Problem Booklet
2015/16

Czech Astronomy Olympiad

Board of Organizers of the Czech Astronomy Olympiad

Prague, 2016

Contents

Introduction

Foreword

Dear friends,

it is our pleasure to present you with a booklet containing the last year's harvest of the Czech Astronomy Olympiad problems, this being the first such compilation in the Olympiad's 13 years of existence. Despite the fact that 13 is in many cultures considered an unlucky number, the students were yet again left enriched by numerous fascinating problems, observations, practical experience and new friendships.

Czech Astronomical Society, the main organizer of the Olympiad, celebrates its centenary in December 2017. At the time it was founded, its main goal was to build an observatory in Prague. Since then it has developed into a respected society joining amateur and professional astronomers in a single organization. Its 600 members work in 17 branches, covering the most important areas ranging from solar astronomy over variable stars to cosmology. Aside from the research topics, the society concentrate immense effort into work with young astronomers. In addition to the Olympiad (which itself attracts about 10 000 participants annually) it organizes summer schools and camps for children and youth together with astronomical courses, clubs and more.

Czech Astronomy Olympiad was founded in autumn 2003 and quickly established itself as a recognized competition, comparable to the other main scientific olympiads. It is divided into four age groups (called – from the oldest to the youngest – AB, CD, EF and GH), each having three stages. The first round takes place at school with its main objective being to attract pupils to astronomy and motivate them for further work. In the second (regional) stage, participants have to solve more complex problems at home as well as to perform observational tasks. The best participants go forward to the national rounds held in Opava and Prague in March and May, the winners of which then meet at the selection camps for the IAO and IOAA.

After obtaining an official accreditation from the Czech ministry of education in spring 2006, the Olympiad received an invitation to participate at the XI$^{\text{th}}$ IAO in 2006. In 2010 we joined the IOAA and won the first gold medal in astronomy for the Czech Republic. Since then, over 40 students accompanied by 9 team leaders got hold of 5 gold, 11 silver and 18 bronze medals.

As the Czech language belongs to rather difficult and less popular ones (try to read this: Strč prst skrz krk!), we hope that you pick up our offer of this English written booklet and dive into the presented problems, maybe finding further inspiration for you or your students.

We wish you dark skies and nice reading!

Organizers of the Czech AO

Legend and acknowledgements

Each problem presented in this booklet comes equipped with its name and ID code containing information about the place of its original use in the Olympiad. For instance, "CD/R/2" denotes the second problem in the regional round of the CD category. Finally, all problems have their answers shown in small print below their statement.

Most of the competition problems that appear in Czech Astronomy Olympiad are original work of its organizers. Credits for the problems presented in this booklet are now given:

Stanislav Fořt: AB/R/3, AB/N/5, CD/N/6; *Tomáš Gráf:* CD/R/1, CD/R/2, CD/N/1, CD/N/2; *Martin Raszyk:* AB/R/2, AB/N/2, AB/N/6, CD/N/4, CD/N/8; *Ondřej Theiner:* CD/R/3; *Jakub Vošmera:* AB/R/1, AB/N/1, AB/N/3, AB/N/4, CD/R/4, CD/N/3, CD/N/5. The problems of the EF category were created in collaboration of *Radek Kříček* and *Václav Pavlík*.

We also gratefully acknowledge contribution from the *Russian Astronomy Olympiad* and *IOAA* whose problems served as an inspiration for CD/N/7, AB/N/8 and AB/N/7.

The reader certainly would not be able to enjoy the problems in their present form were it not for the meticulous work of *Miroslav Randa*, *Ota Kéhar* and *Michal Švanda* who carefully reviewed all questions used in the competition rounds.

Finally, it is here that we choose to express our immense gratitude to *Jan Kožuško*, *Lenka Soumarová* and *Tomáš Gráf* for making the Czech Astronomy Olympiad happen by diligently providing all imaginable kinds of organizational support.

Theoretical problems

Geometry, time and instrumentation

Car lights CD/N/1

A car is travelling along a straight road with the lights (which we consider to be point sources of monochromatic light with wavelength 550 nm) turned on. The distance between the lights is 140 cm. Compute the maximum distance from which we will observe the lights as two individual sources rather than a single one given that

a) we use naked eyes (pupil diameter $d = 5\,\mathrm{mm}$),
b) we use a binocular 10×50.

You can neglect all atmospheric effects. The observation site is located at a high ground relative to the car so that you can neglect effects due to curvature of Earth's surface.

[a) 10.3 km; b) 103 km]

Vehicle CD/N/2

The space probe Exoplanet Prospector is equipped with a landing module named Spider, which is capable of autonomous movement along newly discovered bodies. Spider landed on one of them and found out that by coincidence the local sky possesses its own "Polaris", i.e. a star located close to the direction of the rotational axis of the exoplanet. It measured its location, used it to fix directions and set off precisely to the north.

The algorithm governing Spider's movement had been programmed so that the vehicle was moving with acceleration $a = 0.3\,\mathrm{cm \cdot s^{-2}}$ during the first half an hour. It was then cruising at a constant speed during the next 10 hours, and finally decelerated to a full stop at a constant rate over the course of next half an hour. The vehicle then started moving precisely to the west, accelerated with the rate of $a' = 0.4\,\mathrm{cm \cdot s^{-2}}$ during 15 mins, and afterwards decelerated to a halt at a constant rate for 15 mins.

After these two actions, “Polaris” was located two degrees higher above the horizon than before. Find the diameter of the exoplanet assuming that its shape is a perfect sphere.

[12 000 km]

Satelite AB/N/2

Consider a satellite which is put on a geostationary orbit around the Earth. For an observer O_1 located at a place with longitude $\lambda_1 = 99.6^\circ$ E on Earth's equator, the satellite has astronomical azimuth $A = 90^\circ$ (i.e. westwards from O_1). Assuming that an observer O_2 can see the satellite hovering in the direction of the local meridian, find the longitude λ_2 of O_2. The Earth is to be assumed spherical with radius $R_\mathrm{E} = 6\,371$ km and mass $M_\mathrm{E} = 5.972 \times 10^{24}$ kg. Effects due to atmospheric refraction can be neglected.

[18.3°]

Circumpolar star CD/N/4

In an archive, records of observations of a certain circumpolar star H from an unknown observation site S were found. We only know that S is located on the northern hemisphere. It also follows from the records that the minimum and maximum altitude above the horizon that H could attain when observed from S were $h_1 = 40^\circ$ and $h_2 = 80^\circ$, respectively. Furthermore, a note was discovered which states that the local sidereal time at the moment the star reached the altitude h_1 was $\Theta = 6$ h. Given this information, determine the equatorial coordinates (α, δ) of H and the latitude φ of S. Find all possible solutions.

[$(18\,\mathrm{h}, 70^\circ, 60^\circ)$; $(18\,\mathrm{h}, 60^\circ, 70^\circ)$]

Polaris from Borobudur CD/R/2

It is known that the construction of Borobudur, the largest Buddhist temple in the world built out of stones, began around 750 AD. Several local texts mention that it was possible to see Polaris form Borobudur during its construction. The temple is located on Java, Indonesia (geographical coordinates $\phi = 7^\circ\,36'\,28''$ S and $\lambda = 110^\circ\,12'\,14''$ E). Let us assume for simplicity that the declination of Polaris in 2015 was precisely 90°.

a) Decide whether it was possible to see Polaris above the horizon from Borobudur in 750 AD. Proper motion of Polaris and nutation can be neglected.
b) Was it possible to use Polaris to orient Borobudur by cardinal points?

