8th Bangladesh Olympiad on Astronomy and Astrophysics

National Round 2025 – Senior

18 April 2025

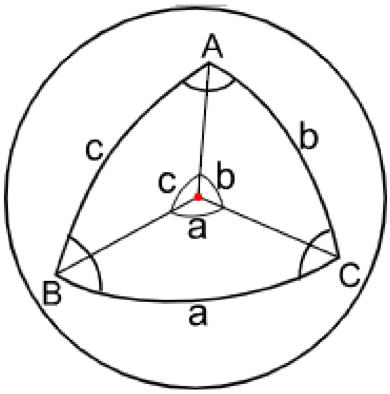
Instructions for the Candidate - পরীক্ষার্থীদের জন্য নির্দেশনা:

- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
  প্রতিটি প্রশ্নের জন্যই উত্তরের চেয়ে সমাধানের প্রক্রিয়া বেশি গুরুত্বপূর্ণ। যুক্তিপূর্ণ অনুমান/অ্যাপ্রক্সিমেশন পুরোপুরিভাবে গ্রহণযোগ্য। সমাধানের বিশদ ও স্পষ্ট ব্যাখ্যা আমাদের প্রত্যাশিত।
- Be sure to calculate the final answer in the appropriate units asked in the question.
  চূড়ান্ত উত্তর প্রশ্ন অনুযায়ী সঠিক এককে গ্রহণযোগ্য।
- Non-programmable scientific calculators are allowed.
  নন প্রোগ্রামেবল সায়েন্টিফিক ক্যালকুলেটর গ্রহণযোগ্য।
- The mark distribution is shown in the [ ] at the right corner for every question.
  প্রতিটি প্রশ্নের শেষে [ ] বন্ধনীতে নম্বর বন্টন দেয়া আছে।
- The exam duration is **2 hour**.
  পরীক্ষার সময় ২ ঘন্টা

নাম (বাংলায়) :	
নাম (In English) :	
শ্রেণি (২০২৫ সাল) :	মোবাইল নং:
প্রতিষ্ঠান :	
জন্মতারিখ (dd/mm/yy) :	

Constants and Formulas

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Radius of the Moon	$R_☾$	$\approx$	1.7374×10^{6} m
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Speed of light	c	$\approx$	2.99×10^{8} m
Astronomical Unit(AU)	$a_\oplus$	$\approx$	1.496×10^{11} m
Distance of Earth to Moon	$a_☾$	$\approx$	3.78×10^{8} m
Solar Luminosity	$L_\odot$	$\approx$	3.826×10^{26} W
Moon's apparent magnitude	$m_☾$	$=$	-12.7^m
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$
1 parsec	$1\,pc$	$=$	3.986×10^{16} m
Stefan's constant	σ	$=$	$5.670 \times 10^{-8}\,\mathrm{Wm^2K^{-4}}$
Boltzmann constant	k_B	$\approx$	1.38×10^{-23} J/K
Pogson's law	$m_1 - m_2$	$=$	$-2.5 \log \frac{F_1}{F_2}$



Idea 1: Cosine rule for spherical triangle:

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$

Idea 2: Any real number x can be expressed as a continued fraction:

$$x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

where a_0 is the integer part, and the remaining terms are determined iteratively.

1 Metonic Photography

Faria was preparing for the BDOAA 2025 National with her friend Rokon. They traveled to the highest peak in Bangladesh, Bijoy (1280 m above sea level), with her new telescope—a gift from her father after she won a Silver Medal in IOAAjr 2024.
During their observation, they witnessed a unique celestial event: the Waning Gibbous Moon was very close to Antares during its upper culmination in the Scorpius constellation, occurring 169 minutes before sunrise. Captivated by the sight, they captured a photograph with the Moon in the background. The position of Antares is given as:

$$\alpha_{(2000)} = 16^{\mathrm{h}}19^{\mathrm{m}}26^{\mathrm{s}}, \quad \delta_{(2000)} = -26^\circ 25' 55''.$$

Faria and Rokon promised to recreate this photograph in the future. Rokon pointed out that they could do so if the tropical year ($T_Y = 365.2422$ days) and the synodic month of the Moon ($T_S = 29.53059$ days) aligned again. Excitedly, Faria exclaimed, "Oh, are you talking about the Metonic Cycle?"

