British Astronomy and Astrophysics Olympiad 2022-2023

Astronomy & Astrophysics Competition Paper

Monday 6th February 2023

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 3 hours ($\sim$ 65 minutes for Q1, $\sim$ 70 minutes for Q2 and $\sim$ 45 minutes for Q3).

Questions: All three questions should be attempted. Each question contains independent parts so that later parts can be attempted even if earlier parts are incomplete.

Solutions: Answers and calculations are to be written on loose paper. **YOU WILL NEED GRAPH PAPER**. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. **EACH QUESTION ANSWERED must be started on a new page.** A standard formula booklet with standard physical constants may be used if desired.

Instructions: To accommodate students sitting the paper at different times, please **do not discuss** any aspect of the paper on the internet until 8 am Saturday 11th February.

Clarity: Solutions must be written legibly, in black pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Eligibility: The International Olympiad will be held in August 2023; all sixth form students are eligible to participate.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Training Dates and the IOAA (Chorzów, Poland, 10th - 20th August 2023)

The team will be selected from students taking this paper. The best students that are eligible to represent the UK at the IOAA will be invited to attend the **Training Camp** *to be held in Oxford from* **Saturday 1st to Wednesday 5th April 2023**. *Astronomy material will be covered; problem solving skills and observational skills (telescope and naked eye observations) will be developed. At the Training Camp a Data Analysis exam along with a Round 3 theory paper will be sat. A team of five students (plus one reserve) will be selected for further training, including additional training camps in the summer.*

Important Constants

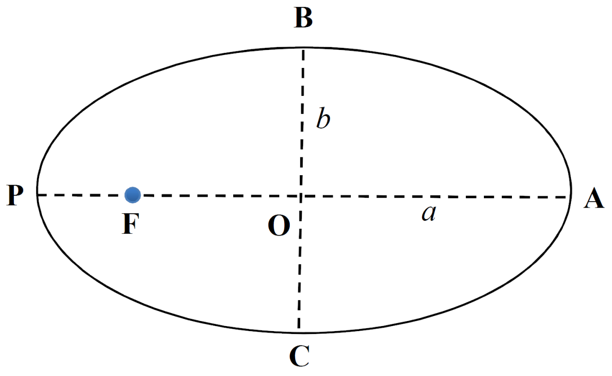
Constant	**Symbol**	**Value**
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_\odot$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_\oplus$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_\oplus$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.85 \times 10^{26}\,\mathrm{W}$
Stephan-Boltzmann constant	σ	5.67×10^{-8} J m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_p	1.67×10^{-27} kg
Electron rest mass	m_e	9.11×10^{-31} kg
Wien's displacement law	$\lambda_\mathrm{max}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Basic calculus formulae:

Chain rule	$\dfrac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = f'(g(x))g'(x)$
Product rule	$\dfrac{\mathrm{d}}{\mathrm{d}x}(uv) = \dfrac{\mathrm{d}u}{\mathrm{d}x}v + u\dfrac{\mathrm{d}v}{\mathrm{d}x}$
Quotient rule	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{u}{v}\right) = \dfrac{\frac{\mathrm{d}u}{\mathrm{d}x}v - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
Integration by parts	$\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$
Standard integral	$\displaystyle\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C$

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM}a^3 \ ,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Vis-Viva Equation: For an elliptical orbit, the speed v of an object at a distance r from the focus is related to the semi-major axis, a, total mass of the system, M, and universal gravitational constant, G, (as defined above), such that

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \ .$$

Magnitudes: The apparent magnitudes of two objects, $\mathscr{m}_1$ and $\mathscr{m}_0$, are related to their apparent brightnesses, b_1 and b_0, via the formula

$$\frac{b_1}{b_0} = 10^{-0.4(\mathscr{m}_1 - \mathscr{m}_0)} \ .$$

The absolute magnitude of an object, $\mathcal{M}$, is the same as its apparent magnitude when viewed from 10 pc, hence the relationship between apparent and absolute magnitude and distance is

$$\mathscr{m} - \mathcal{M} = 5\log\left(\frac{d}{10}\right) ,$$

where d is measured in parsecs.

