

**BAAO**  
British Astronomy and  
Astrophysics Olympiad

## British Astronomy and Astrophysics Olympiad 2025-2026

### Astro Round 2

Tuesday 27<sup>th</sup> January 2026

**This question paper must not be photographed or taken out of the exam room**

#### Instructions

**Time:** 3 hours (roughly 1 hour per question).

**Questions:** All three questions should be attempted. Each question contains independent parts so that later parts can be attempted even if earlier parts are incomplete. **The questions build in difficulty.**

**Solutions:** Answers and calculations are to be written on loose paper. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet may be used. **START EACH QUESTION ON A NEW PAGE.**

**Clarity:** Solutions must be written legibly, in black or blue pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

**Instructions:** To accommodate students sitting the paper at different times, please **do not discuss** any aspect of the paper on the internet until 8 am Saturday 7<sup>th</sup> February.

**Calculators:** Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

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**Training Dates and the IOAA** (Hanoi, Vietnam, 25<sup>th</sup> September - 5<sup>th</sup> October 2026)

*The best students taking this paper eligible to represent the UK at the IOAA will be invited to attend the **Selection Camp** to be held in Oxford from **Tuesday 7<sup>th</sup> to Saturday 11<sup>th</sup> April 2026**. At the camp, problem solving skills and observational skills (including with telescopes) will be developed, and students will sit a Data Analysis exam along with the Round 3 paper. From this, a team of five students (plus one reserve) will be selected for further training, including additional camps in the summer.*

*The BAAO are very proud to be sponsored by G-Research*



## Important Constants

Constant	Symbol	Value
Speed of light	$c$	$3.00 \times 10^8 \text{ m s}^{-1}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16} \text{ m}$
Astronomical Unit	au	$1.50 \times 10^{11} \text{ m}$
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^8 \text{ m}$
Radius of the Earth	$R_{\oplus}$	$6.37 \times 10^6 \text{ m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30} \text{ kg}$
Mass of the Earth	$M_{\oplus}$	$5.97 \times 10^{24} \text{ kg}$
Luminosity of the Sun	$L_{\odot}$	$3.83 \times 10^{26} \text{ W}$
Absolute magnitude of the Sun	$\mathcal{M}_{\odot}$	4.74
Hubble constant	$H_0$	$70 \text{ km s}^{-1} \text{ Mpc}^{-1}$
Stephan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Gravitational constant	$G$	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Boltzmann constant	$k_B$	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Planck's constant	$h$	$6.63 \times 10^{-34} \text{ J s}$
Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C}$
Proton rest mass	$m_p$	$1.67 \times 10^{-27} \text{ kg}$
Electron rest mass	$m_e$	$9.11 \times 10^{-31} \text{ kg}$
Wien's displacement law	$\lambda_{\text{max}}T$	$2.90 \times 10^{-3} \text{ m K}$
Avogadro's constant	$N_A$	$6.02 \times 10^{23} \text{ mol}^{-1}$

### Basic calculus formulae:

Chain rule  $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

Product rule  $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$

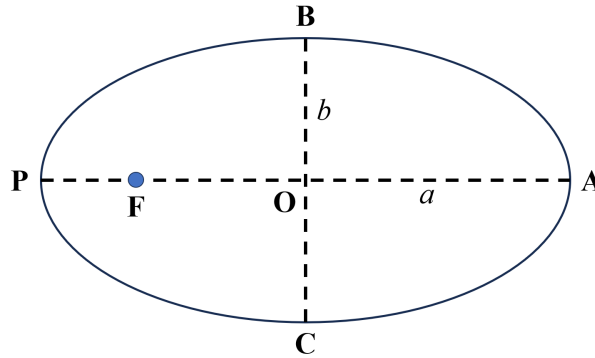
Quotient rule  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

Integration by parts  $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Standard integral  $\int \frac{1}{x} dx = \ln|x| + C$

## Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



**Elements of an elliptic orbit:**

- $a = \text{OA} (= \text{OP})$  semi-major axis
- $b = \text{OB} (= \text{OC})$  semi-minor axis
- $e = \sqrt{1 - \frac{b^2}{a^2}}$  eccentricity
- F** focus
- $\text{PF} = a(1 - e)$  periapsis distance (shortest distance from **F**)
- $\text{AF} = a(1 + e)$  apoapsis distance (longest distance from **F**)
- $\pi ab$  area of the ellipse

**Kepler's Third Law:**

$$T^2 = \frac{4\pi^2}{GM} a^3$$

**Vis-Viva Equation:**

$$v^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right)$$

**Wien's Displacement Law:**

$$\lambda_{\text{max}} T = \text{constant}$$

**Stephan-Boltzmann Law:**

$$L = 4\pi R^2 \sigma T^4$$

**Brightness (Intensity):**

$$b = \frac{L}{4\pi d^2}$$

**Magnitudes:**

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5 \log \left( \frac{d}{10} \right)$$

