# $4^{\text{th}}$ Bangladesh Olympiad on Astronomy and Astrophysics

National Round 2021

May 20, 2021

**Instructions for the Candidate:**

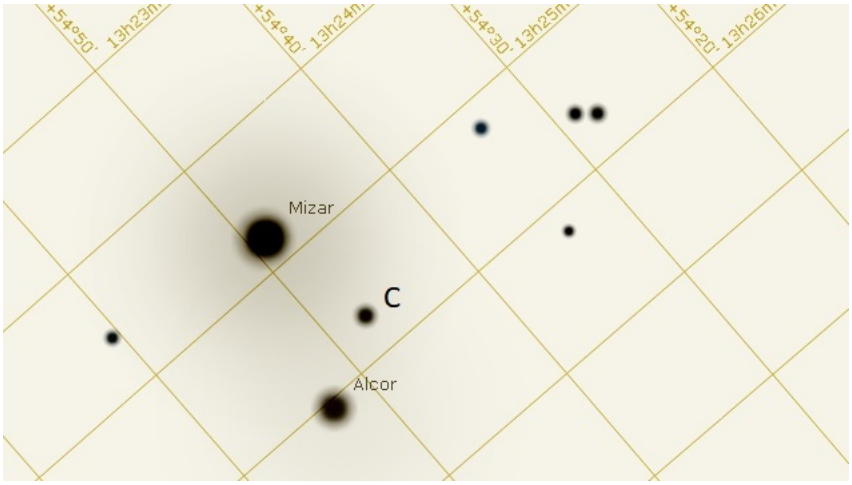
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Be sure to calculate the final answer in the appropriate units asked in the question.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.

Table 1: **Useful Constants and Formulas**

| | | | |
|---|---|---|---|
| Mass of the Sun | $M_\odot$ | $\approx$ | $1.989 \times 10^{30}\,\text{kg}$ |
| Mass of the Earth | $M_\oplus$ | $\approx$ | $5.972 \times 10^{24}\,\text{kg}$ |
| Mass of the Moon | $M_☾$ | $\approx$ | $7.347 \times 10^{22}\,\text{kg}$ |
| Radius of the Earth | $R_\oplus$ | $\approx$ | $6.371 \times 10^{6}\,\text{m}$ |
| Radius of the Sun | $R_\odot$ | $\approx$ | $6.955 \times 10^{8}\,\text{m}$ |
| Speed of light | $c$ | $\approx$ | $2.99 \times 10^{8}\,\text{m}$ |
| Astronomical Unit(AU) | $a_\oplus$ | $\approx$ | $1.496 \times 10^{11}\,\text{m}$ |
| Solar Luminosity | $L_\odot$ | $\approx$ | $3.826 \times 10^{26}\,\text{W}$ |
| Gravitational Constant | $G$ | $\approx$ | $6.674 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$ |
| 1 parsec | $1\,pc$ | $=$ | $3.986 \times 10^{16}\,\text{m}$ |
| Stefan's constant | $\sigma$ | $=$ | $5.670 \times 10^{-8}\,\text{Wm}^2\text{K}^{-4}$ |

# 1 Arman's digital Observation

Arman was observing the Ursa Major constellation with Stellarium. He noticed Mizar and Alcor are two stars froms an apparent binary system. The coordinates shown in this diagram is an equatorial coordinate system




a. Estimate the coordinates [Dec, RA] of Mizar, Alcor, and star C. [3]

b. Mizar peaks at a wavelength $b_M$ and the surface magnitude* of Mizar is $\mu_M$. Again, Alcor peaks at a wavelength $b_A$ and the surface magnitude of Alcor is $\mu_A$. Prove that,

$$\mu_M - \mu_A \propto \log \frac{b_M}{b_A}$$

[2]

c. Mizar itself is quadruple star system. The whole system lies about 83 light-years away from the Sun, as measured by the Hipparcos astrometry satellite. Arman has found a new constant which works similarly to Hubble's constant now dubbed as Arman's parameter = $70\,\mathrm{kms}^{-1}\mathrm{pc}^{-1}$,by finding the radial velocity conclude if the whole system is red shifted or blue shifted. [2]