Hint: You may find it useful to draw an appropriate spherical triangle. The law of cosines for spherical triangles reads

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma\,,$$

where a, b, c are the sides of the triangle (in radians) and γ is the angle between a and b.

[a) max. altitude 7°; b) probably not]

Solar system

Planet EF/R/2

a) Write down the name which is used to refer to the primary direction of the heliocentric ecliptic coordinate system and draw the symbol by which it is usually denoted. In what direction is it located (specify by giving a constellation)?

b) A planet is marked by an arrow in a picture of the sky taken on March 1, 2009 (fig. 1). Which planet is it? Write down its name and the length of the semi-major axis of its orbit. Name the source you used to obtain the answer.

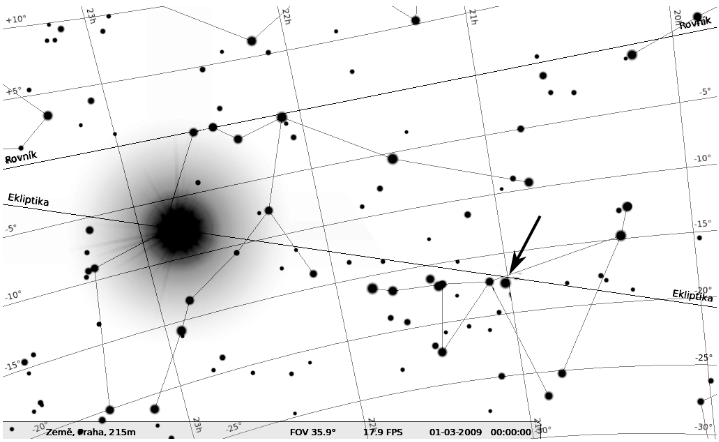

Figure 1: Sky on March 1, 2009. The planet is marked by an arrow.

c) On March 1, 2009 the heliocentric longitude of the Planet and Earth were $l_\mathrm{P} = 307°28'$ and $l_\mathrm{E} = 160°33'$. Draw two diagrams of the same size, both of them containing the Sun, the primary direction of the heliocentric ecliptic coordinate system and the orbits of Earth and the planet, simplified as circles, with the correct ratio of their radii. In the first diagram, mark the location of Earth and the planet on March 1, 2009. In the second diagram, mark the location of both bodies at opposition. All angles should be drawn rounded to a whole number of degrees.

d) The location of the planets as well as the angle between the lines planet–Sun and planet–primary direction are varying with time. The angle swept out by the line joining the Sun and a planet in one day is called the mean motion. Calculate the date of the upcoming opposition assuming that the mean motion of Earth and the planet is $n_\mathrm{E} = 59.1'$ and $n_\mathrm{P} = 5.0'$, respectively.

[a) First point of Aries, Pisces, ♈; b) Jupiter, 5.2 au; d) August 11, 2009]

The Great Comet of 1997 EF/R/3

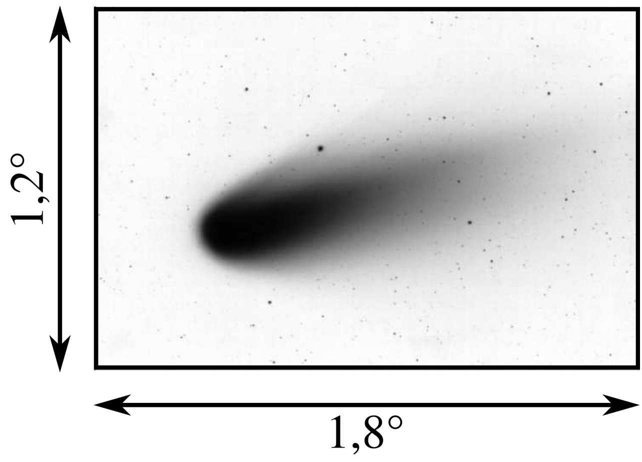

Figure 2: A picture of the comet C/1995 O1 (Hale–Bopp) with colours inverted. Displayed is the size of field of view.

a) On fig. 2, a picture of the comet C/1995 O1 (Hale–Bopp) is shown. Angular dimensions of the picture are $1.8° \times 1.2°$. Write down the conversion rule between degrees and centimetres for this picture.

b) Measure the diameter of the coma. Write down your answer in centimetres (rounded to tenths of a centimetre) and in degrees (rounded to hundredths of a degree).

c) Calculate the real size of the coma if the comet was in a distance of 1.4 au from the Earth at the time when the photo was captured. Round your answer to multiples of 5 000 km. Compare this value with a typical diameter of a comet nucleus, with the diameter of Earth (approx. 12 600 km) and with a typical size of a coma which ranges from 20 000 to 200 000 km. Use the conversion 1 au = 150 million km. *Hint:* Assume that the angular size of the comet is very small.

d) It is known that the comet passed through the perihelion at a distance $q = 0.914$ au from the Sun and that its aphelion lies at a distance $Q = 370.8$ au from the Sun. Draw a diagram containing the comet's orbit and place the Sun into one of its foci. Mark out the lines q and Q joining the Sun and the comet at perihelion and aphelion, the semi-major axis a of the ellipse and the semi-minor axis b. You should also mark the distance ε between the centre and one of the foci of the ellipse (linear eccentricity). We also write $\varepsilon = ae$, where e is the numerical eccentricity of the orbit.

e) Calculate the length a of the semi-major axis and the numerical eccentricity e of the comet's orbit.

f) The third Kepler's law reads

$$\frac{a^3}{T^2} = \text{const.}$$

Calculate the orbital period T of the comet Hale–Bopp.

g) Determine the year in which the comet will again pass by the Sun given that the last such occurrence happened on April 1, 1997.

[a) $1\,\text{cm} \sim 0.2°$; b) $(1.1 \pm 0.1)\,\text{cm}$, $(0.22 \pm 0.02)°$; c) $(805\,000 \pm 75\,000)\,\text{km}$; e) 186 au, 0.994; f) 2537 y; g) 4534 AD]

Comet CD/R/3

Imagine that on the date of discovery of the Hayakutake comet (January 30, 1996), another comet was observed. That day, exactly at 21:00 UTC, this comet was located near the vernal equinox point and it was moving along the ecliptic in a direction of decreasing right ascension with angular speed $\omega = 2.00\,° \cdot \text{d}^{-1}$ relative to background stars. That comet was just passing relatively close to the Earth so the radar measurements allowed astronomers to determine that radial component of velocity was equal to zero and the distance between the Earth and the comet was $d = 0.30$ au. The Earth's orbit is to be assumed circular with radius $r_\text{E} = 1.00$ au.

a) Draw a diagram depicting the situation described above as viewed from the direction of the north ecliptic pole. Display the following features:

   1. approximate position of the comet relative to the Earth

2. direction towards the vernal equinox point
3. velocities **u** and **v** of the comet and the Earth

b) Determine the distance r_c of the comet form the Sun at the moment of observation. You may find useful the fact that in 1996 the spring equinox occurred on March 20 exactly at 8:03 UTC.

c) Find the speed w of the comet relative to the Earth on the day of observation.

d) Find the speed u of the comet relative to the Sun on the same day.

In part d), do not forget to take into account Earth's motion in its orbit and the fact that its velocity does not have to be parallel to the velocity of the comet.

e) Decide to which family the comet belongs. Your argument should be supported by a calculation.