Qu 1. Solar Analemma

Plotting the position of the Sun in the sky at the same time every day, you get an interesting figure-of-eight shape known as an analemma (see Figure 1). For observers in the Northern hemisphere, you might expect to always see the Sun due South at midday, however on some days the Sun has already passed through that bearing and on others it needs a few more minutes before it gets there. This is due to two effects: the axial tilt of the Earth, and the fact the Earth's orbit is not perfectly circular

Figure 1: The analemma above was composed from images taken every few days at noon near the village of Callanish in the Outer Hebrides in Scotland. In the foreground are the Callanish Stones and the main photo was taken on the winter solstice (when the maximum angle the Sun reaches above the horizon is the lowest of the year, so is at the bottom of the analemma). Credit: Giuseppe Petricca.

The vertical co-ordinate of a point in the analemma is entirely determined by the Earth's axial tilt. This is known as the solar declination, δ, and varies sinusoidally throughout the year. The horizontal co-ordinate of a point in the analemma is determined by a combination of the Earth's axial tilt and the eccentricity of the Earth's orbit. Both of these individually vary sinusoidally, but the superposition of the two is no longer sinusoidal.

We will define α as the angle between due South and the Sun at local midday as seen from Oxford, where a positive value means the Sun has already passed through due South (so is on the right of the figure above) whilst a negative value means the Sun has yet to pass through due South. If α_{tilt} is the contribution due to the axial tilt and α_{ecc} is the contribution due to the Earth's orbital eccentricity, then

$$\alpha = \alpha_{\text{tilt}} + \alpha_{\text{ecc}} \, .$$

If the angle of the axial tilt is ε and the eccentricity of the Earth's orbit is e, and we assume that both are small enough that the sinusoidal approximation of δ, α_{tilt}, and α_{ecc} apply, then we find the following boundary conditions:

- δ has a period of 1 year, an amplitude of ε, is maximum at the summer solstice (21st June) and minimum at the winter solstice (21st December)
- α_{tilt} has a period of 0.5 years, an amplitude (in radians) of $\tan^2(\varepsilon/2)$, is zero at the solstices and the equinoxes (vernal equinox = 21st March, autumnal equinox = 21st September), and (using our sign convention) positive just after the vernal equinox
- α_{ecc} has a period of 1 year, an amplitude (in radians) of $2e$, is zero at the perihelion (4th January) and the aphelion (6th July), and (using our sign convention) negative just after the perihelion

Given the n^{th} day of the year, a value can be calculated for δ and α, and these are the co-ordinates for the analemma (it is drawn by these parametric equations). For the Earth, $\varepsilon = 23.44°$ and $e = 0.0167$.

a. Although α is really an angle in radians (where 2π radians $= 360°$), it is normally more useful to convert it into time units (essentially the time since the Sun was due South, or the time until the Sun reaches due South). Taking the mean solar day to be exactly 24 hours:

   (i) Convert the amplitude of α_{tilt} and α_{ecc} for the Earth into minutes.

   (ii) Determine equations for δ (in degrees) and α_{tilt} and α_{ecc} (both in minutes) as a function of the day of the year, n. Take $n = 1$ to be 1st January and $n = 365$ to be 31st December.

   (iii) Sketch the analemma you would see if the Earth's orbit was circular (i.e. $e = 0$) but with the current value of ε, with α on the x-axis (in minutes) and δ on the y-axis.

b. Using your equations derived in the previous part, sketch on GRAPH PAPER on the same set of axes:

   (i) α_{ecc} (vertical axis) in minutes against n (horizontal axis).

   (ii) α_{tilt} in minutes against n.

   (iii) Hence, carry out the superposition of those two waves to get α against n. [Note: a clear indication of the general shape will do - no need to spend very long on making this perfect]

c. Using your graph to guide you to the relevant point of the year, but using your precise algebraic expressions (rather than reading off the graph):

   (i) What are the dates and values of the maximum and minimum values of α?

   (ii) What are the dates when the Sun is due South at midday?

