and Sports, the Czech Astronomical Society, and the Silesian University – Faculty of Philosophy and Science in Opava.

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