Hint: Orbital speed v of a particle moving along an elliptical orbit with semi-major axis a in a central force field due to a gravitating body with mass M can be written a function

$$v(r) = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

of the distance r between the central body and the particle. Use the following values of relevant constants: astronomical unit $1\,\mathrm{au} = 1.496 \times 10^{11}\,\mathrm{m}$, solar mass $M_\mathrm{S} = 1.9886 \times 10^{30}\,\mathrm{kg}$ and Newton's gravitational constant $G = 6.674 \times 10^{-11}\,\mathrm{N \cdot m^2 \cdot kg^{-2}}$.

[b) 0.83 au; c) $18.1\,\mathrm{km \cdot s^{-1}}$; d) $43.9\,\mathrm{km \cdot s^{-1}}$; e) semi-major axis 4.4 au, Jupiter's family]

Pluto probe AB/R/2

The purpose of this problem is to explore the different stages of a flight to Pluto.

The final stage of a rocket carrying a probe to Pluto left Earth's sphere of gravitational influence in the direction of Earth's orbital motion at speed $v_0 = 12.1\,\mathrm{km \cdot s^{-1}}$ relative to Earth. At this moment, the whole rig (rocket and the probe) had mass $M_0 = 25\,000\,\mathrm{kg}$. So as to place the probe on a Pluto-bound trajectory, it is required that it has to be moving at speed $v = 43.9\,\mathrm{km \cdot s^{-1}}$ relative to the Sun in the direction of Earth's orbital motion at the moment it leaves Earth's sphere of influence. (Earth's orbit is to be assumed circular with radius $a_\mathrm{E} = 1.00\,\mathrm{au}$.)

a) Find the duration Δt of a corrective burn which has to be performed after the probe leaves Earth's sphere of influence so as to put the craft on the envisaged orbit to Pluto. You can assume that the effective exhaust velocity during the burn is $u = 2.00\,\mathrm{km\cdot s^{-1}}$ with mass flow rate $\mu = 800\,\mathrm{kg\cdot s^{-1}}$ and that there is enough fuel to accomplish the maneuvre.

Now we are getting to the longest stage of the probe's flight where, after it separates from the rocket, it travels from the Earth to Pluto under the gravitational influence of the Sun. We are going to assume that at the moment of probe's approach, the distance of Pluto from the Sun is $a_\mathrm{P} = 32.0\,\mathrm{au}$ and that the area swept out by the line joining the Sun and the probe over the course of its journey from the Earth to Pluto is $S = 9.80\times10^{23}\,\mathrm{m^2}$.

b) Find the duration T of the flight from Earth's vicinity to Pluto. Determine the heliocentric speed v' of the probe when it approaches Pluto.

Let us further assume that the probe has total mass $m = 470\,\mathrm{kg}$ out of which $m_\mathrm{f} = 77\,\mathrm{kg}$ is fuel and that the effective exhaust velocity (relative to the probe) during a burn reaches $u' = 1.50\,\mathrm{km\cdot s^{-1}}$.

c) Decide whether it is possible to place the probe on a low circular orbit around Pluto. The heliocentric speed of Pluto during probe's approach was $v_\mathrm{P} = 5.70\,\mathrm{km\cdot s^{-1}}$. You should make a convincing quantitative case for your answer.

[a) 19.8 s; b) 9.46 y, $14.4\,\mathrm{km\cdot s^{-1}}$; c) not possible]

Planet CD/N/6

A planet with mass m is orbiting a star with mass $M \gg m$ along an elliptic trajectory with semi-major axis a and numerical eccentricity e.

a) Express the distances r_p and r_a, between the planet and the star at periastron and apoastron in terms of a and e.
b) Write down the equality of the total mechanical energy at periastron and apoastron in terms of the speeds $v_\mathrm{p}, v_\mathrm{a}$ at periastron and apoastron, semi-major axis a, numerical eccentricity e, masses M, m and physical constants.
c) Express the ratio $v_\mathrm{p}/v_\mathrm{a}$ in terms of e. By which law is this result implied?
d) Using your answers from parts b) and c), express the speeds v_p and v_a in terms of M, a, e and physical constants.
e) Find the total mechanical energy of the planet in its elliptical trajectory in terms of a, M and physical constants.

At the moment the planet is passing through apoastron, the star suddenly changes its mass to $M' = pM$, where $0 < p < 1$ (i.e. the mass decreases).

f) Express the new semi-major axis of the planet a' in terms of a, e and p. For which values of p does the resulting trajectory remain gravitationally bound? Write the corresponding condition as an inequality in terms of p and e.

[a) $a(1-e)$, $a(1+e)$; b) $\frac{1}{2}mv_\mathrm{p}^2 - G\frac{Mm}{a(1-e)} = \frac{1}{2}mv_\mathrm{a}^2 - G\frac{Mm}{a(1+e)}$; c) $\frac{1+e}{1-e}$; d) $\sqrt{G\frac{M}{a}}\sqrt{\frac{1+e}{1-e}}$, $\sqrt{G\frac{M}{a}}\sqrt{\frac{1-e}{1+e}}$; e) $-G\frac{Mm}{2a}$; f) $ap\,\frac{1+e}{2p+e-1}$, $2p > 1-e$]

Mars probe AB/N/5

A spacecraft orbits the Sun (mass $M \doteq 1.99 \times 10^{30}\,\mathrm{kg}$) along a circular orbit with radius $r = 1.0\,\mathrm{au}$ in prograde direction within the ecliptic plane. Having performed a very short burn in the direction of its previous motion it is accelerated by Δv and thereby placed on a parabolic orbit.

In the following you should assume that the spacecraft is influenced solely by the gravity of the Sun.

a) Find Δv. Write your answer in terms of G, M and r, where G is the Newton's gravitational constant.

The trajectory of the spacecraft intersects the orbit of Mars under an angle θ. The orbit of Mars is to be assumed circular with radius $R = 1.5\,\mathrm{au}$ as well as lying within the ecliptic plane.

b) Find the angle θ. Write your answer in terms of G, M, r and R, as well as numerically in degrees.

Let us further neglect the influence of Martian gravitational field and assume that the probe actually hits the planet. Let v_p be the speed of probe's approach relative to Mars and φ the angle between the direction of probe's approach and the direction towards the Sun, as it is seen from the planet's surface.

c) Express v_p in terms of G, M, r and R. Find its numerical value in $\mathrm{km}\cdot\mathrm{s}^{-1}$. Rotation of Mars about its axis and any effects due to its non-zero size should be neglected.
d) Find the magnitude of φ. Write your answer in terms of G, M, r and R, as well as numerically in degrees.

[a) $(\sqrt{2}-1)\sqrt{GM/r}$; b) $\cos^{-1}\sqrt{r/R}$, $35.3°$; c) $\sqrt{GM/R}\sqrt{3-2\sqrt{2}\sqrt{r/R}}$, $20.2\,\mathrm{km.s^{-1}}$; d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\frac{1-\sqrt{2r/R}}{\sqrt{1-r/R}}\right)$, $10.7°$]

Cosmic sphere AB/N/6

A spacecraft with mass M orbits the Sun along a circular orbit with radius r at speed v_c. This orbital motion is affected only by gravity of the Sun. Scientists on Earth intend to launch an absolutely black and perfectly spherical probe from the spacecraft. The probe has radius R, mass m and is made of an unknown material with density ϱ. The sphere is to orbit the Sun along a circular trajectory with radius r (same as for the mother spacecraft).

