Bangladesh Olympiad On Astronomy and Astrophysics 2018

National Round

Organized By: **Bangladesh Olympiad On Astronomy and Astrophysics Committee**
In cooperation with: **Open Space and Jahangirnagar University**

Full Mark:110 **Duration**: 2.30 Hour

Name	:
Date of Birth	:
Address	:
Phone	:
Email	:
Institute	:
Class	:

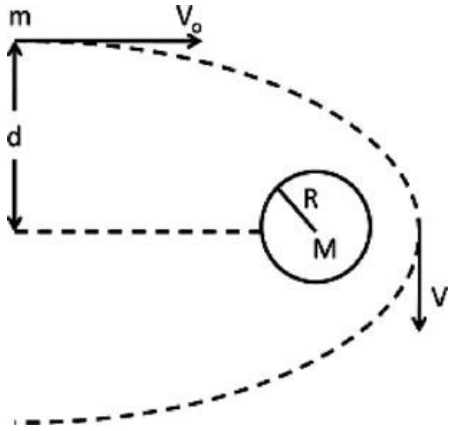
Instruction For the Candidate:

- The candidate must write his/her personal information and registration number on the answer script.
- You will write your answer in the answer sheet provided. If you need more extra paper, ask the exam invigilator.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.

Section 1. Theory

Short Questions

Problem 1: **Evolution of A Comet**



A comet with mass of m and diameter of D is orbiting a star of mass M, radius R and temperature T. The eccentricity of the orbit is e with a major axis 2a. During its closest approach, it gathers the maximum amount of energy from the Star. This energy helps melting the ice which consist of 40% of the total mass of the comet. Latent heat of fusion of ice is L_v.

a. Suppose the comet is moving towards the star from a long distance with initial speed v_0 and impact parameter d. Calculate the minimum value of v_0 such that the comet does not hit the star. [4]

b. Find out how many revolutions, **n** it would take to melt the total ice of the comet and turn it into a meteor swarm. [6]

Problem 2: Eclipsing Binary

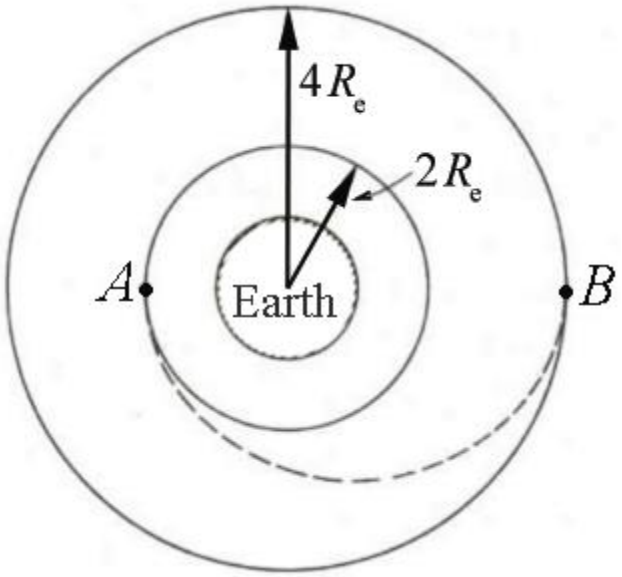
An eclipsing binary consists of two stars of different radii and effective temperatures. Star 1 has radius R_1 and T_1, and Star 2 has $R_2 = 0.5R_1$ and $T_2 = 2T_1$.

(a) Find the change in bolometric magnitude of the binary, Δm_{bol}, when the smaller star is behind the larger star. [3]

(b) An extrasolar planet has been observed which passes in front of (i.e., transits) its Star 1. If the planet is dark (i.e., contributes essentially no light of its own) and has a surface area that is 2% of that of its parent star, find the decrease in magnitude of the system during transits. [3]

Problem 3: Transfer Orbit

A space vehicle is in a circular orbit about the earth. The mass of the vehicle is M_v and the radius of the orbit is $2R_e$. It is desired to transfer the vehicle to a circular orbit of radius $4R_e$. The mass of the earth is M_e.