*__Surface Brightness:__ The total luminosity emitted by a column of linear area $dA = \Omega r^2$. Generally presented as,

$$\Sigma \approx \frac{F}{\Omega} \approx \frac{L}{4\pi d^2\, \pi (R/d)^2} \approx \frac{L}{4\pi^2 R^2}; \qquad \text{Unit} : \mathrm{Wm}^{-2}\mathrm{ster}^{-1}$$

Surface Magnitude: Logarithmic form of Surface brightness, generally presented as,

$$\mu = -2.5 \log \Sigma + C$$

Just like any other magnitude term you can't easily multiply, add or subtract $\mu$. The unit of $\mu$ is mag/arcsec$^2$. To understand this suppose a star has a surface magnitude of $13\,\mathrm{mag/arcsec}^2$, it is actually comparable to a star with angular area of $1\,\mathrm{arcsec}^2$ to seem to have an apparent magnitude of $13^m$.

## 2 Analysis of a Constellation image

Jaber is trying to learn Astrophotography and he is aiming to shoot the constellation Canes venatici. But first he has to locate the constellation. He found an image that can help him which shows an arrow parpendicular to the horizon line!


a. Knowing the angular distance between Merak and Dubhe is 5.4° [The two star in bottom of the yellow line] estimate the latitude of the place with process. [2]

b. Find the time it will take for Cor Caroli (bright star of Canes venatici) to upper culminate. Also what is the local sidereal time during the upper culmination.
(RA = $12^h56^m01.66622^s$) [3]

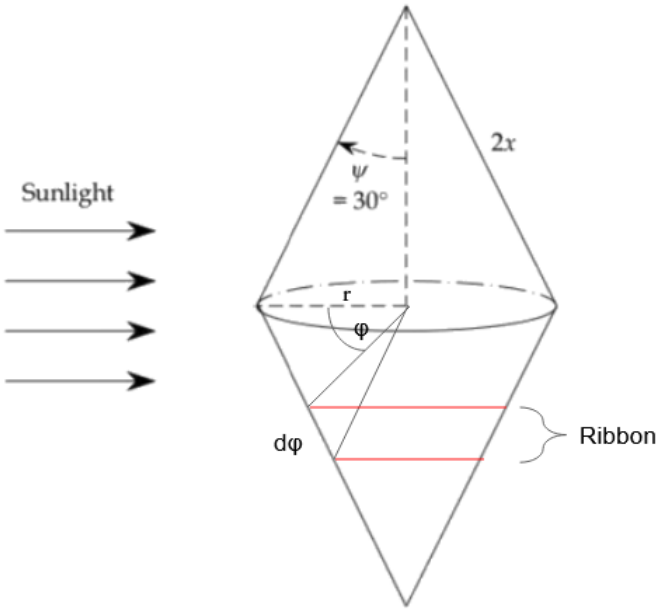
c. Suppose the star has culminated at time $t$, and it has been $d$ days since 21st March. So we can find the right ascension of the sun after $d$ days with following equation,

$$\alpha_\odot = \frac{d}{365.25} \times 24^h$$

Find the time of the day when this picture has to be taken if it is exam day (May 20, 2021). [2]

# 3 Conicalbeeminator

Dr. Doofenshmirtz is planning to launch a weird looking satellite in space which he named "Conicalbeeminator"– A solid body in the shape of two identical cones joined at the base, with dimensions as shown below ($r \sim R_\oplus$), is in a circular orbit with radius $d = 1$ AU about a star with luminosity $L = 1\,L_\odot$.