Any real number x can be expressed as a continued fraction:

$$x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

where a_0 is the integer part, and the remaining terms are determined iteratively.

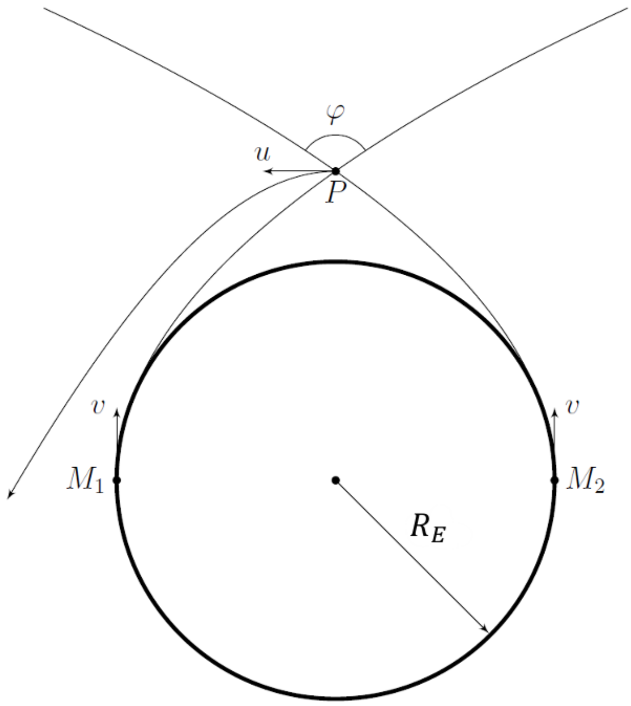
a Calculate the duration of the Metonic cycle in years. Determine how many times this event will occur in the 21st century.

b Determine the exact date and month when this event will take place again this year.

2 Double Satellite [20]

In 1973, a method for launching satellites into a hyperbolic orbit using a gravity assist maneuver was proposed, which you are invited to analyze. Within the framework of this task, the motion of two satellites 1 and 2, launched from opposite poles of the Earth with identical velocities in magnitude and direction, is studied. Satellite 2 is launched into orbit a short time after the launch of the first. If only one of the satellites is launched into orbit, its trajectory will be a parabola.
The mass of satellite 1 is M_1, and satellite 2 is $M_2 \gg M_1$. The resistance of gas in the atmosphere can be neglected, as can the influence of gravitational attraction from other bodies. The mass of the Earth $M_E \gg M_2$, the radius of the Earth is R_E, and the gravitational constant is G. Let us denote by P the point of intersection of the elliptical orbits of the satellites, and by φ the angle between the directions of their velocities at a given point. The motion of satellite 1 consists of three sections:

- Motion along a parabolic orbit;
- Interaction with satellite 2 near the point P;
- Further movement along a hyperbolic orbit in the Earth's gravitational field.



Part A: Trajectory of one of the satellites

In this part of the problem, you will have to approximately describe the trajectory of the satellites to the intersection point P of their orbits.

(a) What is the speed of the satellites v at the moment of launch? Express the answer in terms of G, M_E, and R_E.[2]

(b) How far from the center of the Earth R_0 is the point P intersection of satellite trajectories? Express the answer in terms of R_E.[2]

(c) What is the angle φ between the directions of the velocities of the satellites at point P? [2]

(d) Find the time t_0 through which satellites reach the point P. Express the answer in terms of G, R_E, and M_E.[3]

From here on, consider that $\tau \ll t_0$. As the satellites approach the point P, their interaction with the Earth can be neglected.

Part B: Satellite Interactions

(e) Considering that near the point P the satellites moved along straight lines whose directions coincided with the directions of their velocities at a given point, find the relative velocity of the satellites v_0, as well as the minimum distance b between them, neglecting their gravitational interaction. Express the answer through v and τ.[2]

Next, consider that in the reference frame of satellite 2, the orbit of satellite 1 is a hyperbola with an impact parameter b at their infinitely large distance. Also consider that at an infinitely large distance the relative speed of the satellites is equal to v_0, found in the previous paragraph.