(a) What is the minimum energy expenditure required for the transfer? [2]

(b) An efficient way to accomplish the transfer is to use an elliptical orbit from point A on the inner circular orbit to a point B on the outer circular orbit (known as a Hoffmann transfer orbit). What changes in speed are required at the points of intersection, A and B and on which direction? [6]

Problem 4 : Planetary Nebula

Near the end of its life , a star may blow off its outer envelope leaving only a hot remnant white dwarf star. The ultraviolet radiation from the hot central star can then ionize (lit up) the surrounding matter that has been ejected. The result is called a planetary nebula. Assume the entire ejected matter consists of only neutral hydrogen atoms with uniform density n . There exists a sharp boundary called the Stromgren sphere surrounding the central star inside which hydrogen is nearly completely ionized. The radius of the Stromgren sphere is taken to be the radius of the nebula that is optically bright.

a)If one half of the luminosity of the central star is emitted at wavelengths shorter

than the Lyman limit(91.6 nm) , estimate the number of photons emitted per second with $\lambda < 91.6$ nm. [1 nm = 10^{-9} m] [2]

Assume that every Lyman limit photon goes towards ionizing a neutral hydrogen atom. In other words, no Lyman limit photon travels past the radius of the Stromgren sphere. The rate (per unit volume) at which electrons and protons recombine to form neutral hydrogen atom is given by $n^2\alpha$, where α=

3×10^{-13} cm^{-3} s^{-1}.

b) Using the result in (a) and the information above derive an expression for the radius of the Stromgren sphere R_s . [4]

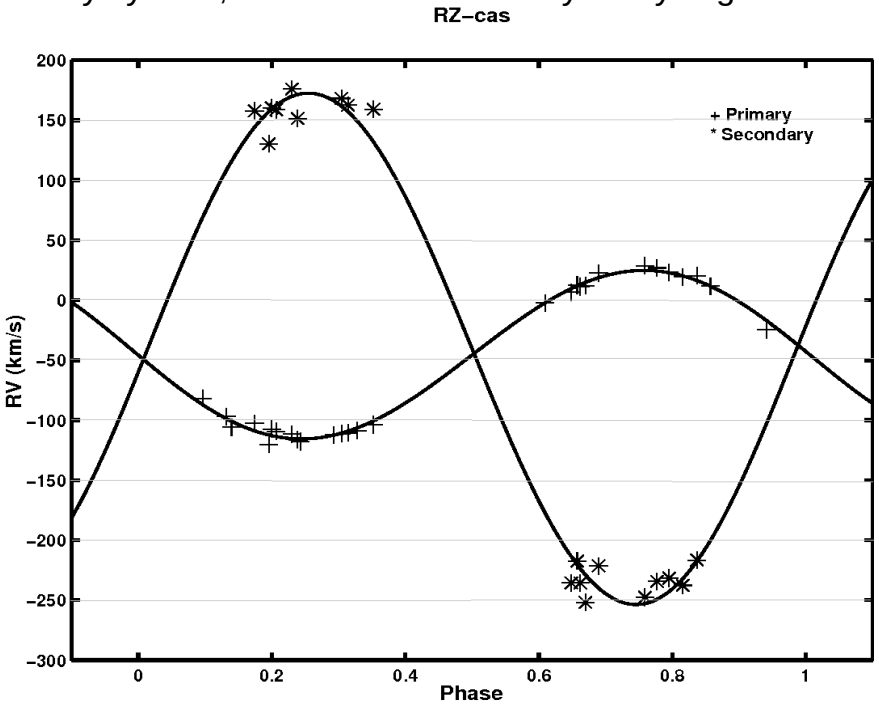
Consider that the central white dwarf just formed and the gas surrounding it initially neutral.

c) What is the timescale for the Stromgren sphere to develop ? That is , how long does the star take to completely ionize the region ? Give only an order of magnitude estimate ignoring radiative recombination (electron proton recombination to form neutral hydrogen). [4]

Long Questions

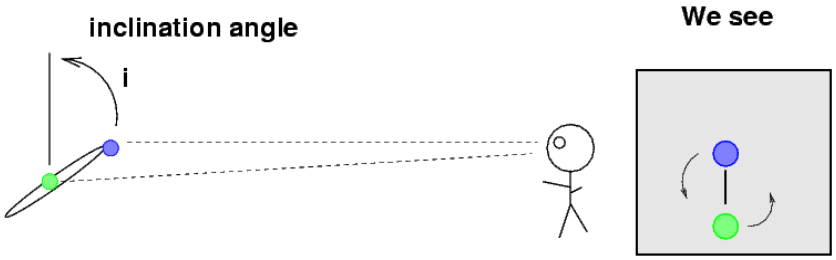
Problem 5: Binary Star

The following curve shows the radial velocity of the two individual stars of a binary system, that was calculated by analyzing the received spectrum.



The orbital period of the stars is 2 years.

Firstly, we assume that the system is edge-on and the stars are orbiting in a circular orbit.