**Distance-Parallax Relation:**

$$d = \frac{1}{p}$$

**Rayleigh Criterion:**

$$\theta = \frac{1.22\lambda}{D}$$

**Redshift:**

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

**Hubble's Law:**

$$v = H_0 d$$

## Qu 1. Poynting Into a Star

All stars begin life in clouds of gas and dust that undergo gravitational collapse. From these, discs of debris develop around new stars, from which planets eventually form. In our own Solar System, the asteroid belt and Kuiper belt are likely the only leftovers from this period of assembly. As well as this large debris, dust particles with radii  $10 - 100 \mu\text{m}$  can also be found in a tenuous disc within the inner Solar System. The effect of this dust is a brightening of the sky near the horizon known as zodiacal light (see Figure 1).



**Figure 1:** Zodiacal light seen from Paranal, Chile. Credit: ESO/Y.Beletsky.

There are many mechanisms by which dust of this size is removed from the Solar System. In this question, we will investigate one such mechanism: radiative forces from the Sun.

Photons are able to exert forces on small particles as they carry a momentum,  $p$ , related to the photon's energy,  $E_{\text{phot}}$ , by

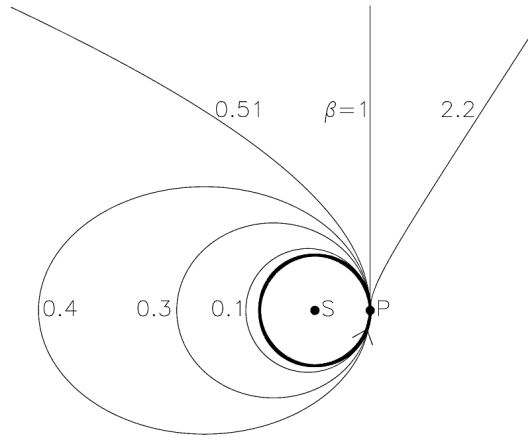
$$E_{\text{phot}} = pc ,$$

where  $c$  is the speed of light. This force is called the radiation force,  $F_{\text{rad}}$ , and can be compared to the size of the gravitational force,  $F_{\text{G}}$ , acting on the particle using the parameter  $\beta$ , given by

$$\beta \equiv \frac{F_{\text{rad}}}{F_{\text{G}}} .$$

Light particles have larger values of  $\beta$ . The value of  $\beta$  determines whether the dust is ejected from the system or can remain in a stable orbit (see Figure 2).

- a. Consider a spherical dust particle with mass  $m$  and cross-sectional area  $\sigma$  at rest a distance  $R$  from a star with luminosity  $L$  and mass  $M$ .
  - (i) Derive an expression for  $F_{\text{rad}}$ , assuming all incident photons are absorbed.
  - (ii) Hence, derive an expression for  $\beta$ .
- b. Consider now that the dust particle is moving in a circular orbit of radius  $R$ . Assuming that  $F_{\text{rad}}$  is acting entirely radially, by considering the resultant force acting on the particle derive an expression for the total energy in terms of  $\beta$  and other relevant variables. (*Hint: You may wish to use the relationship between force and potential energy that  $F = -\frac{dU}{dR}$* )



**Figure 2:** A large planet P on a circular orbit around star S has a massive collision and generates dust of a variety of sizes and hence a variety of  $\beta$ . The subsequent orbits of dust fragments for several values of  $\beta$  are shown.  
Credit: Prof Mark Wyatt (University of Cambridge).

- c. Suppose a rather large dust particle (with  $\beta_0 \approx 0$ ) collides with a similar grain, generating a large number of smaller fragmentary grains (which collide and fragment further, producing a collisional cascade). Assuming the parent particle had an orbit with semi-major axis  $a_0$  and eccentricity  $e_0$ , a fragment will have a new  $\beta$  and new orbital properties

$$a = a_0 \frac{1 - \beta}{1 - 2\beta/(1 - e_0)}$$

$$e = \frac{e_0 + \beta}{1 - \beta}.$$

- (i) Show that all particles with  $\beta > 0.5$  will be ejected from the system on hyperbolic orbits, regardless of initial  $a_0$  or  $e_0$ .
- (ii) Hence, for the Solar System, find the minimum radius of a spherical dust grain of density  $1 \text{ g cm}^{-3}$  that will **not** be ejected. Give your answer in  $\mu\text{m}$ .

We have so far treated the radiation force as acting purely radially, which is a reasonable approximation since the orbital speed  $v \ll c$ . However, since  $v > 0$  there is actually a small tangential component as well, which we can find by resolving  $F_{\text{rad}}$  into components in the particle's rest frame. The radial component is referred to as 'radiation pressure', whilst the tangential component is called 'Poynting-Robertson drag' (PR drag), named after the two scientists that developed the theory, and acts as a resistive force  $F_{\text{PR}}$ . This means that particles with  $\beta < 0.5$  will eventually spiral into their star on a timescale of  $t_{\text{PR}}$ . In practice, the grains never quite reach the star as they sublimate at a distance of  $R_{\text{sub}}$  when their black-body equilibrium temperature reaches  $\sim 1500 \text{ K}$ , and for  $\beta < 0.1$  the grains are more likely to collide with similar sized grains and fragment further before reaching the star.