   (iii) What are the dates when the Sun passes between the large and small loops in the figure-of-eight (i.e. the crossover point)? [Note: $\alpha \neq 0$ at that point, but is small]

Consider an alternative version of Earth, known as Earth 2.0. On this planet, the year is unchanged and the perihelion and aphelion are at the same time, but it has a different axial tilt, a different orbital eccentricity, and a different month for the vernal equinox (although it is still on the 21st day of that month). The analemma as viewed from Earth 2.0 is show in Figure 2 below.

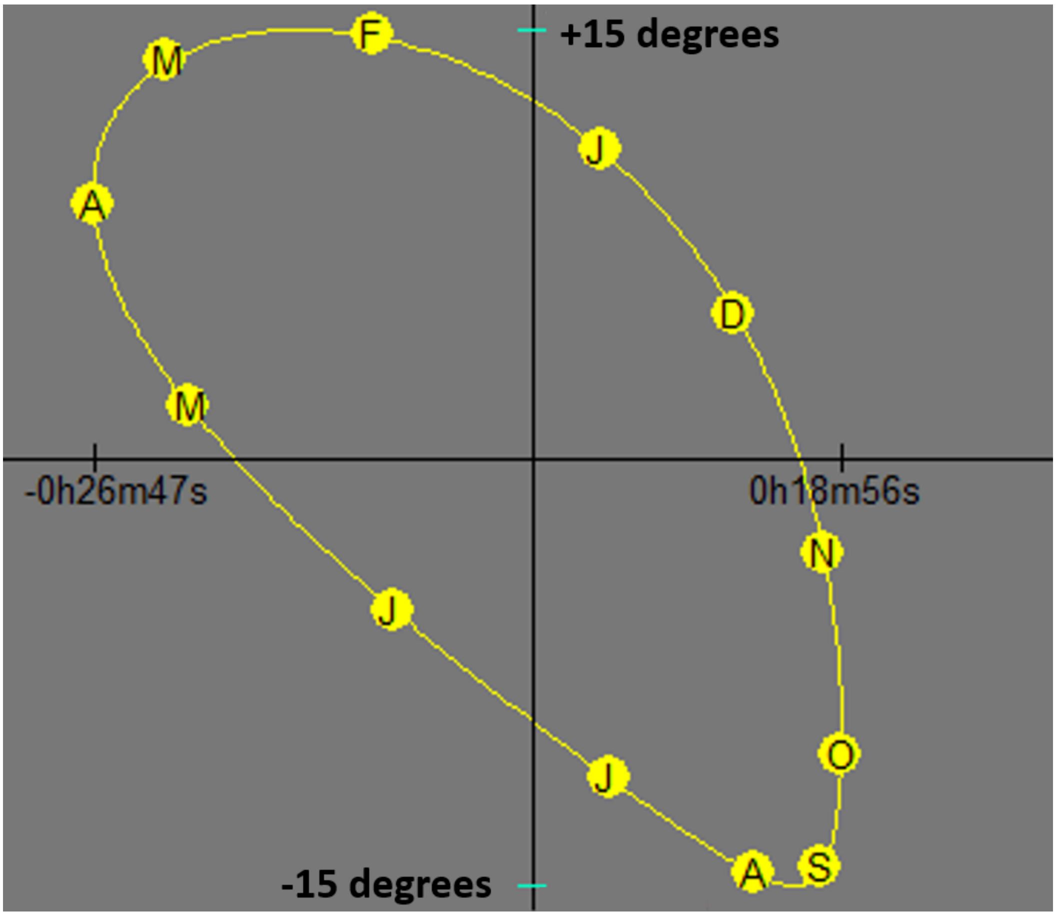


Figure 2: The analemma of the Sun at midday as seen by an observer on Earth 2.0. In this situation, α ranges from -26 mins 47 secs to 18 mins 56 secs. The circled letters correspond to the same (unknown) day of each month (for example 5th Jan, 5th Feb, 5th March etc.). Credit: Bob Urschel.

d. Considering data from Figure 2, determine for Earth 2.0:

  (i) The axial tilt, ε.
  (ii) The month of the vernal equinox.
  (iii) The eccentricity of the orbit, e.