A binary system is called edge-on, if the plane of its orbit is perpendicular to the sky (i.e. the inclination angle, i = 90 degree)

a. Why the radial velocities are changing with times and why periodically? Explain very briefly (in 1-2 line). [2]

b. Does the entire system is moving relative to us? If yes then find the velocity and the direction. [2]

c. Determine the orbital velocity of the stars, corresponding distances from their centre of mass and the total distance between them. [3]

But actually the system is not totally edge-on. It has an inclination angle of 30 degree. So, your answer in c) is need to be corrected.

d. Determine again the orbital velocities and the distances from the center of mass for each of the stars. [3]

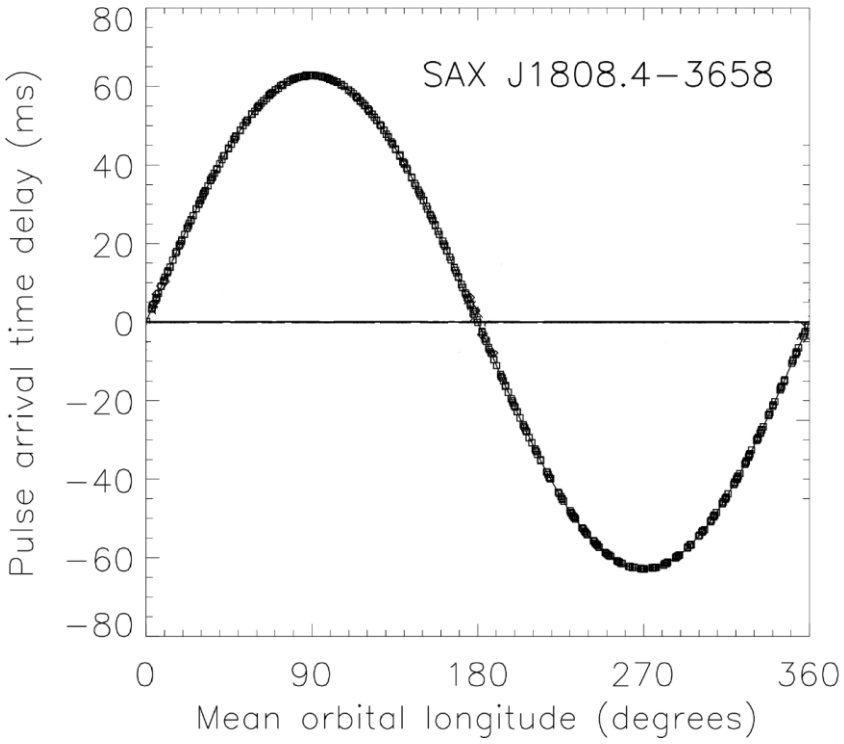
e. Calculate the mass of the two stars individually. [4]

f. If this binary system was face-on, (means its inclination angle, i = 0 degree) then how would be the radial velocity vs time (or phase) graph? [Don't use any graph paper or specific values of velocity or time. Just draw and label the axis and do a rough sketch] [3]

Now we will consider a very special binary system, consisting of a millisecond X-ray pulsar (has a pulsating period in millisecond range) and a companion star. So, we are supposed to receive electromagnetic pulse after a certain fixed period. But observationally, the pulse (that has a velocity of visible light) arriving time was changing due to the small orbital movement around the center of mass of the system. (See the following figure)

The amplitude of the sine curve indicates the projected light travel time across the orbit of the neutron star around the center of mass of the binary.

[Mean orbital longitude is just another expression of its orbital phase]



g. Using the figure, estimate the orbital radius (the distance from the center of mass) of the pulsar if the system is edge-on? [2]

h. Determine the actual orbital radius, considering the fact that the binary system has an inclination angle of 30 degree. [1]

i. Assume that the pulsar has a mass of M = 1.4 M and the orbital period of the binary system is, P = 2 hours. Compute the mass of the companion star, m.
[Hint: in order to solve for mc you will have to make the approximation that m/M << 1 and, equivalently, M/m >> 1.] [5]

Section 2: Data Analysis

In the regional round you have calculated the period of a Cepheids. Cepheids are extremely important in astronomy because of Leavitt's Law, also known as the Period-Luminosity Relationship. In the early 1900s, Henrietta Leavitt showed that **the absolute magnitude M of a Cepheid has a simple linear relationship with the log of its period**. In other words, if you know a Cepheid's period, you can calculate its absolute magnitude M and thus, are able to calculate its distance. This led to the Cepheids to be treated as *Standard Candles* and have been influential in measuring distances of other galaxies.