The axes of the cones are perpendicular to the orbit, and the body is spinning rapidly about this axis. The cone surfaces have uniform Bond albedo $A_b = 0.71$ and infrared emissivity $\varepsilon = 1$.

a. What is the equilibrium temperature $T_{\rm eq}$ at the satellites equator as a function of distance from the star $d$, the luminosity of the star $L$, albedo $A_b$ and emissivity $\varepsilon$? (Do not put in any numbers yet.) [2]

Perry the Platypus is trying to destroy the Conicalbeeminator. He is looking for a weak spot where it can be attacked for which he needs to know lowest heated point on the surface of the satellite.

b. Write an expression for the (infinitesimal) area of a ribbon of the satellite surface at latitude $\phi$ and with width $d\phi$. Write an expression for the area of the shadow of this ribbon that it casts against incident sunlight. (From star's point of view) [3]

c. Derive an expression for the surface temperature of the Satellite as a function of latitude by assuming the rapid rotation to result in a constant temperature for each ribbon, determined by the balance between absorbed sunlight and blackbody radiation. [3]

d. Now put in the numbers: plot the planet's surface temperature against the latitude over the full range of latitudes. [2]

# 4 Jack in a Well

Jack is absent minded all the time but he loves Jill and Astronomy. One day, as he was walking around his home, he remembered that the Sun was just passing through the vernal equinox point. Unfortunately for him, this happened just at the moment when the Sun at the zenith so that he forgot to watch his step and fell into a deep and empty well. As Jill was angry with him she was not around to help him. So until help arrives he decided to wander towards the sky.

Assume that the water well had a circular cross-section and its depth and radius were $H = 15$ m and $r = 1$ m, respectfully. Jack's eyes are $h = 1.75$ m above ground and he always undertakes his observations near the center of the well. Atmospheric refraction can be neglected.

a. What fraction of the sky can Jack see at any point of time. [2]

b. What fraction of the sky can be observed by Jack over the course of year? [3]

c. Find the Number of days during which the Sun is visible from the well (even partially) during an year. The angular diameter of the sun is $D = 32'$. Assume that the Earth orbits the sun along a circle. [2]

   In the mean time somewhere Jill is waiting for Jack and she observes the sunset and sunrise everyday waiting for him. The position of the Sun on the horizon at sunrise and sunset varies throughout the year. At the equator, it wanders $\pm 23.4^\circ$ North and South of due East and West. On the Arctic circle it varies by $\pm 90^\circ$ so that it is due South or North at the solstices. Somewhere between those points there is a latitude at which the Sun varies by $\pm 45^\circ$. At this latitude the positions of the sun at sunrise and sunset at the winter and summer solstices are exactly SE, SW, NE and NW, with the various directions lying at right-angles with respect to each other.

d. Workout the latitude of this place where Jill is waiting. [3]

# 5 An Introduction to Data Analysis

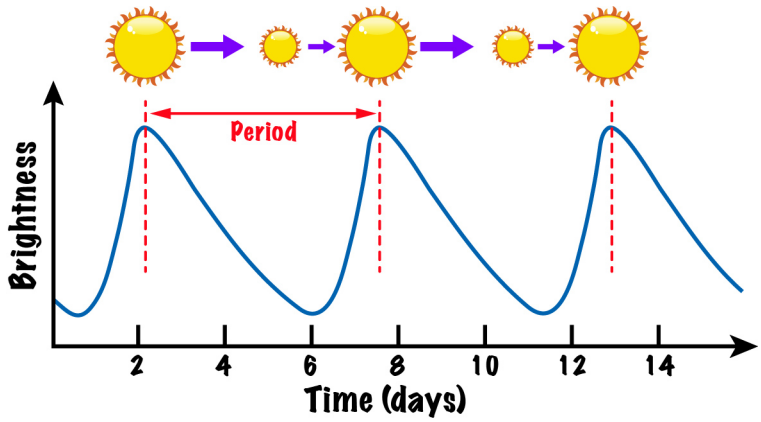
Astronomy has always been a data-driven discipline. From the earliest civilisations to the very bleeding edge of 21st-century humanity, data has been the core of the matter. Humans have obtained and processed data; we have analysed and made conclusions from said data to learn more about the world and the universe we live in.
In other words, astronomy is driven by new observations from data obtained by observing the universe. Scientists employ a very wide variety of techniques and tools to make sense of the data they have obtained. For instance, the CHANDRA X-Ray Observatory observes in X-ray wavelengths, but the data is clearly not recorded as X-ray photons! Instead, a computer on board the satellite encodes the data from the sensor as bits on some storage, which is then sent to ground-based stations.
This data can then be output into an image, or represented on a graph, or something else. Which representations of data are used, is determined by scientists with respect to the context: whether it be a paper in a scientific journal, or a news article in the morning newspaper, in a popular science magazine, or even as a video on some online platform.
At each step of data analysis, however, lies the possibility for bias to creep in: from the very beginning of data acquisition, to the presentation of data.