Qu 2. Wolf-Rayet Dust Shells

Wolf-Rayet (WR) stars are some of the hottest stars known, with very strong stellar winds causing considerable mass to the be lost to the interstellar medium (ISM). In binary systems between a WR star and a very large O or B spectral class star, where their strong stellar winds collide can create the conditions for the formation of dust which goes on to enrich the ISM.

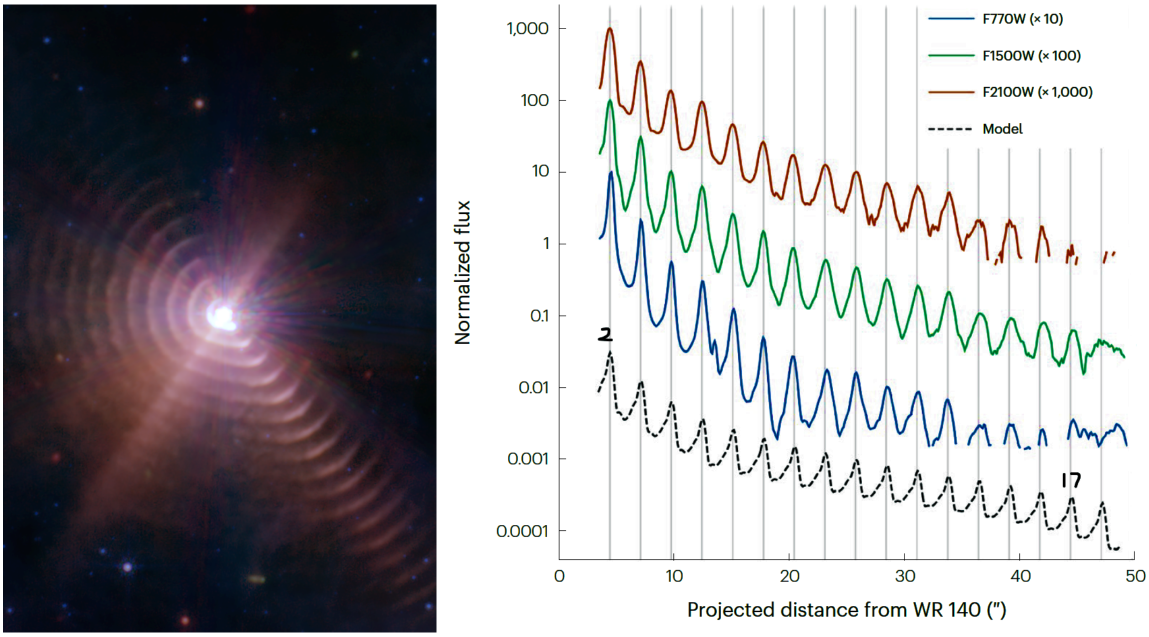


Figure 3: *Left:* A view of the WR140 binary system taken with the James Webb Space Telescope (JWST) in July 2022, showing clearly at least 17 nested dust shells. Credit: NASA/ESA/CSA/STScI/JPL-Caltech. *Right:* A radial plot along the image in the three mid infrared JWST filters used corresponding to 7.7, 15 and 21 μm, as well as the model of the dust production. The peaks correspond to each shell. Shells 2 and 17 are indicated on the model and the median shell separation is shown by the grey vertical lines. The projected distance is given in arcseconds. Credit: Lau et al. (2022).

The WR140 system consists of a WR and an O star which produce dust very regularly when the two stars are close together, around periastron. They are in a highly elliptical orbit ($e = 0.8993$) with a period of 2895 days. Once far from the stars, these dust shells move through space at a remarkably constant speed as indicated by the regularity of the shells in the recent image taken with the James Webb Space Telescope (JWST), shown above in Figure 3. An artist's impression of the two stars in the system and the orbit (in the reference frame of the WR star) is shown in Figure 4 below.