Assume that the sphere conducts heat sufficiently well so that it is able to maintain uniform temperature (which we denote by T) and that this temperature stabilises at $T = 180\,\mathrm{K}$ after the launch. The value of the solar luminosity to be used throughout this question is $L_\mathrm{S} = 3.85 \times 10^{26}\,\mathrm{W}$.

a) Find the value of r in au.
b) Starting from the momentum conservation for photons incident on the sphere, find the magnitude F_r of the force exerted by radiation on the sphere. Write your answer in terms of r, R, L_S and c, where c is the speed of light in vacuum. Momentum p of a single photon with energy E can be expressed as $p = E/c$.
c) Find the speed v'_c of sphere's free orbital motion at radius r. Write your answer in terms of $r, R, L_\mathrm{S}, \varrho, v_\mathrm{c}$ and c. You should assume that the motion of the sphere is affected only by gravity of the Sun and the radiation pressure discussed in b).

We notice that in order to maintain sphere's circular orbit at radius r, we have to slow it down a little. This can be achieved by launching it opposite to the direction of the spacecraft's motion at a speed v relative to the spacecraft. Also, so as to avoid dangerous destabilisation of spacecraft's motion, the maximum admissible impulse the spacecraft can impart on the sphere is $\Delta p_\mathrm{m} = 1\,\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}$.

d) Find (numerically) the maximum radius R_m of the sphere, such that any dangerous destabilisations of the spacecraft during the launch are avoided. Assume that $m \ll M$ and that $v = v_\mathrm{c} - v'_\mathrm{c} \ll v_\mathrm{c}$.

Hint: we have $\sqrt{1-x} \approx 1 - \frac{1}{2}x$ for $|x| \ll 1$.

[a) 2.39 au; b) $\frac{L_\mathrm{S}}{4c}\frac{R^2}{r^2}$; c) $v_\mathrm{c}\sqrt{1 - \frac{3L_\mathrm{S}}{16\pi c\varrho R r v_\mathrm{c}^2}}$; d) 21 cm]

Stellar astronomy

Cepheid CD/N/3

Compute the difference Δm between the bolometric magnitude of a cepheid at its minimum and maximum brightness, knowing that its radius R at maximum is 1.5times larger compared to the minimum and the effective temperature T of the cepheid at maximum is 1.2times larger compared to the minimum.

[1.7 mag]

Mira-type variable **CD/R/1**

A newly discovered Mira-type variable (parallax $\pi = 0.0003''$) has brightness 6.5 mag at its maximum and 10.2 mag at minimum. Interferometric measurements allowed astronomers to determine the angular diameter of the variable at its maximum brightness as $0.0018''$ and $0.0022''$ at minimum.

a) What is the radius of the variable at its maximum and minimum?
b) What is its effective temperature at maximum and minimum?
c) Determine on which wavelengths the variable emits most energy, both at its maximum and its minimum.

Let us now suppose that there is a white dwarf orbiting the variable and that it has been accreting mass from the variable since January 1, 2016. Initially, the mass of the white dwarf was $M_0 = M_\mathrm{S}$ (where M_S denotes solar mass).

d) Calculate the time in which the white dwarf explodes as a supernova (measured from the moment it started accreting mass). You can assume that the accretion rate is constant in time and equal to $0.000\,01 M_\mathrm{S}$ per year.
e) How long would it take the white dwarf to explode if the accretion rate were described by an exponential function as $M(t) = M_0\,\mathrm{e}^{0.000\,01\,t/\mathrm{year}}$?

[a) 3.0 au, 3.7 au; b) 2 770 K, 1 070 K; c) 1 050 nm, 2 710 nm; d) 40 000 years; e) 30 000 years]

Star cluster **AB/N/3**

A star cluster, consisting of stars whose luminosity is similar to that of our Sun, appears to the naked eye as a homogeneous nebulous object with surface brightness $\mu = 20\,\mathrm{mag}\cdot\mathrm{arcsec}^{-1}$. A refracting telescope with aperture diameter $D = 10\,\mathrm{cm}$ is barely powerful enough to resolve individual stars in the cluster. Assuming that $n = 1/\alpha^2$, where n is the number of stars in the cluster per radian squared and α is the average angular distance between stars in the cluster, find the distance d of the cluster in kpc. Visible light has wavelength $\lambda_\mathrm{v} = 550\,\mathrm{nm}$ and the visual magnitude of the Sun is $m_\mathrm{S} = -27\,\mathrm{mag}$. Interstellar extinction is to be neglected.

[8.8 kpc]

Binary stars and exoplanets

Habitable planet EF/N/3

Astronomers discovered an exoplanet orbiting a star similar to the Sun. They also managed to determine that one year on this exoplanet is 0.253 of our (sidereal) years.

a) Find the semi-major axis of the exoplanet. Write your answer in astronomical units.

Although we do not know the exact radius R of the exoplanet, we can assume that it is of spherical shape. The distance between the exoplanet and its central star is large enough so that we can also assume that exactly one hemisphere of the exoplanet is illuminated.

b) Express the area of the cross section of the exoplanet in terms of R.

c) All bodies (including our exoplanet) emit thermal radiation from its surface in all directions. Express (in terms of R) the area through which the exoplanet emits this radiation.

Existence of most forms of complex life crucially depends on water in liquid form. In the rest of this problem, assume the following standard form of energy balance between the received and emitted energy for the exoplanet:

$$S_{\mathrm{e}}\sigma T^4 = S_{\mathrm{a}}\alpha\frac{L}{4\pi d^2}\,,$$

where, on the left-hand side, S_{e} is the area through which the radiation is emitted, σ is Stefan–Boltzmann constant and T is the equilibrium temperature of the exoplanet's surface. On the right-hand side, S_{a} is cross-sectional area through which the radiation energy is absorbed by the exoplanet, α says what fraction of the incoming radiation is absorbed (approximately 70 % in our case so $\alpha = 0.70$), L is the luminosity of the star (which is approximately solar) and d is the distance of the exoplanet from the star.

d) Find the distance d for which the surface temperature of the exoplanet reaches the boiling point of water at standard conditions.

e) Is the exoplanet capable of harbouring Earth-type organisms? Why?

[a) 0.400 au; b) πR^2 ; c) $4\pi R^2$; d) 0.467 au; e) no]

Spectroscopic binary AB/N/1

The H_α line (laboratory wavelength $\lambda = 656.28$ nm) in the spectrum of a close binary periodically splits into two components symmetrically from its mean position. The two components reach maximum separation $\Delta\lambda = 0.77$ nm once in $T = 2\,\mathrm{d}$. Assuming that the observer's line of sight lies in the orbital plane of the binary and that the orbits are circular, find the masses M_1 and M_2 of both components in the units of solar mass.

[both $9.0M_\mathrm{S}$]

Binary star CD/N/5

A visual binary (right ascension $\alpha = 19^\mathrm{h}\,12^\mathrm{m}\,33^\mathrm{s}$, declination $\delta = 34^\circ\,52'\,2''$) located at an unknown distance d from the Earth, consists of two identical components of unknown masses $M_1 = M_2 \equiv M$. It follows from long-term astrometric observations that the two stars revolve around a common center of mass with period $P = 1.2\,\mathrm{y}$ along a circular trajectory whose radius r appears to the astronomers on the Earth under an angle $\rho = 0.01''$.