It is of great significance in Astronomy to know the distance to a star or celestial object to facilitate many astronomical studies. You are part of a team of Astronomers that is attempting to; using calculations, determine the distance to the Redwood Cluster to facilitate further studies of the cluster.

a. The star RZ-1034 is found within the Redwood cluster. The Hipparcos satellite has measured its parallax to be $0.05 \pm 0.001''$. With an illustration, calculate the distance to RZ-1034 in light years. [2]

   Dissatisfied with the measurement from the parallax, your team decides to utilise a different measure. Making use of a standard candle, namely the Cepheid variable CV-1992, you attempt to utilise it to find the distance.

b. Using the graph below, calculate the period of oscillation of the brightness of the cepheid. [1]



c. The following data is from various known Cepheids. Plot these values on graph paper and demonstrate the period-luminosity relationship. Write out the equation for the relationship. Use your value of period [From **b.**] to extrapolate from the graph and determine the luminosity of CV-1992.

- For Graph plotting **6 marks** [1 for each 5 points].
- Right Axis determination **2 marks**.
- Spacing **1 mark**.
- Finding the equation and Luminosity **2 marks**.

| Star | Period (d) | $M_V$ | Star | Period (d) | $M_V$ |
|---|---|---|---|---|---|
| U Aq1 | 7 | -3.66 | SY Aur | 10.1 | -4.12 |
| Sz Aq1 | 17.1 | -4.7 | YZ Aur | 18.2 | -4.86 |
| TT Aq1 | 13.8 | -4.51 | FF Aur | 2.1 | -2.15 |
| FF Aq1 | 4.5 | -3.55 | AD Cam | 11.3 | -4.25 |
| FM Aq1 | 6.1 | -3.48 | SS CMa | 12.4 | -4.37 |
| FN Aq1 | 9.5 | -4.04 | VY Car | 18.9 | -4.91 |
| IU Aq1 | 22 | -5.1 | WZ Car | 23 | -5.16 |
| V493 Aq1 | 3 | -2.58 | YZ Car | 18.2 | -4.86 |
| V496 Aq1 | 6.8 | -3.62 | KN Cen | 34 | -5.65 |
| V526 Aq1 | 4.2 | -3.02 | CP Cep | 17.9 | -4.84 |
| V800 Aq1 | 20.1 | -4.98 | VX Cyg | 20.1 | -4.99 |
| V1162 Aq1 | 5.4 | -3.32 | V396 Cyg | 33.2 | -5.62 |
| V340 Ara | 20.8 | -5.03 | V609 Cyg | 31.1 | -5.53 |
| V475 Ara | 1.5 | -1.72 | T Mon | 27 | -5.36 |
| RX Aur | 11.6 | -4.29 | X Pup | 26 | -5.31 |

d. The apparent magnitude of the Cepheid from Earth is 1.53. Given the absolute magnitude calculated in part **c.**, find the distance of CV 1992 from earth in light years. [2]

e. The 2 methods have given significantly different measurements. Suggest potential sources of error from each method. Then estimate the error in your calculation using both method. [3]