Figure 4: *Left:* The relative size of the Sun, upper left, compared to the two stars in the system WR140. The O-type star is $\sim 30\,M_\odot$, while its companion is $\sim 10\,M_\odot$. Credit: NASA/JPL-Caltech. *Right:* The projected orbital configuration of WR 140 in the reference frame of the WR star. The red solid region around the periastron passage is where the O star is when dust is being formed. Credit: Lau et al. (2022).

a. Take the distance to the system to be 1.64 kpc.

(i) By taking measurements from Figure 3, show that the average radial expansion velocity between shells 2 to 17 is ~ 2600 km s^{-1}. [1 arcsecond $= 1''$ is a measurement of angle, where $3600'' = 1°$.]

(ii) Shell 1 was observed by JWST on 27th July 2022 to be $1.63''$ away from the central stars. It was formed during the last periastron passage of the O star, which (as viewed from Earth) took place in December 2016. Taking light travel time into account, in what year was the periastron passage responsible for shell 17?

The average radial expansion velocity of shell 1 is less than that seen for shells 2 to 17, suggesting a mechanism accelerates the dust up to that speed from some lower speed resultant from the bow shock between the two colliding stellar winds. A prime candidate is radiation pressure, where photons deposit all of their momentum onto an optically thick dust cloud, before it moves too far from the original stars and becomes optically thin (so the transfer of momentum becomes less and less efficient) as shown in the right panel of Figure 5.

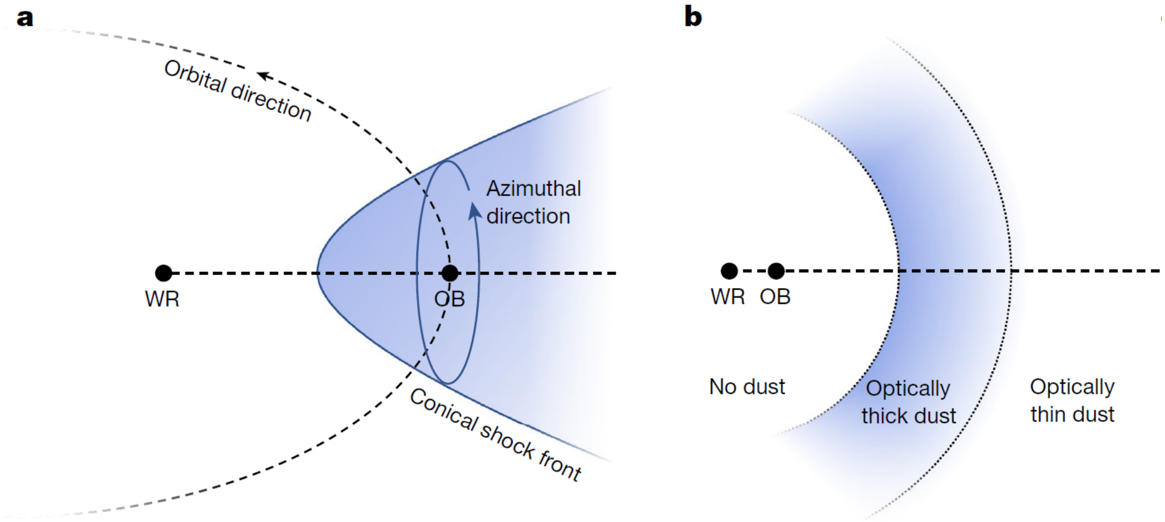


Figure 5: *Left:* The collision of stellar winds between the WR and O star creates a conical shock front in which the conditions are right for dust production. This cone is shown when at periastron in the reference frame of the WR star, although dust production is not limited to periastron so the cone can have a wide range of orientations. The place where the shock front is closest to the WR star is known as the stagnation point.
Right: Close to the stars no dust is made in the conical shock fronts, however once far enough away the dust forms and is optically thick so experiences strong radiation pressure accelerating it. Eventually a distance is reached where the dust becomes optically thin and the acceleration decreases. Credit: Han et al. (2022).