On May 12, 2015 at 12:00 CEST, proper motion in right ascension and declination of one of the components was $\mu_\alpha = 0.003^\mathrm{s} \cdot \mathrm{y}^{-1}$ and $\mu_\delta = 0.03'' \cdot \mathrm{y}^{-1}$. At the same time, the wavelength of the line H_α in the spectrum of this component was measured as $\lambda = 656.26\,\mathrm{nm}$. The laboratory wavelength of this line is equal to $\lambda_0 = 656.28\,\mathrm{nm}$.

Proper motion of the center of mass of the binary system as well as the effects of a non-zero parallax should be neglected.

a) Find the numerical value of the radial velocity v_r of the component at the above-given time in $\mathrm{km} \cdot \mathrm{s}^{-1}$.
b) Compute the numerical value of the proper motion μ of the component at the above-given time in arcseconds per year.
c) Find a general relationship between the quantities d, ρ, P, μ and v_r. It could be helpful to express the orbital speed v of the components relative to the center of mass of the system in different ways.
d) Determine the numerical value d of the distance of the binary from the Earth in pc.
e) Determine the mass M of each component in units of solar mass.

[a) $9.14\,\mathrm{km} \cdot \mathrm{s}^{-1}$; b) $0.05'' \cdot \mathrm{y}^{-1}$; c) $2\pi\rho d/P = \sqrt{v_\mathrm{r}^2 + \mu^2 d^2}$; d) $88\,\mathrm{pc}$; e) $1.9M_\mathrm{S}$]

Galactic astronomy

Crab nebula EF/N/1

Crab nebula (M1) has equatorial coordinates $5^\mathrm{h}34^\mathrm{m}31.94^\mathrm{s}$, $22^\circ0'52.2''$. It is widely assumed that this is an example of a type II supernova remnant. According to the measurements from 2009, M1 appears to have an oval shape with dimensions $420'' \times 300''$ and it is $l = 2\,\mathrm{kpc}$ away. We also know that in the narrower axis of its oval shape, the remnant expands with a speed of $1\,500\,\mathrm{km\cdot s^{-1}}$. Our goal is to estimate in which century the supernova exploded.

a) Find the real size of Crab nebula. Denote it by d and write your answer in parsecs.
b) Considering that the linear size of Crab nebula is now given in parsecs and our objective is to estimate the time elapsed from the explosion of the corresponding supernova, then $\mathrm{km\cdot s^{-1}}$ is not a suitable choice of a unit for the expansion speed. Suggest a better unit. You should also state the numerical value of the expansion speed in your new unit.
c) Determine the century in which the star exploded. Assume that the speed of expansion has remained constant since the explosion occurred.

[a) 2.90 pc; b) $0.001\,53\,\mathrm{pc\cdot y^{-1}}$; c) 11th century]

Ancient supernova AB/R/1

Medieval Egyptian astronomer Ali ibn Ridwan reported about the supernova SN 1006 that the "spectacle was a large circular body, two and a half to 3 times as large as Venus. The sky was shining because of its light. The intensity of its light was a little more than a quarter that of Moon light". Ali observed this supernova over the course of summer 1006 towards the constellation of Lupus.

Another testimony comes from the monks of St. Gallen, who even managed to record an approximate light-curve of the supernova. Based on this data we can conclude that SN 1006 would probably be classified as a type Ia supernova. This type of supernovae occur in symbiotic binaries containing a white dwarf: after the mass of the dwarf exceeds the so-called Chandrasekhar limit, a supernova explosion is triggered. The uniformity of physical conditions prior to these explosions means that at their maxima, all type Ia supernovae have the same absolute bolometric magnitude. In the V-filter this is equal to $M_\mathrm{V} = (-19.5 \pm 0.4)\,\mathrm{mag}$.

During the last two decades, astronomers managed to amass enough observational data about the SN 1006 remnant in order to succeed at inferring its distance. Comparing pictures of the remnant taken several years apart, it was found out that the spherical envelope forming the remnant expands at current speed $\alpha = (0.280 \pm 0.008)\,\mathrm{arcsec\cdot y^{-1}}$. Further, measuring the Doppler shift of spectral lines H_α, He I and He II, the radial velocity of envelope's expansion was determined as $v_\mathrm{r} = (2900 \pm 100)\,\mathrm{km\cdot s^{-1}}$. Finally, from reddening of the spectra of nearby stars, the interstellar extinction towards the SN 1006 remnant was found as $A_\mathrm{V} = (0.31 \pm 0.10)\,\mathrm{mag}$.

a) Find the distance d to the SN 1006 remnant and its uncertainty.
b) Assuming that the distance of SN 1006 when it exploded was approximately equal to d, find the visual magnitude m_V (together with its uncertainty) of the supernova at its maximum. Compare your result to Ali's testimony.

To conclude this question, imagine that a sudden increase of brightness lasting $\Delta t = 11.5\,\mathrm{d}$ was observed during the explosion of SN 1006.

c) Find the duration Δt_{220} of this outburst as observed by an astronomer based in the galaxy Arp 220, whose distance is $d_{220} = 79\,\mathrm{Mpc}$. Take $H_0 = 68\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$ to be the current value of the Hubble's parameter.

[a) $(2.2 \pm 0.1)\,\mathrm{kpc}$; b) $(-7.5 \pm 0.4)\,\mathrm{mag}$; c) $11.7\,\mathrm{d}$]

Cosmology and relativity

Light chasing **EF/R/1**

Galileo Galilei (1564–1642) is remembered in astronomy, physics and mathematics. He was the first to observe the phases of Venus, sunspots and he also described mountains and craters on the Moon.

a) What instrument is named after Galilei and what elements does it contain?

However, some of his attempts at making breakthroughs were unsuccessful. One of the experiments designed by Galilei was measuring the speed of light. Two experimenters were supposed to climb neighbouring hills with lamps, where both lamps would be covered by a shield at the beginning of the experiment. Then one of the experimenters would uncover his lamp and allow the light to escape. At the moment the second experimenter would see the light, he would uncover his own lamp. The light would then travel back to the first experimenter. From the resulting time delay and the known distance between the two hills, one would then be able to calculate the speed of light.

b) It does not take a big effort to find the catch. Calculate the distance between the hills which is needed to overcome the problem with a non-zero reaction time of the experimenters (say 1 second).

Today we already know that the speed of light is $c \doteq 300\,000\,\mathrm{km \cdot s^{-1}}$.

c) What is the ratio of this distance to the length of Earth's equator and to the mean Earth–Moon distance?

[a) Galilei telescope, convex lens (objective), concave lens (eye-piece); b) 150 000 km; c) 3.74, 0.39]

Space comparisons EF/N/3

The following comparison was inspired by a similar one used by A. C. Clarke in his book *2001: A Space Odyssey*. (Here we will use modern estimates.) There are seven billion people living on the Earth and for each of us there are 15 more who do not live any more.

a) Calculate how many stars in our Galaxy belong to every person ever born on Earth. You may assume that there is an estimated number 100 to 400 billion stars in our Galaxy.
b) If atoms were as big as humans, what actual cosmic body would humans match by size? The size of an atom is about 10^{-10} m.

Recently, the detection of gravitational waves was announced: the passing wave caused an extension of a 4 km long arm of the LIGO interferometer by approximately 10^{-18} m.

c) Find an analogous change in the distance to α Cen (about 4 ly away) assuming that the same relative change is applied. Write your answer in millimetres.

