Total Marks: 90 **Time: 3 Hour**

BDOAA 2019

BANGLADESH OLYMPIAD ON ASTRONOMY AND ASTROPHYSICS

Name:
Class:
Institution:
Registration no:
Email:
Phone Number:

Instruction for the Candidate:

- The candidate must write his/her personal information and registration number on the answer script.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.

Table of Constants: ▪ *Luminosity of Sun,* $L_{\odot} \approx 3.826\times10^{26}$ W ▪ *Mass of Sun,* $M_{\odot} \approx 2\times10^{30}$ kg ▪ *Radius of Sun,* $R_{\odot} \approx 7\times10^{8}$ m ▪ *Absolute Magnitude of Sun,* $M_{\odot} = 4.74$ ▪ *Radius of the Earth,* $R_{\oplus} \approx 6.371\times10^{6}$ m ▪ *Gravitational Constant,* $G \approx 6.674\times10^{-11}$ Nm^2/kg^{-2} ▪ *Distance of Sun,* $d_{\odot} \approx 1.496\times10^{11}$ m	▪ *Wien's Constant,* $\lambda_{max} T = 2.898 \times 10^{-3}$ m K ▪ 1 **pc**= 3.086×10^{16} m = 206265AU ▪ *Pogson's law of magnitude and brightness,* $m_1 - m_2 = -2.5 \log (F_1/F_2)$ ▪ *Astronomical Unit,* **AU** $= 1.496 \times 10^{11}$ m ▪ *Stefan-Boltzmann Constant* , $\sigma = 5.67 \times 10^{-8}$ W/m^2 ▪ *Radio Wavelenght* $\approx 1 \times 10^{-3}$ m

1. Radiation pressure in Main Sequence Star

The Luminosity of stars can be expressed as a function of time where $L=\frac{\Delta E}{\Delta t}$; where $\boldsymbol{E}$ is the total amount of energy radiated by $\boldsymbol{t}$ amount of time by a star. $\boldsymbol{\eta}$ is the percentage of the total mass of a star that it burnt in nuclear fusion through its main-sequence lifetime. The outer layer of the star remains stable due to the balance of outward force of radiation pressure and inward force of gravity. For this problem, assume the luminosity, radius and mass of the star is $\boldsymbol{L}$, $\boldsymbol{R}$ and $\boldsymbol{M}$ respectively and the mass of the outer layer is $\boldsymbol{m}$.

Recall that the momentum of a photon is $p = E/c$.

a) Determine the main-sequence lifetime of the star in terms of M, L, η and other constants. [2]
b) Derive an expression for the radiation pressure, defined as the outward momentum per unit time per unit area, delivered to the outer layers of the star. [3]
c) If $L = 10,000L_{\odot}, M = 10M_{\odot}, m = 10^{-5}M_{\odot}$, estimate the radius of the star, R. [3]
d) Estimate the effective temperature of the star, T based on your answer on question c. [2]

2. Stellar magnitudes

Two stars are observed that appear to be companions (i.e. a binary star system). They are very close in the sky and both have an annual parallax of 0.1 arcsec. One of the stars is much brighter than the other.

a) The brighter of the two stars has an apparent bolometric magnitude of 4.74 and its spectrum peaks at 0.5 m. [3]
   A1. Estimate the surface temperature of the star in Kelvin.
   A2. Estimate the luminosity of the star in $L_{\odot}$
   A3. Find the radius of the star in $R_{\odot}$.

b) The dimmer companion has an apparent magnitude of 10.69 and its intensity peaks at 0.29 m. Estimate the surface temperature and the radius of the star (in units of K and $R_{\odot}$) and indicate whether or not it lies on the main sequence. [4]

c) How far should the binary system be to cause extinction level temperature (333K) in Earth assuming we receive stellar flux from both our sun and the binary system. Observations show that the average temperature of the Earth during the last century was 287 K. [The solar constant is $S_{\odot}$= 1366 w/m^2] [4]

3. Weird Planet!

You are on a planet where an explosion has left a portion of the interior of the planet hollow. The explosion didn't occur right at the center of the planet, and the structure of the planet can be modeled as a solid sphere with a radius 2R with a tangential mass gap of radius R, as shown in the figure. The mass of the planet is the same as the mass of the Earth, and R is the same as the radius of the earth.