The radiation force, F_{rad}, experienced by a single dust grain at a distance r is given as

$$F_{\text{rad}} = \sigma_g \frac{L_{\text{bin}}}{4\pi r^2 c} ,$$

where σ_g is the cross-sectional area of the dust grain, c is the speed of light and L_{bin} is the combined luminosity of the binary system. Han et al. (2022) suggest the following acceleration model to match the observations:

$$a(t) = \begin{cases} 0 & 0 \leq r < r_{inner} \\ a_{\max} & r_{inner} \leq r < r_{outer} \\ \frac{a_{\max} r_{outer}^2}{r^2} & r \geq r_{outer} \end{cases}$$

Here, r_{inner} is the inner radius of the optically thick dust shell and r_{outer} is the distance at which it becomes optically thin. $a_{\max}$ is the constant acceleration experienced by the dust whilst optically thick, and is equal to the acceleration of a dust grain by radiation pressure at $r = r_{outer}$.

b. In the Han et al. (2022) model they use $r_{inner} = 50$ au, $r_{outer} = 220$ au, and assume spherical dust grains with a radius of 40 nm and density 1.6 g cm^{-3}, as well as a dust-to-gas ratio of 0.019. Take the apparent magnitude of the system (after correcting for substantial extinction by the dust shells and the general ISM) as $m = 0.426$, and you are given the absolute magnitude of the Sun is $\mathcal{M}_{\odot} = 4.74$.

   (i) Show that $a_{\max}$ is ~ 940 km s^{-1} yr^{-1}.

   (ii) Hence, sketch $a(r)$ from $0 \leq r \leq 2500$ au.

   (iii) Given shell 1 is measured to be moving at 2540 km s^{-1}, calculate the initial speed of the gas streaming away from the stars in the conical shock front, v_0, before it experiences the acceleration due to radiation pressure.

The stellar winds from each star are initially thermally accelerated up to a terminal velocity, v_∞, following the relationship $v(r) = v_\infty(1 - R_\star/r)$, where $R_\star$ is the radius of the star. The shape of the conical shock front is well approximated as a hyperbolic cross-section so long as the speed of each star's wind at the shock front is $\approx v_\infty$ where they meet. This is true for much of the orbit and the cone, however due to the extreme eccentricity of the WR140 system at periapsis it may not hold close to the stagnation point (the closest the cone gets to the WR star, as seen in Figure 5).

c. The measured semi-major axis of the WR140 system from radio interferometry is 8.82 milliarcseconds. Investigation of spectral lines shows that for the O star $v_{\infty,O} = 3100$ km s^{-1} and for the WR star $v_{\infty,WR} = 2800$ km s^{-1}, and measurements of X-ray absorption lead to suggested mass loss rates of $\dot{M}_O = 3.7 \times 10^{-7}\, M_{\odot}$ yr^{-1} and $\dot{M}_{WR} = 1.7 \times 10^{-5}\, M_{\odot}$ yr^{-1} respectively for each star. In the shock front, the pressure from each stellar wind is balanced. Considering the system at periapsis:

   (i) Show that the distance from the O star to the stagnation point is

$$r_O = \frac{\sqrt{\eta}}{1 + \sqrt{\eta}} r_{peri} \,,$$

   where η is the ratio of the momenta of the two stellar winds, $\eta \equiv \frac{p_O}{p_{WR}}$ and r_{peri} is the separation between the stars at periapsis.

   (ii) Hence, calculate r_O in au.

   (iii) Taking the WR star to be approximately 2 times more luminous than the O star, and the effective surface temperatures of the stars to be $T_{\mathrm{eff,O}} = 35$ kK and $T_{\mathrm{eff,WR}} = 60$ kK, calculate the radius of each star in solar units.