[a) 1 to 4 stars per each person ever born; b) a star; c) 0.01 mm]

Cosmic neutrino background AB/N/4

Let us assume that our universe is flat, so that its average density is equal to the critical density $\rho_c = 3H^2/(8\pi G)$, where G is the gravitational constant and H is the Hubble's parameter (present value $H_0 \doteq 68\,\mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$). Further, assume that the fraction of dark matter in the universe is $X \doteq 26\,\%$ and that relic neutrinos are nowadays cold (i.e. their average kinetic energy is much less than their rest energy). We also have $n_{\nu_e} = n_{\nu_\mu} = \ldots = (3/11)n_\gamma$, where n_γ and $n_{\nu_e}, n_{\nu_\mu}, \ldots$ are the number densities of relic photons and relic neutrinos, respectively, and today we have $n_\gamma \doteq 410\,\mathrm{cm^{-3}}$.

Assuming that relic neutrinos are the sole component of dark matter in our universe, find the sum $m_0 = m_{\nu_e,0} + m_{\nu_\mu,0} + \ldots$ of the rest masses of all neutrino species. Write your answer in eV/c^2. Is it an upper or lower bound for the actual value of m_0?

[$11\,\mathrm{eV}/c^2$, upper bound (Gershtein–Zel'dovich)]

Orbits around a black hole AB/R/3

Schwarzschild solution can be modelled in simple terms by a gravitational force acting on a test particle with mass m at radius r from the central object with mass M. The magnitude of this force can be calculated using the same formula as in Newtonian physics, namely GMm/r^2. The curved geometry of spacetime will, however, have an effect on the magnitude of the centrifugal force which arises in the frame of an observer orbiting the central object with angular velocity Ω at radius r: instead of $m\Omega^2 r$, we have $m\Omega^2 r(1 - 3\mu/r)$, where $\mu = GM/c^2$. The time coordinate will also cease to be universal and will depend on the observer. However, for the purpose of this problem, you will be fine if you stick to the naive Newtonian point of view.

A massive test particle with negligible mass m moves in the vicinity of a non-rotating, uncharged black hole with mass $M \gg m$. The radius r_S of such a black hole can be written as $r_\mathrm{S} = 2\mu$. Let us assume that the particle orbits the black hole on a circular trajectory with radius $r > r_\mathrm{S}$.

a) Find the angular velocity $\Omega_\mathrm{N}(r)$ and period $P_\mathrm{N}(r)$ of particle's motion according to Newton's theory.
b) What is the actual angular velocity $\Omega_\mathrm{S}(r)$ and period $P_\mathrm{S}(r)$ of a particle orbiting a black hole?
c) Find the ratio $P_\mathrm{S}(r)/P_\mathrm{N}(r)$ and comment.

You might have noticed that in order to equate the magnitudes of the gravitational and centrifugal force, we need $\Omega^2 < 0$ for certain values of r. Such trajectories are unphysical and therefore have to be discarded.

d) Determine the range of values of r (write its boundaries as multiples of r_S) for which a circular orbit with radius r around a black hole *cannot* exist.

Just as in the Newton's theory, the magnitude $L = m\Omega r^2$ of angular momentum of a massive particle is conserved in Schwarzschild's spacetime. Such a particle, initially moving along a circular orbit with radius r around a black hole, is suddenly perturbed to a new radius $r' = r + x$ (where $x \ll r$) in such a way so as to preserve the magnitude of its angular momentum.

e) Find the magnitude $F_\mathrm{N}(x)$ of a force acting on the particle in its perturbed state in the co-rotating system according to Newton's theory. Does the particle have a tendency to come back to radius r? Comment on the stability of Newtonian circular orbits.
f) Find the magnitude $F_\mathrm{S}(x)$ of a force acting on the particle in its perturbed state in the co-rotating system according to our treatment of GR. Classify circular orbits around a black hole based on their stability.

Hint: For small ε (i.e. $|\varepsilon| \ll 1$) we have $(1+\varepsilon)^a \approx 1 + a\varepsilon$.

[a) $\sqrt{GM/r^3}$, $2\pi\sqrt{r^3/(GM)}$; b) $\sqrt{GM/r^3}\,(1-3\mu/r)^{-1/2}$, $2\pi\sqrt{r^3/GM}\,(1-3\mu/r)^{1/2}$; c) $(1-3\mu/r)^{1/2} \to 1$ as $r/\mu \to \infty$; d) $r_\mathrm{S} < r < (3/2)r_\mathrm{S}$; e) $-G\frac{Mmx}{r^3}$, all orbits stable; f) $-G\frac{Mmx}{r^3}\left(\frac{r-6\mu}{r-3\mu}\right)$, unstable for $3\mu < r < 6\mu$, stable for $r > 6\mu$]

Practical problems

Quadrant CD/R/4

In this practical problem, we will try out measuring the altitude of astronomical objects above the horizon. Following the instructions below, you will construct a simple paper quadrant to help you perform these measurements.

First, glue the angular scale (which can be downloaded here[1]) to a cardboard of rectangular shape such that line segments making the corner (centre) of the quadrant are parallel to the sides of the cardboard. Second, attach a piece of string with a plumb bob on its end to the corner of the quadrant. The string should be long enough so as to be capable of hanging freely over the edges of the cardboard. The plumb bob should as well be heavy enough so that the string marks the vertical direction clearly.

a) Describe a method of measuring the altitude of objects above the horizon with a quadrant of the construction described above.
b) Following the steps above, construct your own paper quadrant. Do not be shy to do some tweaks to increase accuracy of your measurements (e.g. by adding a pointer).

Now choose a bright star which culminates above the southern part of the horizon during your observation night. Your task will be to determine the time of culmination of that particular star by measuring its altitude above the horizon several times over an interval near to the culmination.

c) Select several times before and after the assumed time of culmination. For each of these times, perform several measurements of star's altitude and record their average.
d) Plot the altitude h of the star above the horizon against the time of measurement. Read off the maximal altitude and the time of culmination.

Do not forget to give a detailed description of the procedure followed during your measurements. Finally, do not forget to indicate the **date** of the

[1] `http://olympiada.astro.cz/zadani/AO_2015_16_CD_2_kolo_kvadrant.pdf`

measurement and the **geographical coordinates** of your observation site in your answer.

Sun and asteroid CD/N/7

a) In fig. 3 an all-sky picture is shown. The picture was taken at an unknown location on the northern hemisphere. It shows the positions of the Sun in hourly intervals during both solstices and also during an equinox day. Find the latitude ϕ of the observer. It may come in useful that the scale of the picture is least deformed near zenith.

b) In fig. 4, we can see a couple of radio telescope images showing the asteroid 136617 (1994 CC) and its two satellites (the actual shape of the asteroid is marked on the left hand image). The images were captured during a fly-by of this asteroid near Earth at the times displayed below. Based on earlier observations, we know that the satellite which is positioned in the bottom part of both images orbits the asteroid on a circular orbit with radius a which is 5 times larger than the radius R of the asteroid itself. We also assume that this satellite traversed a negligible part of the circumference of its orbit during the interval t between both captures. Assuming that the shape of the asteroid is perfectly spherical, estimate its mean density ρ. You should disregard proper motion of the asteroid during the interval t.

[a) 38°; b) $100\,\mathrm{kg\cdot m^{-3}}$]

Stefan–Boltzmann law CD/N/8

The goal of this problem is to use given laboratory data sets to exhibit validity of Stefan–Boltzmann law and determine an approximate value of Stefan–Boltzmann constant.

We are given the outcome of 10 measurements (tab. 1) of certain characteristics of a sphere (which we say to be absolutely black) with radius $R = 0.5\,\mathrm{m}$. During each measurement, the total luminosity and the wavelength of maximum irradiance of the sphere were L_i and $\lambda_{\mathrm{max},i}$, respectively. It can be assumed that the Wien's law constant b is equal to $b = 2.898\,\mathrm{mm\cdot K}$.