When the interaction of satellites 1 and 2 can again be neglected, the velocity of satellite 1 in the Earth's frame of reference is equal to u and is directed perpendicular to the line connecting it with the Earth.

(f) Find u. Express the answer in terms of v. [4]

(g) Find τ. Express the answer in terms of G,M_2,M_E and R_E. [5]

3 The Universe at Reionization [16]

You observe a galaxy's **Lyman-alpha emission line** (rest wavelength $\lambda_{\text{rest}} = 121.6\,\text{nm}$) at $\lambda_{\text{obs}} = 850\,\text{nm}$. This galaxy existed during the **epoch of reionization**, when the first stars ionized the universe. Assume a **matter-dominated universe**.
You are given the Hubble–Lemaître Law:

$$v = Hr$$

where v is the velocity of recession, H is the Hubble parameter and r is the distance to the galaxy. Hubble constant today: $H_0 = 70\,\text{km/s/Mpc}$

a. Calculate the **redshift,** z of the galaxy. We shall call this the Epoch of Reionization (EoR). [2]

b. The expansion of the universe stretched the wavelength out and caused the redshift. To explain the expansion, we define the **scale factor,** a as the ratio of the size of the Universe at any time to its present size. What was the scale factor at the EoR? [2]

c. The universe cools proportionally as it expands. If the current temperature of the universe is $T_0 = 2.73\text{K}$, what was the **temperature**, T at the EoR? [2]

d. Let us define the critical density, ρ_c as the matter density required to halt the expansion of the universe after infinite time. Find an expression of the critical density, in terms H and G. Calculate the present critical density ρ_{c0}. [6]

e. Matter density reduces with expansion. Calculate the critical density at the EoR. [2]

f. Hence or otherwise, calculate the Hubble parameter at the EoR. [2]

4 AGN 2778

Congratulations! Your childhood dream has come true. You're now an astronomer studying Active Galactic Nuclei (AGN), the most powerful engines in the universe. These cosmic monsters are distant galaxies with supermassive black holes at their cores. These core destroy matters and blast out enough energy to outshine galaxies. What makes them truly special? They're among the most distant sources we can detect (second only to the Big Bang's afterglow, the Cosmic Microwave Background). By studying AGNs, you're essentially using them as time machines as their light has traveled billions of years to reach us, carrying secrets about how the first galaxies and black holes formed in the early universe. That's what makes AGNs so extraordinary!

Your New Discovery: AGN 2778

You've identified a fascinating new AGN called AGN 2778 that's emitting tremendous amounts of light, particularly in ultraviolet (UV) and optical wavelengths. You have data from two powerful telescopes, one is space based another ground based:

- **Hubble Space Telescope (HST) COS Spectrograph** – (1150–3000 Å)
- **Hobby-Eberly Telescope (HET) Spectrograph, USA** – (3650–10000 Å)

AGN 2778 has a measured redshift of $z = 0.5$.

Analyzing AGN 2778's Light

AGN 2778 emits two important emission lines:

- **Lyman-alpha (Lyα)**: 1216 Å (far UV)
- **Hydrogen-alpha (Hα)**: 6563 Å (red light)

1. Calculate the observed wavelengths of both Lyα and Hα after accounting the redshift. [1]
2. Determine whether each line could be observed with HST and HET. [0.5]

Dealing with Earth's Glow

Since HST orbits Earth, it also detects geocoronal emission, which is a strong UV glow from Earth's atmosphere at exactly 1216 Å, with a width (FWHM) of about 20 Å.

1. Will this geocoronal emission interfere with our observations of AGN 2778's Lyα line? Why or why not? [0.5]

Whirlpool of Gas

The AGN's Broad-Line Region (BLR) is a swirling cloud of hot gas orbiting the black hole at about 5,000 km/s. This rapid motion broadens the Hα line in the spectrum.