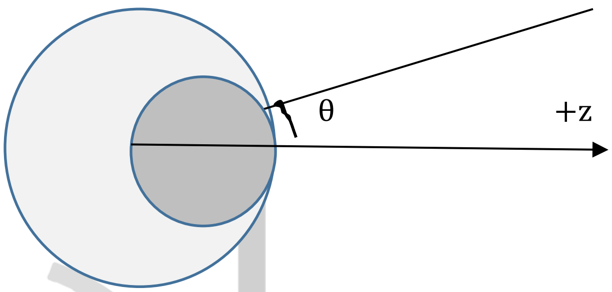


Figure 1: The cross section of the planet

a) Locate the center of mass of the planet in the coordinate system as shown in figure. [2]
b) Let θ be defined as the angle a point makes with the z-axis. If you move along a sphere of 10R centered at the center of the planet, how does the value (and direction) of g varies as a function of θ? [4]
c) What is the gravitational potential of the planet at a distance 2R from the center, as a function of θ? [4]
d) If you were to travel to the outer space, which part on the surface of the planet would you choose to take off your spaceship? Why? [Hint: what velocity is needed for an object to travel to the outer space?] [2]

4. Nebula Expansion

An earth sized radio telescope is observing the expansion of a nebula, 8pc away from the earth. The nebula is expanding at a rate of 1km per minute.

a) How long would the telescope take to detect a measurable change of the nebula? [4]
b) Using a baseline of the diameter of the earth's orbit, how far away will the telescope be able to determine distance using trigonometric parallax, assuming the source is bright enough. [3]
c) From the previous answer, what would be the apparent magnitude of the sun be from that distance? [1]

6. Tidal Force

The tidal force is an apparent force that stretches a body towards the center of mass of another body due to a gradient (difference in strength) in the gravitational field from the other body. In simpler words, tidal force is a difference between the force acting on the center and the surface of a gravitational body by another gravitational body. This force between earth is also responsible for this tide

[The following series can make your life easier:

$(1 + b)^n = 1 + nb + \frac{1}{2}\, n(n-1)\, b^2 + \ldots ..$]

a) Find the tidal acceleration of the earth due to the moon in terms of **M,** mass of moon; **d,** distance between moon and earth and **h**, distance of any point of the surface from the center. [4]
b) Show that, the tidal of the earth depends only on the density of moon, ρ and its angular diameter, θ. [3]
c) The moon, made of rocks, has an average density of 3300 kgm^{-3}, which is about 2.4 times as dense as the sun. Calculate the relative strength of the sun's and moon's tidal forces on the earth. [3]