The ansatz we make is that the intensity I of the black body radiation (defined as a power emitted per unit area of the body's surface) and its temperature T are related by a power law

$$I = \sigma\, T^k\,, \tag{1}$$

where σ, k are positive real numbers (σ dimensionful).

Basic SI units are to be used for all numeric manipulations in this problem.

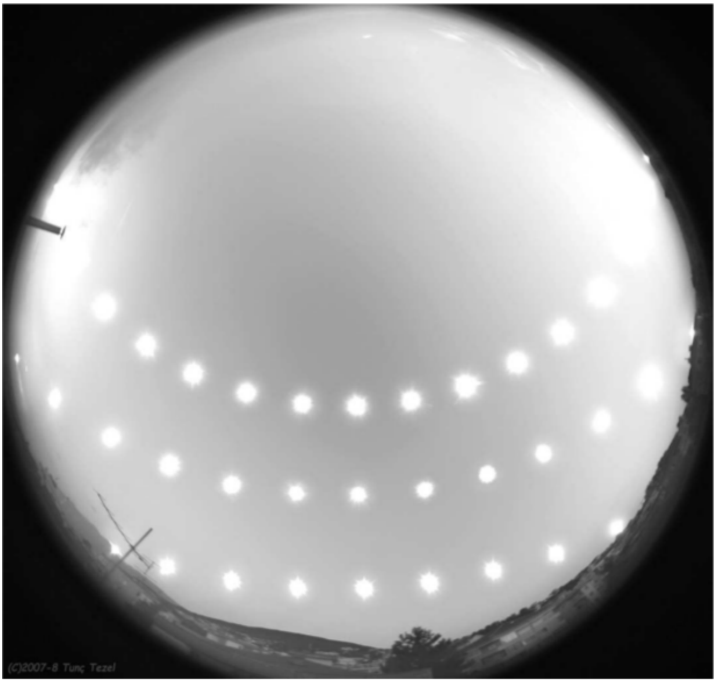

Figure 3: Figure relating to CD/N/7, part a).

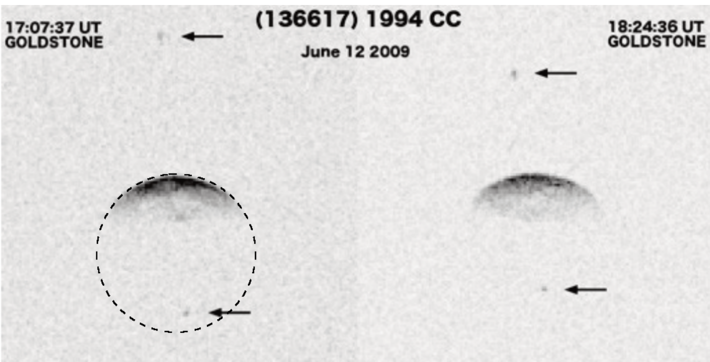

Figure 4: Figure relating to CD/N/7, part b).

a) By taking a log of both sides of the equation, rewrite (1) in the form $y = ax + b$, where x and y are to be expressed in terms of T and I, respectively. Also, a and b are parameters which should be expressed in terms k and σ, respectively. You may use the following identities which are valid for all positive real numbers p and q

$$\log pq = \log p + \log q\,, \qquad \log p^q = q \log p\,.$$

b) Using the relations derived in a), express x, y in terms of the measured quantities $L, \lambda_{\max}$. Find the numerical values of (x_i, y_i) for $i = 1, \ldots, 10$.

c) Plot the data points (x_i, y_i) for $i = 1, \ldots, 10$. Draw the line of best fit and read off approximate values $\hat{a}, \hat{b}$ of the parameters a, b.

NB: Think carefully how to scale the axes of your plot. Then estimate the slope of the line of best fit and find its intercept.

d) Using the relations for a, b derived in part a), find the approximate values $\hat{\sigma}, \hat{k}$ (these will involve the values of $\hat{a}, \hat{b}$ estimated in c)) of the parameters σ and k, thus rewriting (1) by substituting $\hat{\sigma}, \hat{k}$ for σ, k.

Table 1: Laboratory data from the measurement of an absolutely black sphere with radius $R = 0.5\,\text{m}$.

i	$\frac{L_i}{\text{W}}$	$\frac{\lambda_{\max,i}}{\mu\text{m}}$	i	$\frac{L_i}{\text{W}}$	$\frac{\lambda_{\max,i}}{\mu\text{m}}$
1	991.53	10.610	6	1942.5	8.9680
2	1145.6	10.235	7	2194.6	8.6988
3	1316.0	9.8857	8	2470.3	8.4453
4	1504.7	9.5596	9	2771.4	8.2061
5	1713.3	9.2544	10	3099.0	7.9802

[a) $\log\left(\frac{I}{\text{W}\cdot\text{m}^{-2}}\right) = \log\left(\frac{\sigma}{\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-k}}\right) + k\log\left(\frac{T}{\text{K}}\right)$; b) $x = \log\left(\frac{b/\lambda_{\max}}{\text{K}}\right)$, $y = \log\left(\frac{L/(4\pi R^2)}{\text{W}\cdot\text{m}^{-2}}\right)$; d) $5.68\times\ 10^{-8}\,\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$, 4]

Cepheids AB/N/7

The cepheids are a class of very luminous variables whose mean absolute magnitude is a function of their pulsation period. This allows for their luminosities to be determined simply by measuring these periods. Table 2 contains assorted data for several cepheids. P_0 denotes the pulsation period and $\langle M_\text{V}\rangle$ is the mean absolute magnitude.

a) Plot the data from tab. 2 putting $\log(P_0/\text{d})$ on the horizontal axis and $\langle M_\text{V}\rangle$ on the vertical axis.

Table 2: Data for several cepheids.

cepheid	$\frac{P_0}{\mathrm{d}}$	$\frac{\langle M_\mathrm{V}\rangle}{\mathrm{mag}}$	cepheid	$\frac{P_0}{\mathrm{d}}$	$\frac{\langle M_\mathrm{V}\rangle}{\mathrm{mag}}$
SU Cas	1.95	−1.99	DL Cas	8.00	−3.80
V1726 Cyg	4.24	−3.04	S Nor	9.75	−3.95
SZ Tau	4.48	−3.09	ζ Gem	10.14	−4.10
CV Mon	5.38	−3.37	X Cyg	16.41	−4.69
QZ Nor	5.46	−3.32	WZ Sgr	21.83	−5.06
α UMi	5.75	−3.42	SW Vel	23.44	−5.09
V367 Sct	6.30	−3.58	SV Vul	44.98	−6.04
U Sgr	6.75	−3.64			

b) Using the method of least squares (see below) find the mean values $\hat{\alpha}, \hat{\beta}$ of the parameters α, β determining the line $\langle M_\mathrm{V}\rangle = \alpha + \beta \log(P_0/\mathrm{d})$, which is to be fitted to the data plotted in a). Draw this line in the same plot obtained in a).

This functional dependence allows us to find the luminosity of any cepheid given its pulsation period.

c) Figures 5 and 6 show the light-curves of two cepheids. Use the data provided in tables and plots to find the distances $d_{1,2}$ in Mpc of the two cepheids. You should also estimate the corresponding uncertainties. You can assume that these are dominated by the contribution from reading off the mean magnitudes $\langle m\rangle$ of the cepheids from the plots.

d) Find the minimum possible mutual distance of these two cefeids which is consistent with the data provided. Is it possible that the two stars come from the same galaxy?