   (iv) Hence, determine for which star(s) the assumption of $v \approx v_\infty$ at the stagnation point is reasonable, and suggest (if any) the change in the position of the stagnation point taking in the real values of v at r_O.

d. The conditions for dust production turn on when the stars get within ≈ 7.9 au of each other, corresponding to the solid red region in the right panel of Figure 4. A line from the WR star to the start of dust production region makes an angle of 45° with the semi-major axis. By using Kepler's second law, or otherwise, estimate the rough duration of dust production during each orbit. Give your answer in years to 2 decimal places, making any simplifying assumptions clear. [Hint: Kepler's second law states that the area swept out by one object orbiting another is equal in the same amount of time, such that $\mathrm{d}A/\mathrm{d}t = \mathrm{constant} = \pi ab/T$ where T is the period]

Qu 3. Planetary Migration in Protoplanetary Discs

Young, Earth-like planets interact with the protoplanetary discs in which they form, and as a result migrate to different orbital radii. The aim of this question is to quantify this migration in a simple model. We shall think of protoplanetary discs as consisting of nested circular orbits of gas and dust around a central star. For a thin disc (with small vertical extent), we assign to the disc a surface density (or mass per unit area) Σ, and semi-thickness H, which in general vary over the disc's extent. The disc's 'aspect ratio' at radius r from the central star is denoted $h = H/r$. This question is concerned with the migration of 'small' planets, such that $M_p/M_\star = q \ll h^3$.

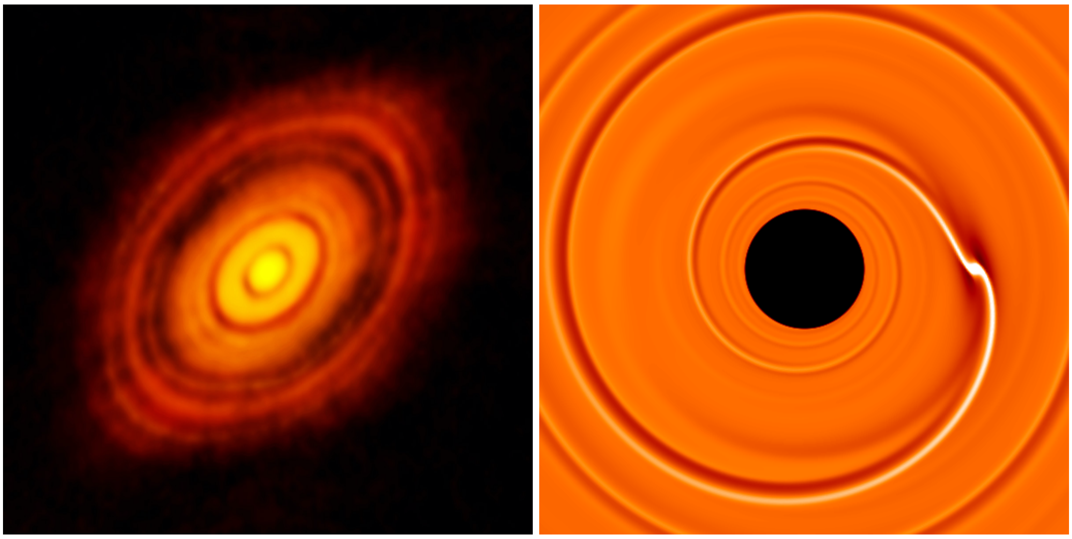


Figure 6: *Left:* ALMA image of the young star HL Tau and its protoplanetary disk. This image of planet formation reveals multiple rings and gaps that herald the presence of emerging planets as they sweep their orbits clear of dust and gas. Credit: ALMA (NRAO/ESO/NAOJ) / C. Brogan, B. Saxton (NRAO/AUI/NSF). *Right:* A small planet orbits whilst embedded in a protoplanetary disc, exciting a 1-armed spiral density wave. Credit: Frédéric Masset.

a. Consider a planet of mass M_p in a circular orbit of radius r_p about a star of mass $M_\star$, with $M_p/M_\star = q \ll 1$. Show that it has angular velocity and angular momentum about the central star given by:

$$\Omega_p = \sqrt{\frac{GM_\star}{r_p^3}}, \quad L = M_p\sqrt{GM_\star r_p}$$

b. Consider now a rotationally symmetric disc with surface density profile $\Sigma = \Sigma_0\,(r/r_0)^{-3/2}$, and outer radius $r_{\text{out}} = 9r_0$. Find the mass of the disc M_{disc} in terms of Σ_0 and r_0 assuming its inner radius $r_{\text{in}} \ll r_0$.