Data Analysis

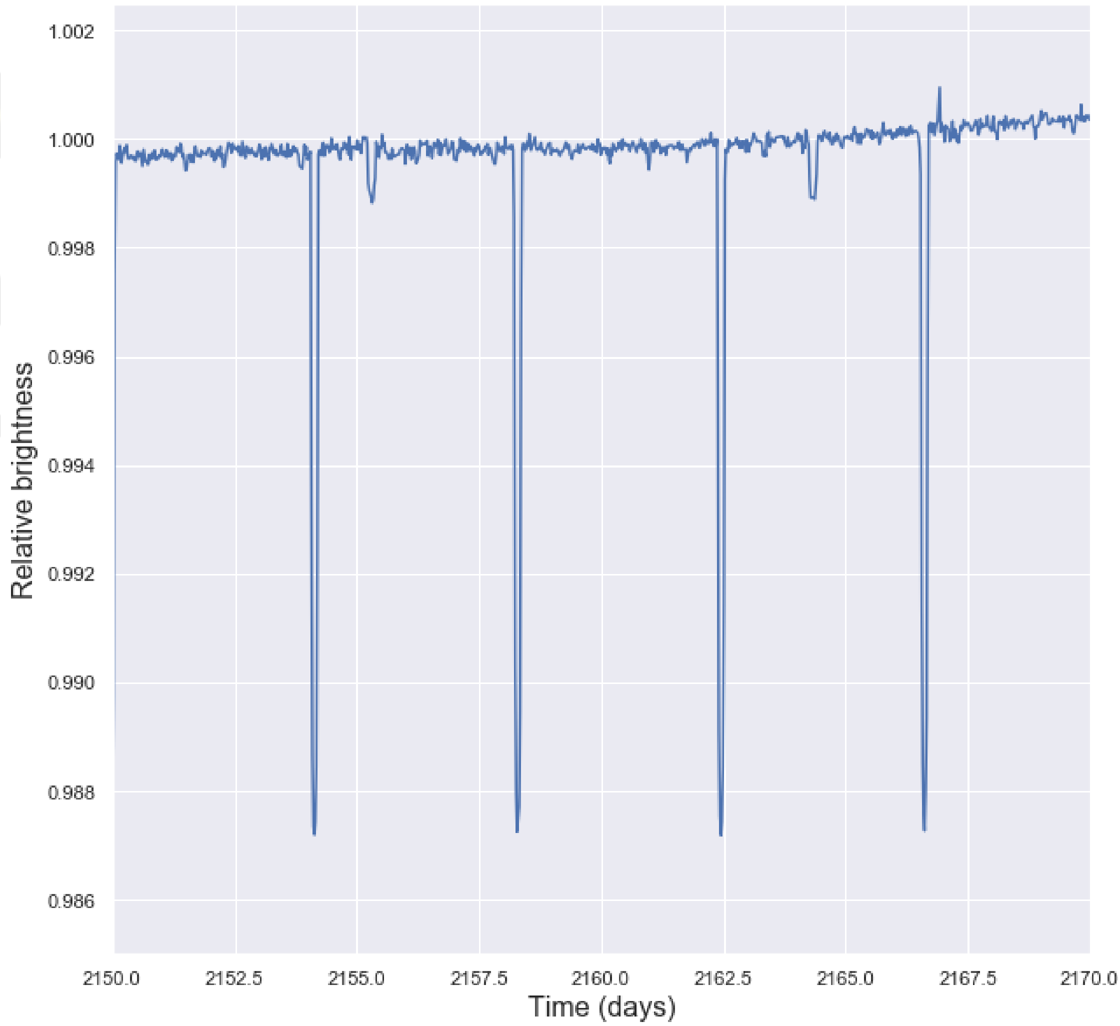
7. Period-Distance relationship of the exoplanets

In this problem, we will explore the relationship between orbital period and semi major axis of the planets using the known period and semi-major axis of some currently discovered and confirmed exoplanet collected from the open exoplanet catalogue at kaggle.com. The host stars of all these exoplanets have a mass of $M_{\odot}$.

The following table shows the semi major axis in AU and the orbital period in years. The log_{10}value for the semi major axis is also calculated in the table.