- d. With a suitable diagram and clear reasoning, show that  $F_{\text{PR}} \approx F_{\text{rad}} \times v/c$ .
- e. Consider our earlier dust particle from part b. travelling in a circular orbit of initial radius  $R$  as having  $0.1 < \beta < 0.5$  such that it will not be ejected but instead will be affected by  $F_{\text{PR}}$  and spiral into the star. Assume  $t_{\text{PR}}$  is long enough that at any given moment on its spiral its orbit can be assumed to be effectively circular.
- (i) For the Sun, calculate a value for  $R_{\text{sub}}$ . Give your answer in units of  $R_{\odot}$ . Take the Sun's effective surface temperature to be  $5780 \text{ K}$ . Treat the dust particle as a spherical black body.
- (ii) Derive an expression for  $t_{\text{PR}}$  for a particle starting at  $r = R$  and ending at  $r = R_{\text{sub}}$ , giving your answer in terms of  $\beta$ ,  $M$  and relevant constants. (*Hint: find an expression for  $\dot{r}$  first.*)
- (iii) Hence show that  $t_{\text{PR}}$  for a dust grain with  $\beta = 0.1$  starting in a circular orbit at  $R = 1 \text{ au}$  in our Solar System is  $\sim 4000$  years.

- f. Instead of a very slow spiral, we could instead model the cumulative effect of PR drag as a single instantaneous impulse  $I$  applied to the dust particle, tangential to its velocity but in the *opposite* direction, to make its circular orbit elliptical. If the dust grain at  $R = 1$  au has mass  $4.9 \times 10^{-14}$  kg, calculate the minimum value of  $I$  needed to ensure it sublimates. If you did not get a value of  $R_{\text{sub}}$  earlier, use  $R_{\text{sub}} = 8R_{\odot}$ .

This timescale  $t_{\text{PR}}$  is short compared to astronomical timescales (although long compared to ejection timescales), so effectively all the dust in the inner Solar System is recently generated rather than left over from the Sun's protoplanetary cloud. It is sparse due to the small output of dust-generating events (asteroid collisions, comet tails, dust swept up from planets, and interstellar dust passing through the Solar System) and the relatively large number of possible routes by which dust is lost (stellar wind, accretion onto a planet, collisions between grains, ejection by a planet's gravity, ejection by radiation pressure, and of course PR drag). Determining the relative dominance of each of these routes is difficult, but a good approximation can be made by looking at the particle number density (determined from measurements of the zodiacal light by space probes moving through the Solar System) as a function of heliocentric distance and comparing the data to what you would expect from each regime.

- g. By mass conservation within the Solar System's dust cloud

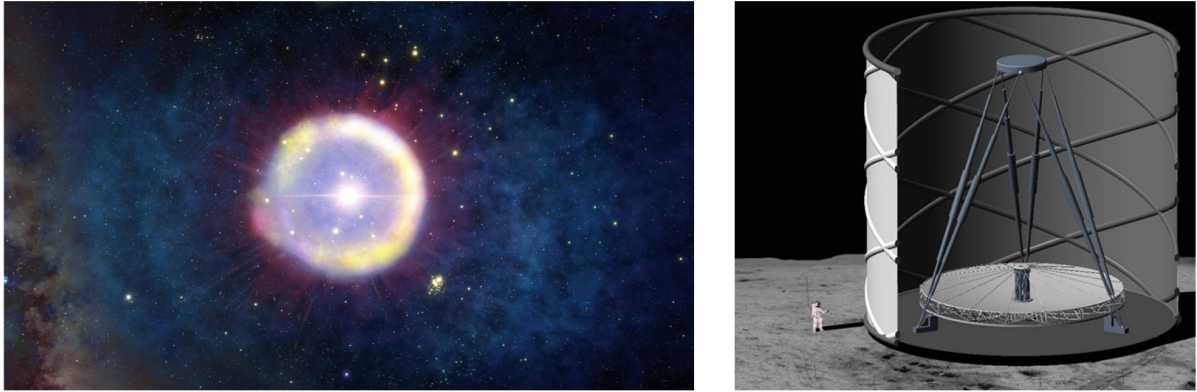
$$4\pi r^2 n(r) \dot{r} = \text{constant} .$$

Here  $n(r)$  is the dust particle number density and is assumed to follow a power law with heliocentric distance as  $n(r) \propto r^\alpha$ , and  $\dot{r}$  is the rate of change of  $r$  for individual particles at that distance.