Since the planet is assumed small ($q \ll h^3$), its interaction with the gas in the disc constitutes the excitation of a spiral density wave, and redistribution of matter in the co-orbital region (that is, matter orbiting at radii $r \approx r_p$), as shown in Figure 6. The resulting non-uniform density distribution induced in the disc exerts a net gravitational force, and hence a torque on the planet, which has been estimated using analytical methods. This torque, Γ, acts to change the planet's angular momentum, and hence its orbital radius, causing it to 'migrate', via:

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \Gamma$$

It is convenient to write the torque in terms of the reference value

$$\Gamma_0 = \left(\frac{q}{h}\right)^2 \Sigma_p r_p^4 \Omega_p^2\,.$$

c. From 2-dimensional steady fluid-dynamical disc models, it is predicted that the total torque Γ has two main contributions: from the spiral wave, the ‘Lindblad torque’, Γ_L, and from the co-orbital region, the ‘Corotation torque’, Γ_C. For a disc of uniform entropy ($\mathrm{d}s = 0$), and with surface density profile $\Sigma \propto r^{-\alpha}$, and pressure profile $P \propto r^{-\delta}$, Tanaka et al. (2002) and Paardekooper & Papaloizou (2009) find these torques are given by:

$$\Gamma_L = (-3.20 + 0.86\alpha - 2.33\delta)\Gamma_0$$

$$\Gamma_C = 5.97\,(1.5 - \alpha)\,\Gamma_0$$

We assume the gas in the disc obeys the ideal gas law, so that:

$$\frac{P}{\Sigma T} = \text{constant}, \quad \mathrm{d}s = \text{constant} \times \left(\frac{1}{\gamma - 1}\frac{\mathrm{d}T}{T} - \frac{\mathrm{d}\Sigma}{\Sigma}\right),$$

where T is the absolute temperature and γ is the adiabatic index (the ratio of the heat capacity at constant pressure to the heat capacity at constant volume). Show that for a disc of uniform entropy,

$$\Gamma = \Gamma_L + \Gamma_C = (5.76 - (5.11 + 2.33\gamma)\alpha)\,\Gamma_0 \,.$$

[Hint: if $\frac{\mathrm{d}y}{y} = \lambda\frac{\mathrm{d}x}{x}$*, then* $y \propto x^{\lambda}$*.]*

d. Assume the disc model from part b. with $h = \text{const}$. Find the time taken for the orbital radius of the planet to halve from initial radius r_0. Take $\gamma = 1.4$, and give your answer in terms of the migration timescale,

$$t_{\mathrm{m}} = \frac{1}{h}\left(\frac{h^3}{q}\right)\frac{M_\star}{M_{\mathrm{disc}}}\tau_0 \,,$$

where τ_0 is the orbital period at radius r_0.

e. If the disc has mass $M_{\mathrm{disc}} = 0.01M_\star$ and aspect ratio $h = 0.05$, and if $q = 5 \times 10^{-6}$ and $\tau_0 = 10$ years, find the elapsed time in years for the migration described in part d. to occur. Is this a feasible mechanism for planetary migration given the lifetime of the disc is roughly 10 Myr?

f. Whilst the planet excites a spiral density wave over the whole disc, in fact the dominant contribution to the torque from the spiral is from only a small neighbourhood around the planet, so that α may be replaced by

$$-\frac{r_p}{\Sigma_p}\left.\frac{\mathrm{d}\Sigma}{\mathrm{d}r}\right|_p \,.$$

Near the inner radius of the disc, the surface density departs from the above power-law behaviour and smoothly decreases to 0. Does the migration stop before the planet reaches the disc’s inner radius?

Helpful equations:
The moment of inertia, I, of a point mass m moving in a circle of radius r is $I = mr^2$.
The angular momentum, L, of a spinning object with an angular velocity of Ω is $L = I\Omega = r \times p$, where p is the linear momentum of a point particle a distance r from the axis of rotation.

END OF PAPER

Questions proposed by:
Dr Alex Calverley (Surbiton High School)
Josh Brown (University of Cambridge)

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