In this problem, we have provided you with a table of 20 Cepheids, their period and their V band absolute magnitudes based on Fouque et al. 2007.

Period	**Log_{10} Period**	**V band Absolute Magnitude**
3.73		-2.84
4.32		-3.31
4.44		-3.06
5.15		-3.34
5.33		-3.22
5.37		-3.04
5.77		-3.23
5.90		-3.09
6.64		-3.45
7.60		-3.89
8.38		-3.86
9.28		-3.71
9.75		-4.02
9.84		-3.93

10.15		-3.93
10.89		-4.14
11.64		-3.89
17.12		-4.62
21.85		-4.60
23.01		-4.61

a. Calculate the $\log_{10}$ Period in the given table. **Make sure to keep significant figures in mind, i.e., do not write MORE digits after decimals than necessary. [4]**

b. Use the provided graph paper to plot the Leavitt's Law **and label the axes**. Remember that in astronomy brighter objects have lower value so it is a convention to plot the y-axis inverted. [4]

c. Estimate a line of best fit based on your plot. This estimation is qualitative so do not overthink this part. A rough sketch is sufficient. [1]

d. Determine the slope and the intercept of your line of best fit. [5]

e. When we invert the subject of a equation, we call it an inverse law. For example, for the equation

$$y = sx + b$$

The inverse would be

$$x = y\text{-}bs$$

Thus, we are able to invert Leavitt's Law, i.e. express the Period as a function of the absolute magnitude M. Write down the functional form of the inverse Leavitt's Law, i.e. how would you express Period as a function of its absolute magnitude M? *Hint: Think about what you plotted. What type of graph is this? [4]*

f. Let us assume that someone discovers a new Cepheid that is 1000 parsecs from the Earth. The astronomer notes that the apparent magnitude of this star is +2. What is the absolute magnitude of this Cepheid? What is its pulsating period? You may ignore any cosmological redshift effect for this calculation. [7]

Section 3: Observation

The star charts below shows the night sky (UTC +08:00) on August 2018 at an unknown latitude and longitude 116^{o} E recorded by Vasco. These star charts are recorded 3 seconds apart to see if Vasco can find any apparent changes in the sky while he is observing .Complete the questions on the following page.

a. On the star chart **"Image 1"**, mark out the asterism, the Winter Hexagon, with solid lines. [2]
b. Find the latitude of the location and justify your answer. [3]
c. Mark the first point of Aries in the "**Image 1**" with A. [2]
d. Vasco noticed some unusual dots in the star chart of image 1, which drastically changed their position in image 2. He then understand that a meteor shower is happening. Find the five meteors in each image and determine the radiant of the meteor shower. Mark it with "R" and identify which meteor shower is that. [4]
e. Based on your findings what is the average angular velocity (Arcmin/second) of the meteor showers. [2]
f. The following four deep sky objects (DSOs) are visible in the star chart. Mark out any three of these DSOs, each with a hollow circle in the "**Image 2**", and write that DSO's catalogue designation adjacent to it. [3]
(i) M45 - Pleiades;
(ii) M31- Andromeda Galaxy;
(iii) M57- Ring Nebula;
(iv) M82- Bode's Galaxy
g. Calculate the local hour angle of the first point of Aries. [2]
h. Estimate the local solar time and the local sidereal time, when the star *Betelgeuse* will be setting. [4]