- (i) By using your earlier expression for  $\dot{r}$  (from assuming particles in circular orbits spiral into the Sun through PR drag), predict what  $\alpha$  would be if PR drag was the dominant route of dust loss in the inner Solar System.
- (ii) Levasseur-Regourd et al. (1991) found that  $\alpha = -0.93 \pm 0.07$  for the local part of the dust cloud. Comment on whether this suggests PR drag is locally dominant or if other routes of dust loss must be more important.

## Qu 2. Seeing the First Stars

The very first stars in the Universe were born from pristine gas, which was effectively only hydrogen and helium, since that was all that was made in the Big Bang. Consequently, stars we see today differ significantly from these early ‘Population III’ or ‘Pop III’ stars. Pop III stars are suspected to have had a much larger average mass than the stars of subsequent generations, in the tens or perhaps hundreds of solar masses.



**Figure 3:** *Left:* An artist’s impression of Population III star, showing clearly the glowing nebula it creates about itself due to ionisation. Credit: NOIRLab/NSF/AURA/J. da Silva/Spaceengine/M. Zamani.

*Right:* An artist’s impression of the possible design for a telescope on the Moon, with large primary mirror and a sunshield. This design has as 20-m diameter mirror, but the design is scalable to 100-m diameter. Credit: Thomas Connors (Steward Observatory, University of Arizona)

Whilst the James Webb Space Telescope (JWST) may be able to find the spectral signatures of early galaxies filled with Pop III stars, some have speculated that Ultra-Large Telescopes (ULTs) of 100-m mirror diameter may be large enough to resolve the glowing ionisation nebulae around individual Pop III stars, if built on the Moon to avoid atmospheric effects (see Figure 3). Building such a telescope on the Moon’s rotational axis on one of its poles would allow for very long exposures that might allow for the detection of these very faint objects.

In this question we will consider an extreme example of a Pop III star, with a mass of  $1000 M_{\odot}$ , so that it is as bright as possible (while still within the limits allowed by some theories). Stellar models show that Pop III stars with masses  $100 - 1000 M_{\odot}$  have similar behaviour and scale similarly. They all have an effective surface temperature  $T_{\star} \approx 100 \text{ kK}$  and a luminosity that scales linearly with mass following

$$L_{\star} = 1.26 \times 10^{31} \left( \frac{M}{M_{\odot}} \right) \text{ W} .$$

Due to the very high temperature and luminosity, they are excellent sources of hydrogen ionising photons (i.e. photons with an energy  $> 13.6 \text{ eV}$ ), and so during their short lifetimes can carve out a bubble of ionised gas around them. This bubble heats up and glows and is the main object that a ULT could resolve (as the star itself is too small). The number of ionising photons produced every second is

$$Q_{\star} = \frac{8\pi^2 R_{\star}^2}{c^2} \left( \frac{kT_{\star}}{h} \right)^3 \times I(x) ,$$

where  $R_{\star}$  is the radius of the Pop III star,  $c$  is the speed of light,  $k$  is the Boltzmann constant,  $h$  is the Planck constant, and  $I(x)$  is a complex integral that takes a value of 1.69 in this case.

- Assume our  $1000 M_{\odot}$  star is born in a gas cloud of neutral hydrogen with uniform number density  $100 \text{ cm}^{-3}$ . Ignoring any recombinations (i.e. assuming each hydrogen atom needs just one photon to ionise and remains so for the star’s lifetime), determine in pc the radius of the ionised gas bubble it would have produced by the end of its lifetime. You should assume that during its time as a main sequence star 0.7% of its mass is converted to energy by nucleosynthesis and that its luminosity remains constant throughout.

In reality, recombinations are important and the gas is not of uniform density so we have to rely on numerical simulations to model it properly. These suggest a somewhat smaller maximum bubble radius of  $\sim 80$  pc for the ULT to resolve and detect. We will conservatively assume that a ULT has similar CCD technology, filters and throughput to the JWST (given a theoretical ULT would be built in the second half of the century). The JWST has greatest sensitivity in the F200W filter which is centred on  $2 \mu\text{m}$ . The brightest part of the spectrum of a Pop III star will be the  $\text{Ly}\alpha$  emission line at  $121.6 \text{ nm}$ , meaning we would be looking at stars at redshift  $z = 15$ .

At this redshift, we have to take into account cosmological effects when working out the physical size of a given angle on the sky. The angular size distance,  $d_A$ , is the physical distance between two objects at redshift  $z$  that now appear to be separated by 1 radian and is calculated as

$$d_A = \frac{r_C}{(1+z)},$$

where  $r_C$  is the ('comoving') radial distance, calculated as

$$r_C = \frac{c}{H_0} \int_0^z \frac{1}{\sqrt{\Omega_m(1+\zeta)^3 + \Omega_\Lambda}} d\zeta$$

(in a flat Universe with only matter and dark energy). Here,  $H_0$  is Hubble's constant,  $\Omega_m$  is the current proportion of mass in the Universe and  $\Omega_\Lambda$  is the current proportion of dark energy in the Universe. This integral cannot in general be done analytically, so instead must be done with numerical methods. We will take values of  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ .

- b. Show that at  $z = 15$  the angular scale is  $\sim 3 \text{ kpc arcsec}^{-1}$ .
- c. Consider our 100-m diameter telescope, operating at  $2 \mu\text{m}$ , and using a CCD so that every 1-D resolution element is 2 pixels across (so that the flux is well sampled).
  - (i) What is the approximate number of pixels the 80 pc radius bubble occupies in the image on the CCD?
  - (ii) It is expected that a lower limit for the number of Pop III bubbles that can be seen at this redshift is  $\sim 70 \text{ arcmin}^{-2}$ . If you have a square detector recording gigapixel images, approximately how many Pop III bubbles would you expect to see in each image?

Having confirmed that a 100-m telescope could resolve the glowing bubble around an individual Pop III star, we should ask how bright this object would be, and whether the telescope would be sensitive enough to image it. We will only receive a small portion of the star's spectrum in the filter of the telescope, so it is only the luminosity near the  $\text{Ly}\alpha$  line at  $121.6 \text{ nm}$  (in the rest frame),  $L_\nu$  (in units  $\text{W Hz}^{-1}$ ), that matters for our calculation. From this, we can get a spectral flux density

$$f_\nu = \frac{L_\nu(1+z)}{4\pi d_L^2},$$

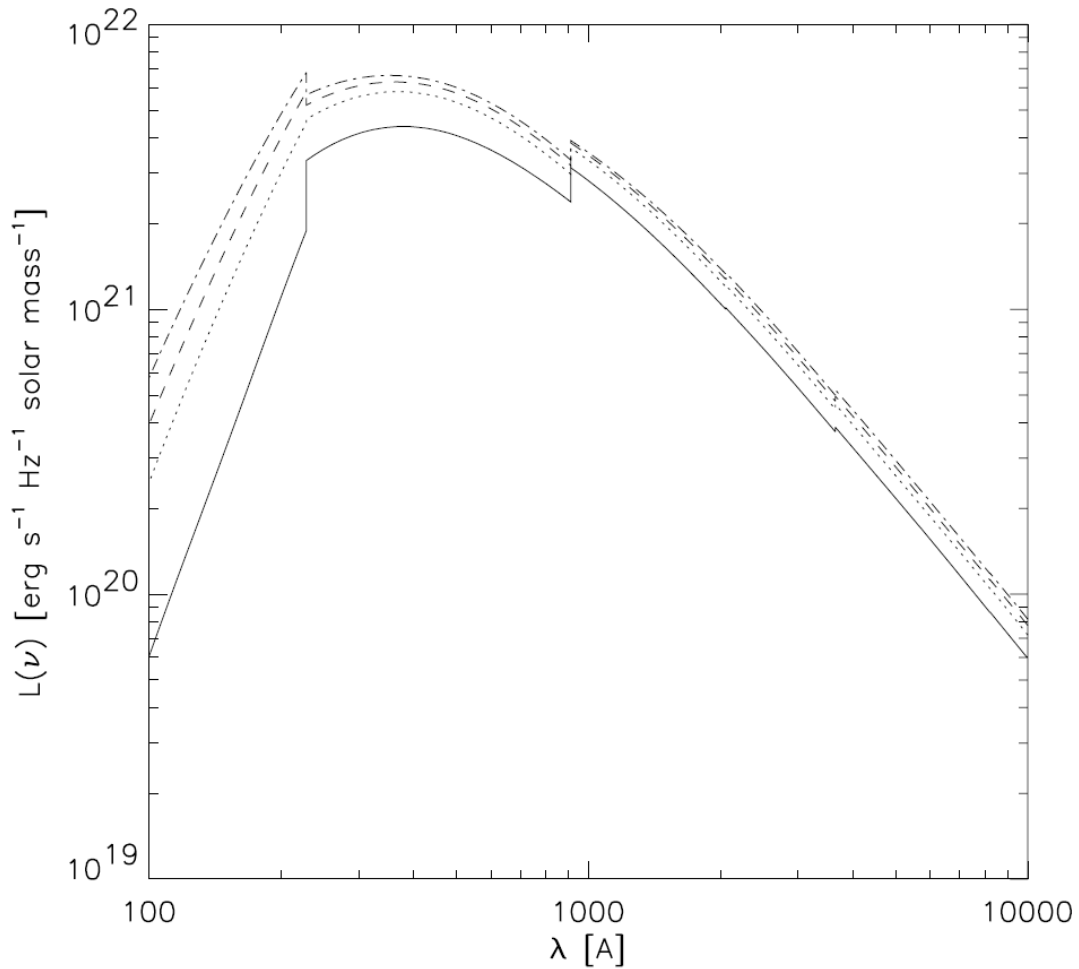
where  $d_L$  is the luminosity distance, also affected by the cosmology of an expanding Universe, given by