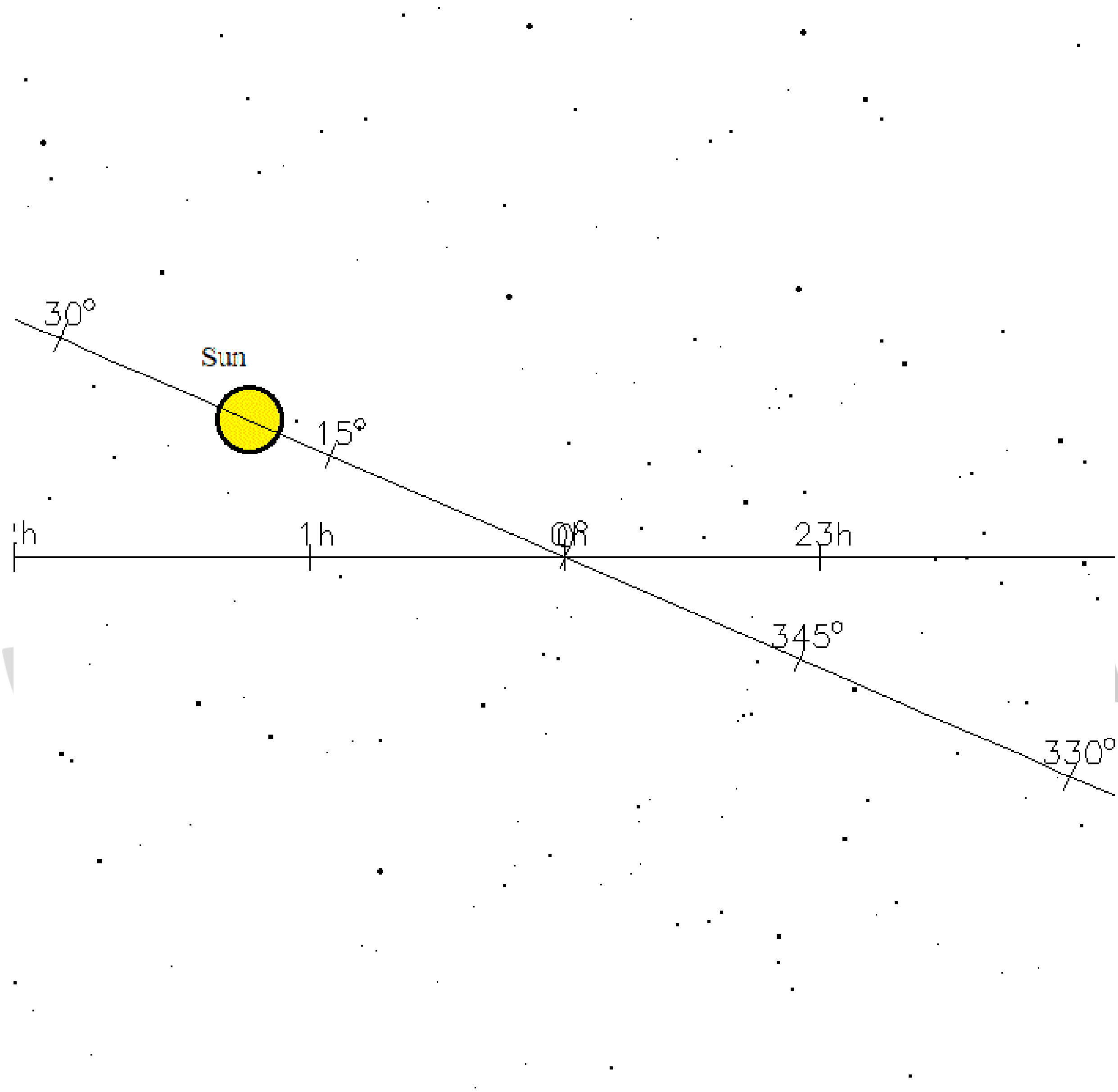
Semi Major Axis (AU)	Period (Years)	log_d	log_p
0.0575	0.015928	-1.24	
0.0628	0.015738	-1.20	
0.6540	0.528143	-0.18	
5.6000	13.55539	0.75	
0.0900	0.028283	-1.05	
1.3200	1.505850	0.12	
2.3950	3.706034	0.38	
0.0474	0.010308	-1.32	
0.3300	0.186917	-0.48	
0.0458	0.017301	-1.34	
0.0590	0.014260	-1.23	
0.2300	0.109516	-0.64	
2.8640	4.835148	0.46	
0.0964	0.029743	-1.02	
1.0510	1.082569	0.02	
0.0362	0.006891	-1.44	
0.0490	0.010872	-1.31	
3.3000	6.108276	0.52	
0.2931	0.159107	-0.53	
0.0499	0.011221	-1.30	

a) Calculate the log_{10} of periods and fill up the table. Keep in mind the significant figure while writing your values. [3]
b) Plot the $log_{10}\ d\ vs\ log_{10}\ p$ in the provided graph paper and label the axes. Follow the best practices of plotting a graph. [4]
c) Draw a best fit line around your plotted points. [2]
d) Determine the slope and intercept of the best fit line. [2]
e) Express $log_{10}\ d$ period as a function of $log_{10}\ p$ [2]
f) Make necessary changes in your equation to express the semi-major axis of an exoplanet (in AU) as a function of its orbital period (in years). [3]
g) The following diagram is the light curve of a host star. Measure the orbital period of the possible exoplanet(s). [3]
h) Using your derived function, determine the semi-major axis of the exoplanet(s). [3]