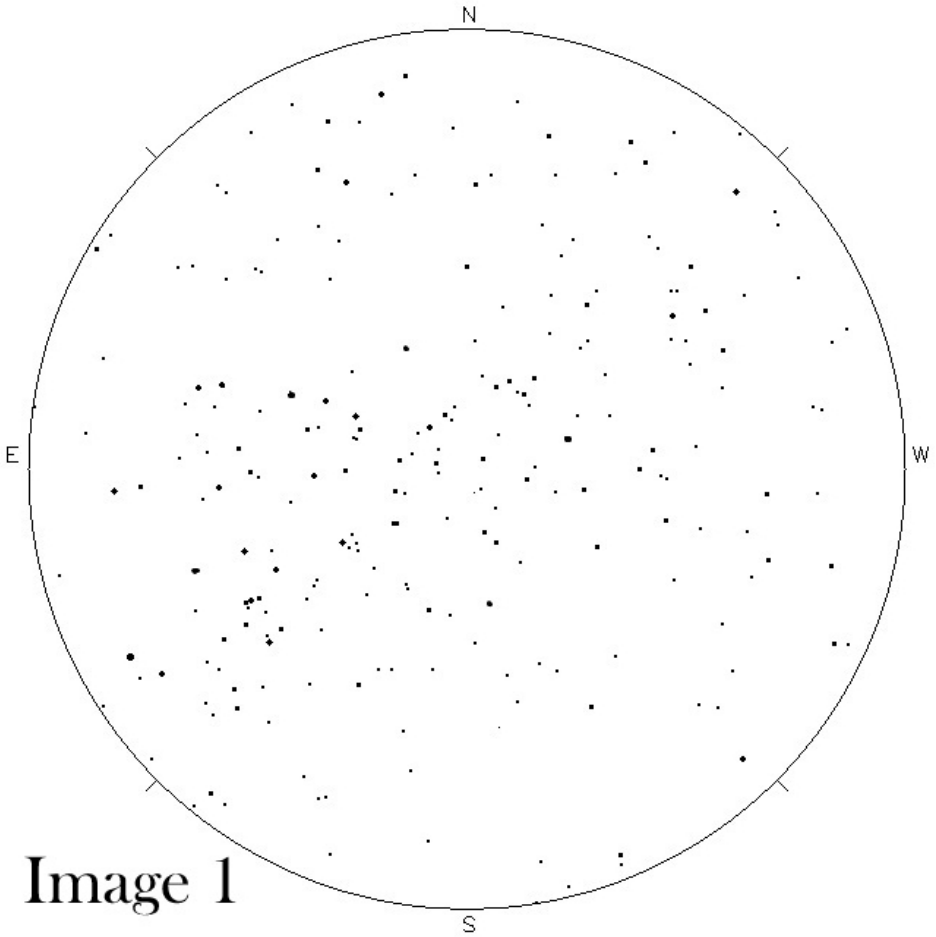


Image 1

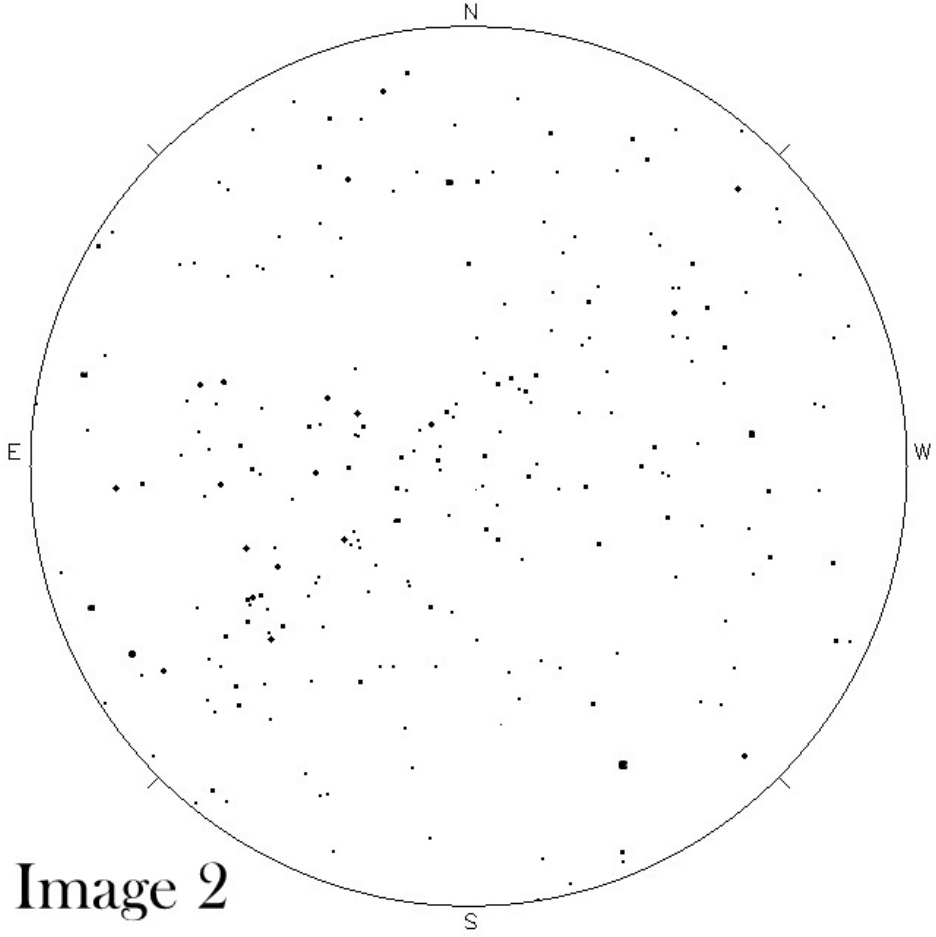


Image 2