1. Calculate how much the Hα line is broadened (in Å) due to this motion. [1]
2. Can HET spectrograph's 2 Å resolution distinguish this broadened line? [0.5]

Estimating the Distance

Let's start by estimating how far away this AGN is using Hubble's Law. The Hubble's constant, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

1. Calculate the approximate distance to AGN 2778 using the simple form of Hubble's Law. [2]

Important Note: This should be a simplified calculation that assumes the universe's expansion rate has been constant over time. In reality, the expansion has accelerated due to dark energy, so this method overestimates distances at higher redshifts like $z = 0.5$. But for the entire problem we will use this distance.

Measuring the Monster's Size

HST's incredible resolution allows us to see the BLR as a tiny speck: just 0.02 arcsecond across.

1. Calculate the actual physical size of the BLR in parsecs. Could HET, with its 0.5 arcsecond resolution, detect this structure? [1.5]

The Hostile Environment Near the Black Hole

The AGN's intense radiation heats surrounding gas to extreme temperatures. The central ionizing luminosity of the AGN's center is a staggering 10^{44} erg/s.

1. Estimate the lowest temperature at the BLR. Could molecular clouds (which need temperatures of 10–100 K to form stars) exist here? [3]
2. There can be planets revolving around the AGN's central accretion disk. These planets doesn't need to be revolving around stars rather they can form from the debris coming from the acretion stream of the AGN's core or any other sources. Determine the habitable zone of the AGN's central accretion disk in parsecs. Habitable zones are the regions where the water can exist in liquid. [3]
3. Suppose there is a planet revolving around a sun like star which is revolving around the central supermassive black hole of the AGN at a distance of 1 pc. Find out the habitable zone of that system. [5]

5 Motion of the Stars

Sifat loves to explore the night sky and look at different constellations. Although the shape of the constellations does not seem to be change, he knows that the stars have their own motion, and as a result, the constellation won't be the same after long time. He looks at Procyon and Sirius and ponders if they ever going to meet at a single point in night sky, and how long it would take to happen. To simplify his calculation, he assumes Procyon has a constant angular velocity of 200 mas/year towards Sirius, and Sirius has a constant angular velocity of 350 mas/year in the direction of Procyon.

Star	Right Ascension (a)	Declination(δ)	Parallax (p)
Procyon	$7^h39^m18.1^s$	$+5°13'29''$	284.56 mas
Sirius	$6^h45^m8.9^s$	$-16°42'58''$	379.21 mas

1. Use Spherical Triangle Formula to find the angular separation of the two stars, Δs. [5 points]
2. Calculate the distance (in parsecs) between the stars. (Use $\Delta s = 10°$ if you have not done question 1). [4 points]
3. According to Sifat's assumption, how long it would take for the two stars to meet at a single point in the night sky? [3 points]
4. What is the Declination and Right Ascention of the point they meet? [4+2 points]
5. Find the proper motion of Procyon e.g find it's rate of change in dec and RA. [4+2 points]

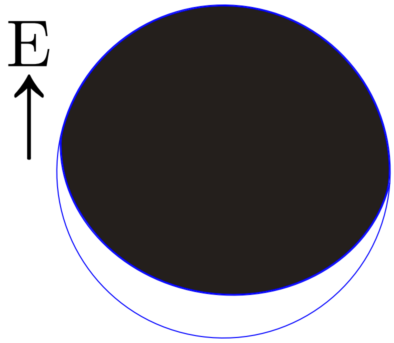
6 Planetary Parade

On February 28, 2025, a rare planetary alignment took place, featuring seven planets—Mercury, Venus, Mars, Jupiter, Saturn, Uranus, and Neptune—all visible in the evening sky. Talha was eager to observe this event, but due to light pollution and obstructing buildings in Dhaka, he struggled to find a suitable location.