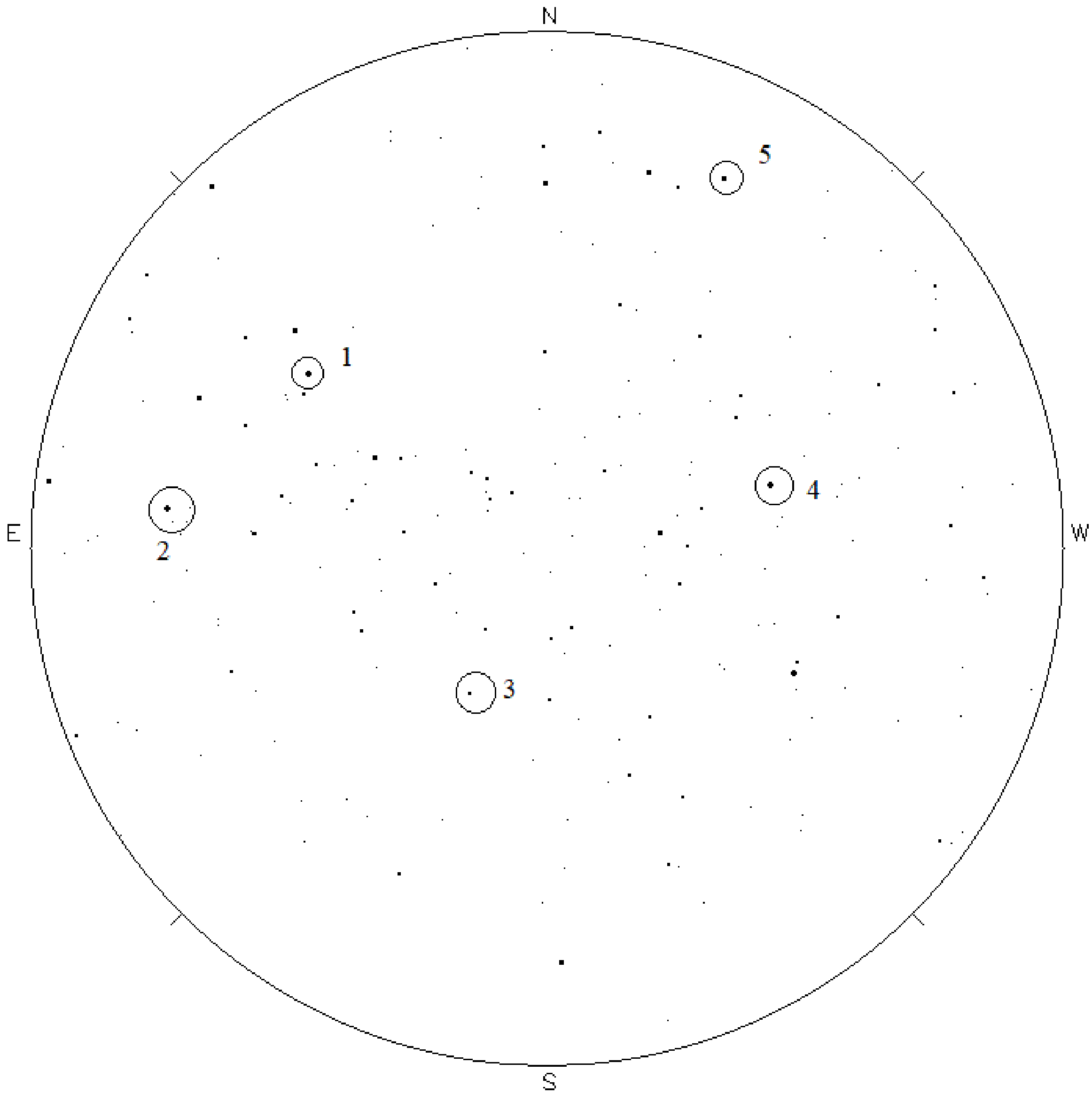
8. Sky Map 1

The sky map provided below shows Sun's apparent position in the sky on a certain day



a) What does the 0 point signify? Based on the map name this point. [1]
b) Calculate the date of the map. [2]
c) If we live on a latitude of 23.5°what would be the Altitude of Sun on that day? [2]
d) What would the coordinates of sun after 6 months? [2]
e) Draw a celestial sphere for north latitude 23.5°indicating clearly the zenith point, the north celestial pole and the horizon. Mark in the cardinal points of the horizon and draw in the celestial equator. The date is March 21; mark in the Sun's position at noon. [3]

9. **Sky Map 2**



a) Name the stars and their constellation marked on the map. [5]
b) Point these celestial objects with a circle in the map: [2]
   i) Andromeda galaxy
   ii) Pleiades
   iii) First point of Aries.