$$d_L = r_C(1+z).$$

We can convert this spectral flux density to janskys (Jy), given that  $1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ , and convert to an apparent magnitude

$$m = -2.5 \log_{10} \left( \frac{f_\nu}{3631 \text{ Jy}} \right).$$

The  $L_\nu$  spectra for  $100 M_\odot$ ,  $300 M_\odot$ ,  $500 M_\odot$  and  $1000 M_\odot$  Pop III stars are shown in Figure 4, scaled by stellar mass to show they are similarly shaped.



**Figure 4:** Normalized spectral energy distribution in the continuum, showing the similarity of shapes for  $100 M_{\odot}$ ,  $300 M_{\odot}$ ,  $500 M_{\odot}$  and  $1000 M_{\odot}$  Pop III stars. The  $1000 M_{\odot}$  star has the dot-dashed line. Both axes are logarithmic, with the  $y$ -axis per unit stellar mass and the  $x$ -axis in angstroms,  $\text{\AA}$ , where  $1 \text{\AA} = 0.1 \text{ nm}$ .

Although the spectra peaks at low wavelengths, everything shortward of Ly $\alpha$  will be absorbed by neutral hydrogen on its long journey to the telescope so the brightest part of the spectra that will still be detectable is at the wavelength of Ly $\alpha$ , which is  $121.6 \text{ nm} (= 1216 \text{\AA})$ . Credit: Bromm, Kudritzki & Loeb (2001).

- d. By carefully reading off  $L_{\nu}$  for our  $1000 M_{\odot}$  star (dot-dashed line) at wavelength of  $121.6 \text{ nm} (= 1216 \text{\AA})$  in Figure 4, show that it would have an apparent magnitude  $m \sim 40$ . (You are given that  $1 \text{ erg} = 10^{-7} \text{ J}$  and should note the logarithmic axes.)
- e. The JWST has an effective mirror diameter of  $6.5 \text{ m}$  and can (at low signal-to-noise) detect objects with  $m = 31.4$  if it has an exposure time  $t_{\text{exp}} = 1 \text{ day}$ . Given that the minimum detectable flux is inversely proportional to telescope area and also inversely proportional to  $t_{\text{exp}}^{1/2}$ , what is the minimum exposure time for our ULT to detect the Pop III star? Give your answer in days.

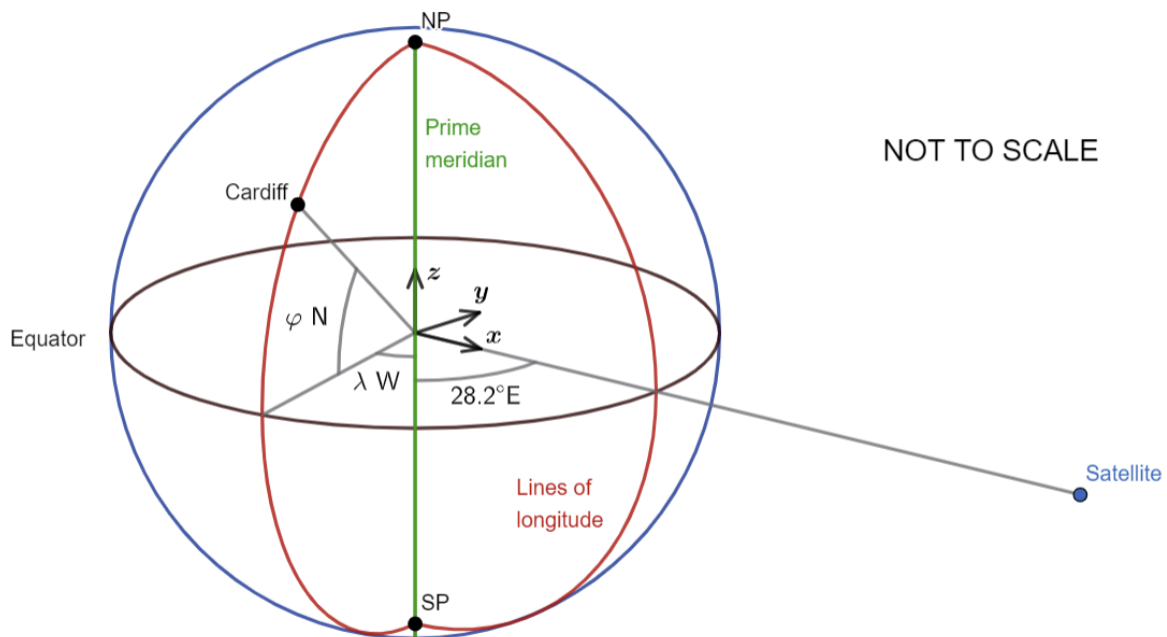
Achieving these long  $t_{\text{exp}}$  is a challenge. One idea would be to have the ULT at the exact North or South Pole of the Moon, and have the mirror rotate exactly in time with the Moon's rotation. However, the Moon's poles precess meaning that the zenith (straight up) from the pole traces out a circle of radius  $1.543^{\circ}$  over a period of 18.60 years.