Method of least squares: When fitting a line $y_i = \alpha + \beta x_i$ through two-dimensional data (x_i, y_i) for $i = 1, \ldots, n$, the mean values $\hat{\beta}$, $\hat{\alpha}$ of the parameters β, α can be found as

$$\hat{\beta} = \frac{n\sum_i x_i y_i - (\sum_i x_i)(\sum_i y_i)}{n\sum_i x_i^2 - (\sum_i x_i)^2} \qquad \text{and} \qquad \hat{\alpha} = \frac{1}{n}\left(\sum_i y_i - \hat{\beta}\sum_i x_i\right).$$

[b) $\langle M_\mathrm{V}\rangle = -1.21 - 2.88\log\frac{P_0}{\mathrm{d}}$; c) $(19 \pm 1)\,\mathrm{Mpc}$, $(18 \pm 1)\,\mathrm{Mpc}$; d) yes, it is possible]

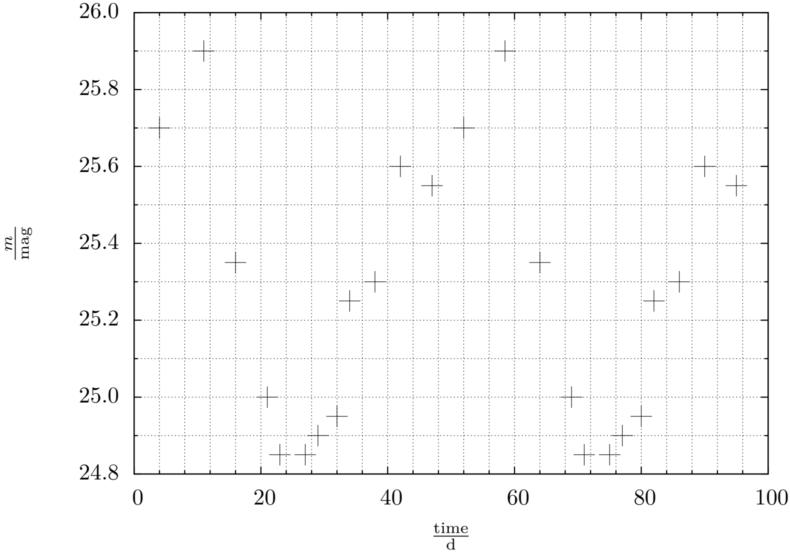

Figure 5: Light-curve of cepheid 1.

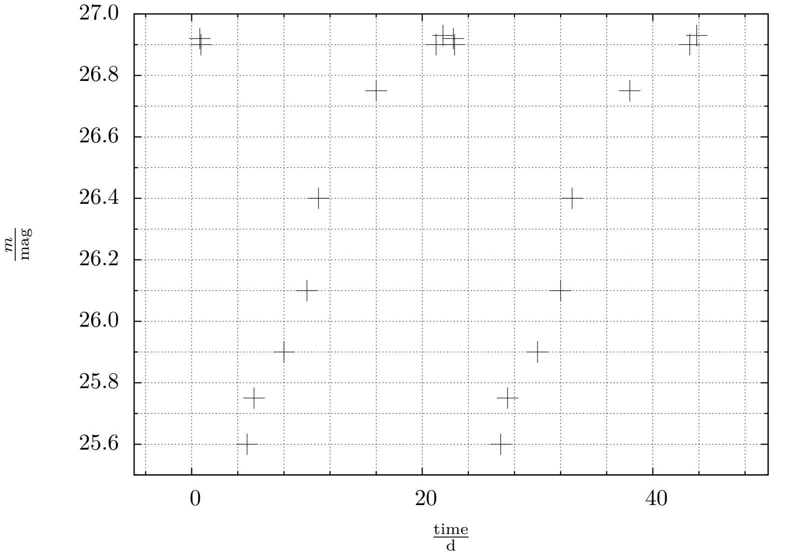

Figure 6: Light-curve of cepheid 2.

Atmospheric extinction AB/N/8

Table 3 shows the measurements of brightnesses of selected stars and planets over the course of one night with stable weather conditions. The brightness J (shown in arbitrary units) of individual objects was measured using filter V for several values of the zenith distance z. Further, the magnitudes m_0 of all the stars we are considering were measured using filter V outside the Earth's atmosphere: 0.03 mag for Vega, 1.25 mag for Deneb, 0.08 mag for Capella and 0.85 mag for Aldebaran.

In the following, you can assume that each of the objects considered satisfies

$$m_0 + \frac{E}{\cos z} = -2.5 \log \frac{J}{J_0} ,$$

where E is the zenith atmospheric extinction (constant for a given night) and J_0 is the brightness of a zero-magnitude object measured outside the Earth's atmosphere. Use the arbitrary units of table 3 for both J and J_0

a) Plot the data from table 3 for each object for which the value of m_0 is known: $1/\cos z$ on the horizontal axis and $2.5 \log J + k$ on the vertical axis where, for now, we put $k = m_0$. Leave enough space in the upper part of your plot, having part c) in mind.
b) Fit an appropriate curve to the plot in a). Read off the value of J_0 in arbitrary units as well as the value of E in mag for the given night.
c) Plot the data for both Jupiter and Venus in the same graph but this time, use $k = -1.8$ mag for Jupiter and $k = -3.3$ mag for Venus.
d) Fit an appropriate curve to the plots obtained in c) and read off the values of m_0 in mag for Jupiter and Venus for the given night.

Table 3: Brightness measurements of selected objects throughout one night.

i	Object	z (°)	J	i	Object	z (°)	J
1	Vega	45.75	5 373	3	Deneb	36.43	1 601
1	Deneb	23.86	1 777	3	Capella	44.67	5 598
1	Capella	55.94	4 885	3	Jupiter	58.36	37 128
1	Jupiter	70.92	34 167	3	Aldebaran	60.65	2 032
2	Vega	49.89	4 892	4	Deneb	44.57	1 482
2	Deneb	28.04	1 900	4	Capella	36.60	5 369
2	Capella	52.33	4 653	4	Jupiter	50.25	40 194
2	Jupiter	66.76	34 271	4	Aldebaran	52.82	2 044
2	Aldebaran	68.98	1 802	4	Venus	70.95	189 588

[b) 7300, 0.25 mag; d) −2.3 mag, −4.3 mag]

Česká Astronomická Společnost

Ministerstvo školství, mládeže a tělovýchovy

Slezská univerzita v Opavě
Filozoficko-přírodovědecká fakulta v Opavě

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History of Astronomy in the Czech lands

1410 Prague Astronomical Clock by Johannes Schindel (see the picture on the front page)

1600–1601 Tycho Brahe lived in Bohemia

1600–1612 Johannes Kepler published numerous books and papers while staying in Prague

1648 Johannes Marcus Marci, his books on optics and mechanics appeared in press in Prague

1842 Christian Doppler was a professor of mathematics and practical geometry at the Prague Polytechnic University when he discovered his famous principle

In the second half on the 19th century, Ernst Mach, professor of physics at the Prague University, contributed enormously to the education of a whole generation of both German and Czech physicists and astronomers.

1886 Astronomical Institute of the Czech University was established. August Seydler, its founder and first director, elaborated sophisticated methods for determination of orbits of minor planets.

1911–1912 Albert Einstein spent one and a half year in Prague appointed as a professor at the German Prague University.