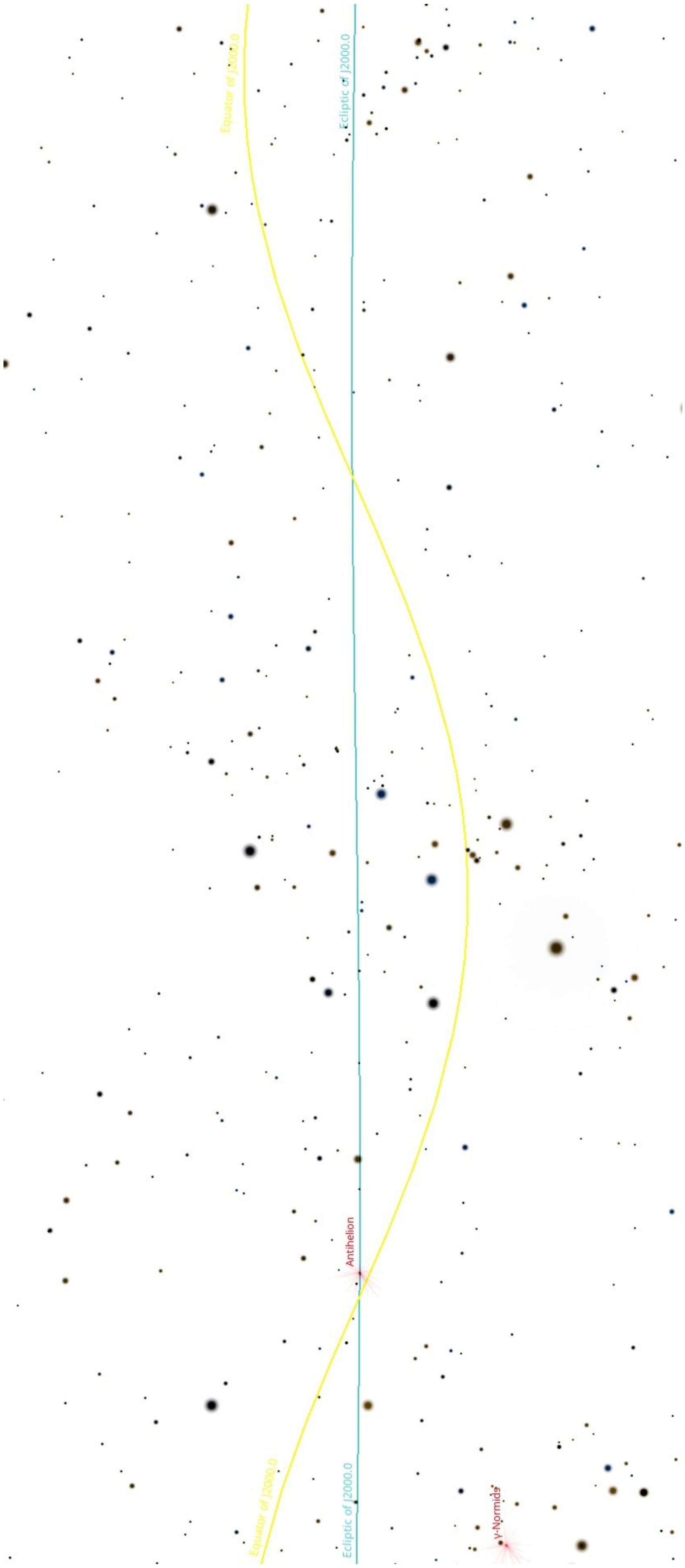
To get a better view, he called his Phaembae[1], who was observing the alignment from Rajshahi under clear skies. Based on their conversation, Talha noted down the following details:

[1]A pseudonym for Fahim vai

- **Mercury** was located 4° west of the First Point of Aries ().
- **Venus** displayed a phase as shown in the figure.
- The **Moon** was 0.5 days old, with just 0.2% illumination.
- **Mars** was seen in the Gemini constellation.
- **Jupiter** appeared above the Hyades cluster.
- The ecliptic longitude of **Saturn** was +350°.
- **Uranus** had a right ascension of $\alpha = 3^h 25^m$.
- **Neptune** was marked on the map.



Your task is to plot the positions of the planets and the Moon on the map, assuming that all of these celestial objects lie along the ecliptic at the given time.


Equator of J2000.0
Ecliptic of J2000.0
Antihelion
Equator of J2000.0
Ecliptic of J2000.0
γ-Normids