- f. Calculate the maximum exposure time possible (in days) with our gigapixel camera on the Moon, based upon the time it takes for an object to move fully through its field of view (i.e start at one CCD edge, and leave from the opposite edge). Will our ULT be able to detect the Pop III star?

### Qu 3. Television Satellite

Monday 26<sup>th</sup> is the 100-year anniversary of the first public demonstration of television by Scottish inventor John Logie Baird. One way television is transmitted is by a system of geostationary satellites, first popularised by science writer Arthur C. Clarke and in use by the late 1960s. Nowadays, satellite television remains widespread and despite a recent surge in internet television usage, still accounts for more than a quarter of TV use in the UK.

Geostationary satellites are those which orbit in the Earth's equatorial plane at such a velocity that they do not move relative to the ground above which they orbit. They prove particularly useful for general public communication, eliminating the need for receivers to track the motion of the satellite across the sky. Major UK TV broadcasters transmit more than 100 national and regional channels to the UK using just three satellites: Astra 2E, 2F, and 2G. All directly above the 28.2° E longitude position on the equator, these satellites provide satellite TV to the UK, much of Europe, and North Africa.



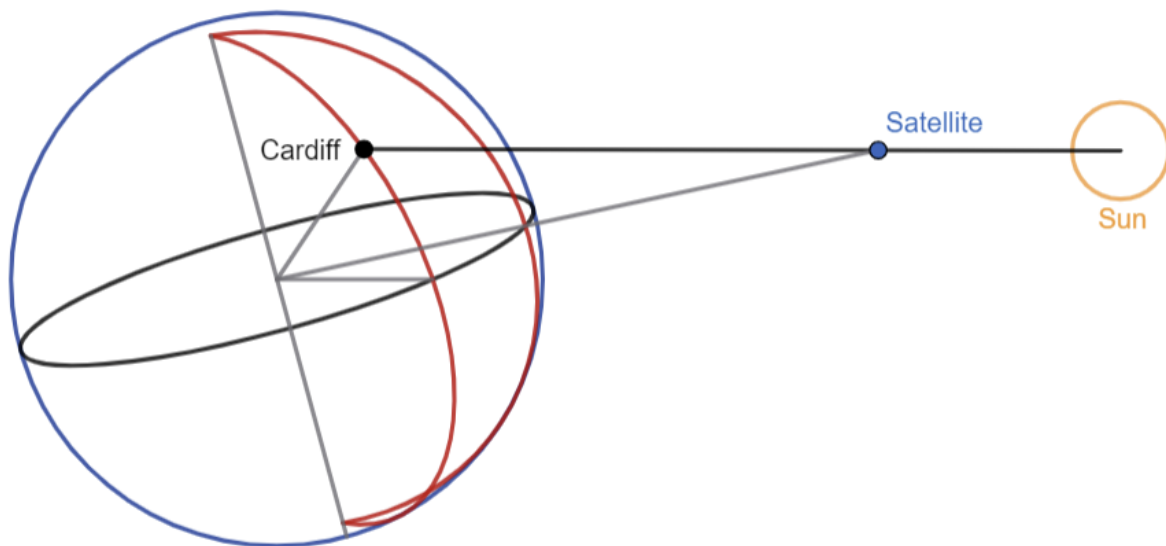
**Figure 5:** A diagram of the Earth and the geostationary satellite with some important angles labelled. Longitude directions of West and East are determined by the side of the Prime Meridian you are on. The Prime Meridian is the line of longitude that passes from the North to South Pole through Greenwich, London. Credit: Dougie Howells.

A family in Cardiff (latitude  $\varphi = 51.48^\circ$  N and longitude  $\lambda = 3.17^\circ$  W) wish to watch *The Sky at Night*. To do so, their satellite dish receiver must point exactly at the satellite. Typically, just two angles are needed to specify the receiver's direction: the angle from the local vertical  $\theta$  (where  $\theta = 0^\circ$  corresponds to straight upwards) and the bearing  $b$  (measured clockwise from North).

- a. The family in Cardiff need to align their satellite dish receiver. Consider the set-up in Figure 5 and its 3D Cartesian co-ordinate system with the origin at the centre of the Earth, satellite on the  $x$ -axis, and North pole on the  $z$ -axis. (You may find the formula  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \gamma$  for the angle  $\gamma$  between vectors  $\mathbf{a}$  and  $\mathbf{b}$  useful.)
  - (i) Show that  $\theta \approx 66^\circ$ .
  - (ii) By projecting the position of the satellite and the North Pole onto a plane parallel to the ground in Cardiff, or otherwise, calculate  $b$ .

Though generally very reliable, satellite television can be affected by Sun outages. Sun outages are temporary losses of signal from a geostationary satellite overpowered by emission from the Sun. They occur predictably and on an annual basis, when the Sun passes close to the satellite from the observer's perspective (see Figure 6). The angle from the satellite within which the Sun will cause an outage is called the outage angle, and is specific to the receiver. For simplicity, we will ignore this outage angle and only consider interference caused by the Sun's disc passing directly behind the (point-like) satellite.

- b. Estimate the times of day between which a Sun outage in Cardiff can occur (i.e. the times of the start and end of an outage) to the nearest second. In order to relate the position of the Sun in the sky to a particular time, you should assume that the Sun appears at its highest position in the sky at exactly 12:00 UTC (GMT) for an observer on the Prime Meridian. You should ignore the effects of the eccentricity of Earth's orbit and effects of axial tilt on the length of the day. That is, you should assume that the projection of the Sun onto the equatorial plane makes one revolution about the centre of the Earth every 24 hours at a constant rate. You may find it useful to continue working in the co-ordinate system described previously.

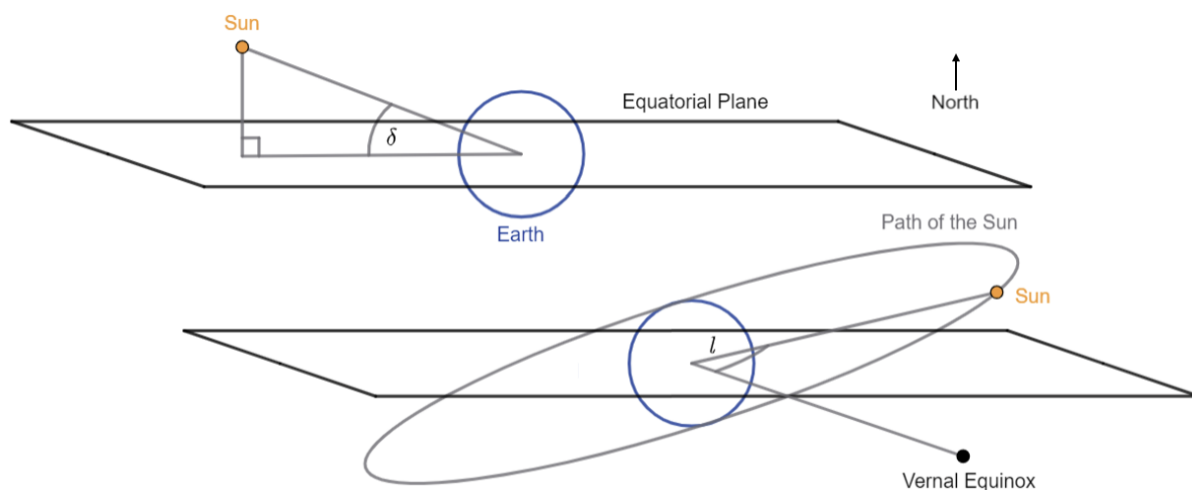


**Figure 6:** A diagram showing the geometry of the situation at the centre of a Sun outage. The outage begins when the satellite moves in front of the edge of the Sun's disc and ends when it's no longer in front of the Sun. An outage can last several minutes. Credit: Dougie Howells.

*This question continues on the next page.*

The position of the Sun relative to the Earth is complicated by the Earth's axial tilt. This means that the plane of Earth's orbit is inclined to the equatorial plane at an angle of  $\varepsilon = 23.44^\circ$ . Consider the following two angles for describing the position of the Sun, as shown in Figure 7.

- Declination,  $\delta$ : The angle between the line that connects the centres of the Earth and the Sun and the plane of the equator. Declination is taken to be positive on the Northern side of the equatorial plane. It is the latitude of the position on the Earth's surface that the Sun is directly above.
- Ecliptic longitude,  $l$ : The angle between the Earth-Sun line and the the Earth-Vernal Equinox line, measured anti-clockwise from the latter. The Vernal Equinox is the point where the Sun crosses the equatorial plane going from the Southern side to the Northern side. This occurs in 2026 at 14:46 UTC on the 20<sup>th</sup> of March.



**Figure 7:** Diagrams showing the Sun's declination  $\delta$  (top) and ecliptic longitude  $l$  (bottom). Credit: Dougie Howells.

- c. (i) By drawing a suitable diagram, derive an expression for  $\sin \delta$  in terms of  $l$  and  $\varepsilon$ .
- (ii) Show that the longest Sun outage in 2026 will occur on 8<sup>th</sup> October. You should assume that Earth's orbit is circular, so that the ecliptic longitude of the Sun increases at a constant rate.

END OF PAPER

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